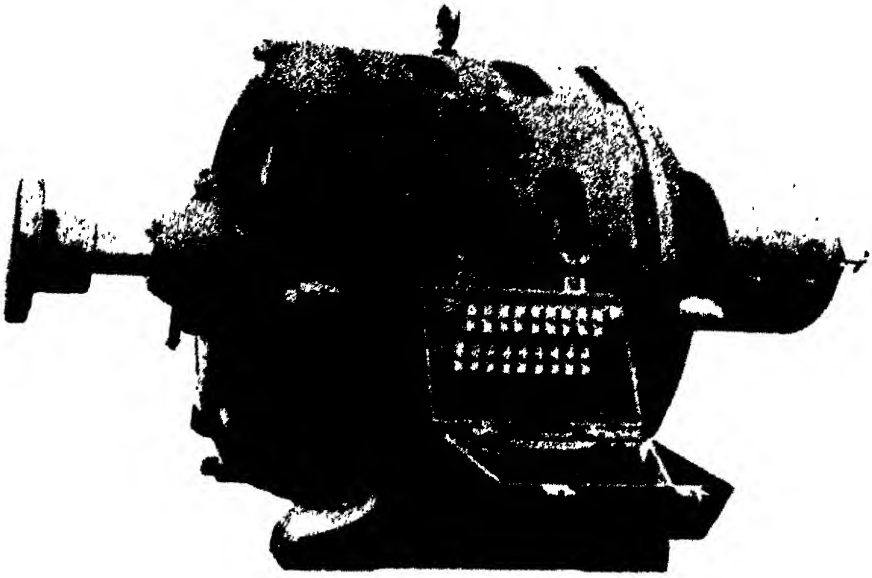# THEORY AND DESIGN
OF
ELECTRIC MACHINES

PLATE I

150 h.p., six-speed motor, three-phase, 440 volts, 50-cycle. Speeds 1,000, 750, 600, 500, 428, 375, calculated by the methods in Chapters XXIX and XXX

THEORY AND DESIGN
OF
ELECTRIC MACHINES

A TREATISE
DEALING WITH THE FUNDAMENTAL PRINCIPLES
OF THE DESIGN AND OPERATION OF ALL TYPES
OF ELECTRICAL MACHINES

CONTAINING A NOVEL METHOD OF ARRIVING AT THE
CORRECT DIMENSIONS FOR ANY PARTICULAR DESIGN

BY

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PREFACE

THE present treatise represents an effort to reduce the study of the dynamo electric machine to a scientific form and to introduce order where, owing to the enormously rapid rate of development, there is little sign of it at present.

A few years ago, when one spoke of a dynamo electric machine, one had in mind a fairly definite idea of either a direct-current machine, a synchronous alternator, or an induction motor. It was thought that, with a few minor modifications, these covered the field. Now, however, it has become clear that the possible types of dynamo electric machine are quite endless in number, fresh possibilities being announced almost every month. It is in order to classify these possibilities that the present treatise has been written, and to carry out a general survey of the subject in order to reveal fresh lines of advance.

The enormous advantages of a general survey of the theoretical possibilities of the future cannot fail to be evident as, by such a survey, we may map out the necessary permanent lines of advance of the future years before they are realized in practice, and in this way save time and money which would otherwise be spent in developing apparatus which the survey shows cannot present the same fundamental advantages as other possible types.

We should never be content with less than the ideal; it may present greater difficulties in realization than a less ideal plan, but when it is realized it is not subject to supersession by some other device, whereupon the time and money spent in developing the less ideal plan are largely wasted, although it may have been temporarily useful.

Such a survey has been considered impossible, but the object of the present treatise is to show that it is possible and to carry it out. This is done as follows—

We start from the most abstract point of view, where the general machine is treated merely as a special case of Maxwell's general equations of the electromagnetic field. We then gradually introduce limitations one after the other, with a critical explanation of the function and necessity of each limitation of generality, thereby dividing our machine into different classes, which we then proceed to describe in detail.

By this procedure we ensure that substantially all possible types are described in principle, thus leaving only the permutations and combinations of individual parts to be developed as occasion requires. These permutations and combinations are, of course,

practically unlimited, but the fundamental types are very few. A demonstration is offered of this point.

The mechanical analogy of each type with a certain type of gear is also developed, which is helpful in showing the essential nature and relationship of each type.

As the result of the principles of classification revealed by the general investigation, we divide our machines into six classes.

1. Fundamental types without commutator or variable pole pitch.

2. Commutator motors operating from fixed frequency generators. A separate volume is devoted to this on account of the complexity of the theory in the most important case.

3. Machines operating from variable and multiple frequency generators.

4. Cascade sets of various types.

5. Multiple polarity machines. This includes cascade machines and machines with multiple polarity windings in general.

6. Variable polarity apparatus, in which the speed of a motor or the frequency of a generator is changed by changing the number of poles.

It is clear that while much is known of Classes 1 and 2, and Class 3 is not of much importance, there are great gaps in our knowledge of Classes 4, 5, and 6. Great attention has been given to filling up these gaps, and a large part of the book is devoted to this. The writer may perhaps be allowed to state that in his opinion we are only at the beginning of this subject.

In addition to these, we may distinguish homopolar machines and electrostatic induction machines, which latter, however, depend on a different case of Maxwell's equations. In each of the classes, owing to the generality of the method of procedure and the critical examination of every limitation as it is introduced, we are enabled to discern literally hundreds of new forms, some of which cannot fail to be of practical use.

In this way, by theoretical criticism, we develop a number of solutions to each problem in a form suitable for experimental trial.

It is not the place of theory to state dogmatically what type should be the solution of a practical problem. Its function is merely to develop a number of likely types and indicate their possibilities and limitations. They should then be tried on a moderate scale experimentally and one or more of the most suitable adopted.

In this way the development of electrical apparatus might be reduced to a scientific form instead of being left to the haphazard efforts of individual inventors, the success of which is determined by natural selection.

It may be pointed out that other engineering subjects might be

treated in the same way; for instance, one can deduce a special case of Euler's equations of hydrodynamics, which is applicable to the general type of hydraulic machine or a generalized hydraulic turbine and centrifugal pump. From these equations a general theory of hydraulic mechanism could be worked out by an hydraulic engineer sufficiently familiar with the subject, which would no doubt reveal a large number of useful forms.

In the same way ordinary mechanism could be derived from Lagrange's equations of rigid bodies, but this would perhaps be too wide a subject for general treatment.

The writer has given some thought to the best title for the subject. The usual definition of a machine is "a device to modify motion and force." In accordance with the modern ideas of energy, we might, perhaps, modify this into the following: "A machine is a device for changing the flow of energy." For instance, the steam engine changes an intermittent flow of pressure energy into a cylinder into a continuous rotary flow of energy. This definition is applicable to all types of what are ordinarily called machines, whether electric, hydraulic, mechanical, or otherwise.

It is clear, moreover, that our subject bears the same relation to electrical machines that the science of "mechanism," developed by Rankine, Reuleaux, and others, does to ordinary machines, and hence "Electric Mechanism" is a suitable title for the subject.

If the present work succeeds in demonstrating the possibility of such a subject and outlining its extent, even in the very roughest way, its object will be accomplished.

Clearly one cannot decide on the practical usefulness of a particular type without a full study of its design. Hitherto each type of dynamo electric machine has been a "special creation," from the designer's point of view, requiring special experience before it could be designed to the best advantage. Sufficiently general methods of designing are described in Part VI of the work to render it possible to carry out a critical comparison of the possibilities of different types in advance of experiment. Of course, it still remains true that special types involve special problems—for instance, in the internal cascade machine there is the problem of the relation of the leakage coefficient to the different pole combinations, and in commutator types there may be special commutation problems, and, of course, in other types other problems. The methods of designing in Part VI, since they leave no scope for guesswork or estimating, throw into relief the existence of problems of this nature, so that experimental research may be concentrated on them. But, above and beyond these applications, the main advantage of these methods of design is that they separate the use of engineering judgment from the necessity of numerical calculation, so that this latter may be handed over to a calculating machine

operator without engineering knowledge and the whole of the drudgery of designing lifted from the engineer's shoulders.

A rational procedure in developing a new type of machine would be the following: Starting from an industrial problem, say, for instance, the problem of printing-press operation or the driving of textile machinery, a report could be prepared on the various means of carrying this out to the best advantage, accompanied by designs of the most likely types of apparatus and a statement of what experiment, if any, would be needed. This report itself would, of course, be derived from general study, such as that of the present work, and would involve no experiment. It could be considered by a committee of experts, including manufacturers, users, and selling experts (the B.E.A.M.A. Research Association in Great Britain, one would think, could supply such a committee); and if thought sufficiently promising, further work could be authorized with a view to the practical development and placing on the market of the new type.[1]

The success of a new type on the market depends on a great many things besides its technical merits. Some of these are as follows—

1. A new type must at first be made in small quantities. A manufacturer's costing system may be such as to penalize enormously everything which is not made in large quantities. If such a costing system is rigidly adhered to, it renders the marketing of new types practically impossible, because of the price it attaches to them. In any case, owing to the small initial turnover the price tends to be high at first.

2. At the start its existence and merits are unknown to potential users. There is, therefore, no demand, since users cannot ask for what they have never heard of. (" There is never any demand for a new thing."—*Henry Ford*.) This demands considerable propaganda and highly skilled salesmanship. It may be that the manufacturer's salesmen have insufficient technical knowledge to explain

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[1] It is quite possible to estimate in advance the cost of such a development. In the most difficult case of all, where it involves a fundamentally new problem, such as a new type of commutation, involving extended oscillographic research or studies in leakage, etc., it will cost not more than £2,500 to £3,000 (1929 prices) to place on the market a line of machines up to, say, 50 kW., with all technical problems solved, all designs made with the utmost economy of material, and tabulated for all the ratings (all voltages and frequencies) likely to be in commercial demand. All expenses of whatever kind likely to be incurred other than selling costs are included, establishment charges on the experimental machines, special testing, and reasonable remuneration for all those engaged on the work. For simpler developments, these costs may be halved or quartered. Those unaccustomed to this work often attempt to employ " short cuts," skipping necessary experimental work or trying to do it at the customer's expense. Where this is attempted, one may put the additional cost incurred at £500. Clearly an unskilful and fumbling development must be much more costly than one which is carried out in the best possible way.

the new type effectively, though well enough on standard apparatus, or may commit other errors in commercial policy.

3. Manufacturers of the second rank who find, owing to imperfect organization, difficulties in reducing their costs to those of competitors, are apt to believe they can make higher profits out of a new type which would become a specialty. But a new type requires more skill in manufacture, not less skill, than standard types, since it is inherently handicapped by the small initial demand.

4. One may be entirely misled as to potential demand, unless a very careful canvass is made of the importance of the industrial problem solved by the new type and of alternative solutions.

All these points require to be carefully weighed before embarking on experimental work. If they could be considered by a central co-operative organization as suggested above, with its wider sources of information, they are clearly rendered much easier than when each manufacturer has to consider them for himself, and this is the course which would undoubtedly be pursued if the public interest alone were considered. But, as things are, competitive jealousies make it very difficult ; but if the manufacturers of any country (or even one manufacturer of large capital) could agree to the systematic study of the possibilities of progress, its rate could be very much accelerated and very large sums of money saved.

The author is indebted to the *Journal of the Institute of Electrical Engineers*, the *Journal of the American Institute of Electrical Engineers*, and the publishers of the *Electrician*, from papers and articles in whose columns extracts appear.

<div align="right">F. CREEDY.</div>

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THEORY AND DESIGN OF ELECTRIC MACHINES

PART I

CHAPTER I

INTRODUCTORY

BEFORE proceeding to the main portion of our investigation we must first formulate a preliminary definition of the dynamo-electric machine from which to start.

The " dynamo-electric machine " consists essentially of two members capable of mutual rotation, a single magnetic flux common to the two, and two electric circuits one on each member, interlinking the flux. In one or two exceptional cases only one electric circuit is used, but these we shall not consider.

Two forms of mechanical construction are in use, one in which revolving and stationary parts are separated by a short air-gap uniform throughout the periphery, and another in which salient poles are used. This latter type, however, is not of general application, being suitable only for continuous current and some forms of single-phase machine, both of which types may, from the technical point of view, be equally well built with uniform air-gap. We shall, therefore, consider the uniform air-gap type to be the standard. In order to minimize magnetic leakage the conductors in both elements are disposed in slots as close as possible to one another, that is, as close as possible to the air-gap, since they are in relative rotation. These conductors are interconnected in various ways, but as it is the purpose of our investigation to study these different methods of connection, we must not prejudge the question by assuming the machine to be connected in any particular way.

Finally, although in actual machines the air-gap surface is almost invariably cylindrical, we shall assume it to be plane, i.e. in the usual phraseology, that the machine is developed. Later we shall be able to formulate an exact and scientific definition of the dynamo-electric machine.

CHAPTER II

FUNDAMENTAL LAWS OF THE ELECTRIC MACHINE

WE have seen in Chapter I that omitting everything non-essential in our dynamo-electric machine, assuming a constant air-gap all round the periphery with all the reluctance of the machine concentrated in it, and developing the machine so that the gap surface is a plane instead of a cylindrical surface, it may be represented as in Fig. 1.

It consists of two magnetic elements separated by the air-gap and capable of relative motion along the axis of x. Each of these elements bears a number of electric conductors perpendicular to the plane of the paper or parallel to the axis of z, say. The axis of
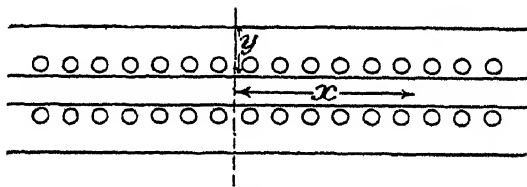
FIG. 1

y may be taken perpendicular to the air-gap surface. We shall have very little occasion to use the axes of y and z, but shall be constantly using that of x.

In consequence of the assumption of a constant air-gap all round, with all the reluctance of the machine concentrated in it, we get our first fundamental equation

$$B = \mu H = \frac{\mu \cdot I}{\int r \, dt} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

"Magnetic density is proportional to magnetomotive force at every point of time and space."

We here neglect saturation, of course, and μ must be taken to represent not the permeability, but the permeance of unit area of the air-gap.

Our second fundamental equation is still more obvious, if possible. The ampere-turns included within any stretch of the machine are a measure of the magnetomotive force H in accordance with the well-known equation

$$\text{ampere-turns} = \frac{10}{4\pi} H \text{ in electromagnetic units.}$$

The ampere-conductors are just twice the ampere-turns; hence they also are a measure of H. In "rational units," which we shall

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adopt, the constant $10/(8\pi)$ is got rid of and H is identical with the ampere-conductors.

Now it seems a truism to say that

ampere-conductors per cm.
= current density ~~per em.~~ of periphery.

Nevertheless, putting this into symbols we get the equation $dH_1/dx = \Delta_1$ for one member, and $dH_2/dx = \Delta_2$ for the other. If we use Δ_1 for the current density in the first member, and H_1 to measure the ampere-conductors, dH_1/dx will be the ampere-conductors per cm.

Putting $H = H_1 + H_2$ to measure the resultant current density we get our second equation

$$dH/dx = \Delta_1 + \Delta_2 \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

We can illustrate this equation by reference to particular cases.

We see in Fig. 2(c) that Δ is zero everywhere along the periphery except for certain small lengths where it has a constant

FIG. 2

value. It is shown at (b) in the form of a curve ; (a) shows the magnetomotive force due to it, a square-topped wave as is obvious, having a constant value everywhere except where there is current. It is clear that the curve (b) is the slope or differential coefficient of curve (a), as stated by our equation.

Another example we may take from an ordinary continuous-current armature winding developed. The current density here is everywhere the same, so that it is represented by the square-topped curve marked Δ. H, however, rises to a maximum at the point where the current reverses, and then decreases. It is clear that the slope of H is constant, reversing after passing the apex, and hence that it is represented by a square-topped curve such as Δ.

A third fundamental equation is the following—

Let E be the electromotive force per bar, i.e. potential difference between opposite ends of the same bar at the point x. Then the potential difference dE between adjacent bars will be equal to the time rate of change of magnetic density dB/dt between them multiplied by dx, or

$$dE_1/dx = dB/dt \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (3)$$

which may be read " Potential difference between adjacent bars = time rate of change of magnetic density between them."

If in addition one of the members be moving with respect to B at speed v, we shall have a further potential difference between adjacent bars equal in absolute units to v times the difference of density dB in the lines cutting them at the given instant.

Thus we get in this case

$$dE_2 = (dB/dt)dx + v(dB), \text{ or } dE_2/dx = dB/dt + v(dB/dx) \quad . \quad (4)$$

which may be read " total potential difference between bars = time rate of change of magnetic density + v times the difference of magnetic density at each bar."

The equations which we have just deduced form a set of differential equations characterizing the dynamo-electric machine of the most general type, homopolar or multipolar.

In order to illustrate the meaning of these equations we may study a very important case by considering a flux wave stationary in space, so that $dB/dt = 0$, while the two elements of the machine move past it with speeds v_1 and v_2 respectively. The equations then take the form

$$\frac{dH}{dx} = \Delta_1 + \Delta_2 ; \quad \frac{dE_1}{dx} = v_1\frac{dB}{dx} ; \quad \frac{dE_2}{dx} = v_2\frac{dB}{dx}$$

The last two equations may be integrated immediately, giving

$$E_1 = v_1B + C_1 \text{ and } E_2 = v_2B + C_2$$

C_1 and C_2 relate to a possible superposed homopolar distribution and may be neglected.

In general, such a wave of fixed form moving as a whole at a constant speed may be represented by an arbitrary Fourier series.

$$B = \Sigma_r B_r \sin \{r(mx - pt) + a\}$$
$$dB/dt = -p\Sigma_r rB_r \cos \{r(mx - pt) + a\}$$
$$dB/dx = m\Sigma_r rB_r \cos \{r(mx - pt) + a\}$$

so that $dB/dt = -(p/m)(dB/dx)$, and we can reduce our general equations to

$dH/dx = \Delta_1 + \Delta_2 ;$

$dE_1/dx = -(p/m)(dB/dx)$, or $E_1 = -(p/m)B + C$; and $dE_2/dx = [v_2 - (p/m)](dB/dx)$ or, neglecting arbitrary constants, $E_2 = [1 - v_2(m/p)]E_1.$

When $v_2(m/p) = 1$, $E_2 = 0$, which is another way of saying that when the member corresponding to E_2 runs at the same speed as the flux there is no electromotive force in it.

Another general law arises from the fact that our machine is in part a moving body and, therefore, subject to the general laws of

motion, and in particular to the law that " action and reaction are equal and opposite."

The force acting on any conductor at the point x is $B\Delta dx$ in absolute units.

The total force, therefore, acting on any stretch of one element from zero to a, say, is

$$F = \int_0^a B\,\Delta dx \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (5)$$

F is usually given, being the load on the machine.

If F is equal and opposite to the force on the other member we get

$$\int_0^a B\,\Delta_1 dx = -\int_0^a B\,\Delta_2 dx, \text{ or } \int_0^a B(\Delta_1 + \Delta_2)dx = 0 \qquad . \qquad (6)$$

This, however, is not necessarily the case except for the whole machine.

We may also express these equations in terms of the flux Φ through the core of our machine. Let us assume that the core length along the axis of Z is unity. Then the flux crossing the air-gap in the distance $x_1 - x_2 = \Phi_1 - \Phi_2$. But we know that the mean air-gap density B in any length $x_1 - x_2$ is the flux crossing the gap in that length divided by the length, or

$$B = \frac{\Phi_1 - \Phi_2}{x_1 - x_2} = \frac{d\Phi}{dx} \text{ in the limit.}$$

We also have, of course, $H = B/\mu = (1/\mu)(d\Phi/dx)$. Substituting these values in the equations given above, and integrating the two E.M.F. equations, we get

$(1/\mu)(d^2\Phi/dx^2) = \Delta_1 + \Delta_2; \; E_1 = d\Phi/dt; \; E_2 = d\Phi/dt + v(d\Phi/dx);$

a form of the equations which is often much more convenient than the original form.

Note to Chapter 2. Maxwell's equations, in the form given by Heaviside, are

$$\text{curl } (\mathbf{E} - \mathbf{e}) = -dB/dt,$$

and $\quad$ curl $\mathbf{H} = \Delta$, the current density (a vector),

where e is the " motional " electromotive force VqB, and q is the velocity vector.

If we take the components of $\mathbf{H}$ and $(\mathbf{E} - \mathbf{e})$ parallel to x, y, and z, as H_1, H_2, H_3, and $(E - e)_1$, $(E - e)_2$, $(E - e)_3$ respectively, these equations in Cartesian co-ordinates will be as follows—

$$\frac{dH_3}{dy} - \frac{dH_2}{dz} = \Delta_1 \; ; \; \frac{dH_1}{dz} - \frac{dH_3}{dx} = \Delta_2 \; ; \; \frac{dH_2}{dx} - \frac{dH_1}{dy} = \Delta_3 \; ;$$

Fig 3

and

$$\frac{d(E-e)_3}{dy} - \frac{d(E-e)_2}{dz} = -\frac{dB_1}{dt} ;$$

$$\frac{d(E-e)_1}{dz} - \frac{d(E-e)_3}{dx} = -\frac{dB_2}{dt} ;$$

$$\frac{d(E-e)_2}{dx} - \frac{d(E-e)_1}{dy} = -\frac{dB_3}{dt} .$$

Now if we take $H_1 = H_3 = 0$, or H parallel to the y axis, and $(E-e)_1 = (E-e_2) = 0$, or $E-e$ parallel to the z axis, while q is parallel to x of length v, we get

$$dH_2/dx = \Delta ; \quad d(E-e)_3/dx = -dB/dt ; \quad e = vB ;$$

so that $dE/dx = dB/dt + v(dB/dx)$.

Thus we have reproduced two of the equations which we obtained before, the third, of course, being obtainable by simply putting $v = 0$.

The second form of the equations in terms of Φ is analogous to Maxwell's form in terms of the vector-potential A. For the case where all the lines of force lie in one plane we may interpret A as the total number of lines of force surrounding a point.

CHAPTER III

GENERAL DISCUSSION OF THE FUNDAMENTAL EQUATIONS

COLLECTING the results of the investigations of the last chapter, we find ourselves in possession of the following equations—

1. $B = \mu H$. Equation of magnetic density.

2. $dH/dx = \Delta_1 + \Delta_2$. Magnetomotive force equation.

3. $dE_1/dx = dB/dt$. Stator E.M.F. equation.

4. $dE_2/dx = dB/dt + v(dB/dx)$. Rotor E.M.F. equation.

5. $F = \displaystyle\int_0^a B\,\Delta\,dx$. Force or torque equation.

6. $\displaystyle\int_0^a B(\Delta_1 + \Delta_2)dx = 0$. Stating that action and reaction are equal and opposite.

Let us see what general conclusions we can draw from these equations—

1. The differential equations are linear equations of the first order in x and t. These equations implicitly assume that the flux
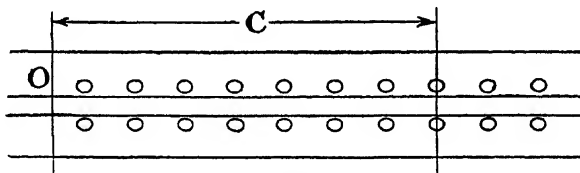
FIG. 4

crossing the gap is due to the current distribution in the conductors. This is not true, for instance, in the homopolar machines, where equation (2) takes the form $(dH - H_o)/dx = \Delta_1 + \Delta_2$.

Before we can deal usefully with these equations we must limit their generality by applying any further general conditions we can discover, necessitated by the nature of our problem. One such condition is the following—

In practice the electric machine cannot be of infinite extent, but must be annular, that is to say, if, starting from any given point we go round the circumference, we return again to the same point. In our development this fact will take the following form.

Starting from a point O (Fig. 4) and proceeding to the right we are compelled to assume that when we have travelled through a

7

distance C equal to the circumference of our annular machine, every quantity mentioned in our equations resumes its original value, that is to say, the solution of our equations is periodic with respect to x and with period C. We only desire to study steady conditions, hence our quantities must be constant or periodic with respect to time also. There is no necessary relation whatever between the time periods and the space period, but the knowledge that the solution is periodic in both time and space enables us to effect a very important simplification.

Hence the general form of solution to which we are limited is

$$H = \Sigma_r a_r H_r \sin (a_r x + \beta_r t + \gamma_r).$$

In the travelling wave discussed in Section 2, which moves at a fixed speed without change of shape, we had $\beta_r/a_r = V$, a constant. In our present wave, however, there is no relation whatever between a_r and β_r, and hence H consists of an unlimited number of waves, the wavelengths of which are sub-multiples of the circumference, but which travel at perfectly arbitrary speeds and are, therefore, entirely independent of one another.

Since we are permitted to assume that some or all of the lines of force crossing the air-gap are closed outside the system discussed, as in a homopolar machine, we cannot deduce the periodic nature of the solution from the fact that all magnetic lines must necessarily be closed. Excluding homopolar machines, however, a periodic solution would be necessitated by this fact also.

Since it may easily be proved that all the different components of the above general wave are independent unless in virtue of some special method of connecting the face-conductors, we may confine our attention in the first place to machines containing a single wave. Such machines may be broadly subdivided into

1. Those containing a forced wave of flux density, etc., i.e. a wave whose wavelength, amplitude, and phase are determined by external circumstances—the interconnection of the face-conductors, the voltage, and frequency of supply, etc. Such machines form the vast majority of practical apparatus. It will be worth while to examine briefly how the wavelength is determined in such a type.

Instead of considering all the conductors as independent, as we have hitherto done, we must now consider how they may be connected among themselves. They may be connected—

(i) All in parallel, forming a squirrel-cage winding.

(ii) All in series, forming a ring winding. In this case the connection from one end of a conductor passes completely around the core carrying the flux to the beginning of the next conductor.

Both these methods of connection are independent of the wavelength of the flux wave. The former, however, is incapable of

connection to an external circuit ; we may, therefore, confine our attention to the latter.

If a periodic wave, such as we proved above necessarily exists in a dynamo·electric machine, is present in a core carrying a ring winding, points a wavelength apart will be at the same potential and may be jointed by a conductor known as an " equalizer." When this has been done it is impossible for any wave to exist having a wavelength different from those determined by the points to which the equalizers are joined. Again, points half a wavelength apart may be joined to opposite sources of potential, constant or
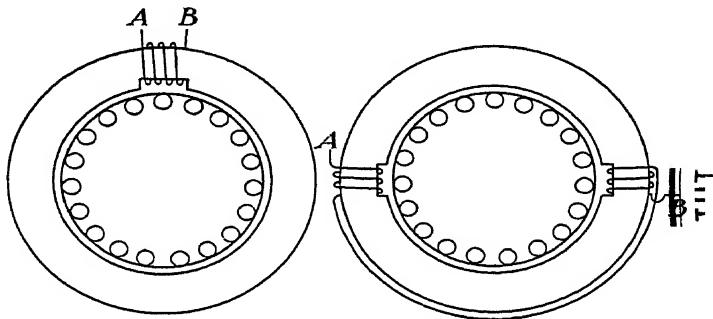
FIG. 5

alternating, or points $1/n$th of a wavelength apart to sources of alternating potential differing in phase by $2\pi/n$.

From these methods of connection as fundamental we may derive by the theory of " armature windings " all the methods of winding employed in actual machines, practically all of which involve a forced wave of flux. The present chapters, however, are devoted to an entirely different class of apparatus, namely,

2. Those containing a free wave of flux, or, in other words, one whose wavelength is determined by internal reactions, and hence is variable instead of being externally impressed. These waves can only occur in a type of winding in which the current in any bar is solely determined by the electromotive force in the same bar, and not by the electromotive force in a series of bars. The only type which fulfils this condition is the squirrel-cage type.

Hence the type of machine with which we shall deal will be one carrying a squirrel-cage winding on one member, and, on the other, merely a primary winding concentrated at one point in the zone which we are studying, or at a limited number of points in the whole machine.

Such a machine is shown in Fig. 5. It consists of an ordinary squirrel-cage rotor and a primary member wound ring-wise around the core and concentrated in a single slot. The diameter ˹of the apparatus is assumed to be sufficiently great to enable the

disturbances on the positive side of the primary member to die away before passing sufficiently far around the machine to meet those on the negative side.

In the following chapters a thorough study is made of such an apparatus, and the influence of resistance, self-induction, and capacity in the rotor is worked out, as well as that of various secondary windings on the stator. Having done this we are in a position to consider the effects of two such coils wound in opposite phases on opposite ends of a diameter and carrying opposite currents. These are equivalent to a drum-wound coil ; and having studied such a coil we can build up by the method of superposition the precise flux and current distribution due to a number of such coils, i.e. to a slotted stator such as is actually used in practice.

Occasionally we wish to consider what is taking place in a moving rotor at a point fixed in space rather than at a point fixed in the rotor. In this case it is helpful to consider the rotor as being ring-wound and connected to a commutator carrying a number of equally spaced brushes at distances apart which are small relative to the wavelength, in fact at distances which we denote by dx. These brushes are assumed to be short-circuited through a resistance, inductance, or capacity. It is shown below that there is a vital difference between the cases in which the inductance and capacity are assumed to be connected across the brushes, and that in which they are contained in the secondary member and revolve with it.

CHAPTER IV

EQUATIONS OF MOTION OF THE WAVES DUE TO RESISTANCE, INDUCTANCE, AND CAPACITY

As above, we shall consider that on one element of our machine there is only a primary winding concentrated at one point, and, consequently, that the three differential equations deduced in Section 2 are reduced to two. These two equations are

$$dH/dx = \Delta \; ; \text{ and } dE/dx = dB/dt + v(dB/dx).$$

We shall find it convenient, as was done in Section 2, to express these equations in terms of the flux. We then get

$$(1/\mu)(d^2\Phi/dx^2) = \Delta \; ; \text{ and } E = d\Phi/dt + v(d\Phi/dx) = d\Phi/dt + vB \; ;$$

or if we suppose both members to be wound

$$(1/\mu)(d^2\Phi/dx^2) = \Delta_1 + \Delta_2 \; ;$$
$$E_1 = d\Phi/dt \; , \quad E_2 = d\Phi/dt + v(d\Phi/dx).$$

There are two possible ways of treating each case. We have a primary and secondary member rotating relatively to one another We may consider the secondary to be rotating and the primary to be stationary (Case 1), or vice versa (Case 2).

Let Z be the symbolical operator.

$$r + LD + 1/KD, \text{ where } D = d/dt.$$

We may use Z_1 for the primary member and Z_2 for the secondary. Corresponding to the first method of treatment (primary stationary) we have the following differential equations -

$$(1/\mu)(d^2\Phi/dx^2) = \Delta_1 + \Delta_2 = E_1/Z_1 + E_2/Z_2 \; .$$
$$E_1 = d\Phi/dt \; , \text{ and } E_2 = d\Phi/dt + v(d\Phi/dx).$$

These equations are partial differential equations, but we assume that every quantity follows a sine law in time, or in mathematical language, when $x =$ constant, $\Phi = Ae^{jpt}$. In this case we get invariably $d\Phi/dt = jp\Phi$ in any stationary conductor. Substituting this value in our equations we find that they are reduced to ordinary differential equations, viz.

$$(1/\mu)(d^2\Phi/dx^2) = \Delta_1 + \Delta_2 \; ; E_1 = jp\Phi \; ; E_2 = jp\Phi + v(d\Phi/dx),$$

t being eliminated.

If by means of some further relation between E and Δ, such as is given above, we eliminate both from our equations, we find invariably that the result is an ordinary differential equation of the second order in Φ whose solution is of the form $\Phi = \Phi_o e^{Sx + jpt}$.

11

This gives $d\Phi/dx = S\Phi$, and permits us to eliminate d/dx from our equation as well, reducing it to an ordinary quadratic equation in complex quantities.

Hence we shall write

$$d/dx \text{ equivalent to } S = a + jm, \text{ say,}$$

and $$d/dt \quad \text{,,} \quad jp.$$

Corresponding to the second method of treatment we have the equations—

$$(1/\mu)(d^2\Phi/dx^2) = \Delta_1 + \Delta_2 \quad E_1/Z_1 + E_2/Z_2 .$$
$$E_1 = d\Phi/dt - v(d\Phi/dx) ; \text{ and } E_2 = - d\Phi/dt.$$

If in Case 1 we assume the rotor to travel in the positive direction (v positive), we must now assume the stator to travel in the negative direction, as we have impressed on the whole system a rotation equal and opposite to that which the rotor had before.

Our limiting condition is now as follows-

When $$x = - vt ; \quad \Phi \quad \Phi_0 e^{jpt}$$

Hence, in general,

$$\Phi = \Phi_0 e^{(a + jm)(x + vt) + jpt}$$

Giving d/dx equivalent to $a + jm$, and d/dt to $jp + v(a + jm)$.

We may illustrate these results by the case of rotor resistance. Put $Z_1 = \infty$ (this corresponds to an unwound stator), and $Z_2 = r_2$. The equations then become--

Case 1. $(r_2/\mu) (d^2\Phi/dx^2) = d\Phi/dt + v(d\Phi/dx)$, or, inserting the values of d/dx and d/dt,

$$(r_2/\mu) (a + jm)^2 = jp + v(a + jm).$$

Case 2. $(r_2/\mu) (d^2\Phi/dx^2) = d\Phi/dt$, or, inserting the values of d/dx and d/dt, $(r_2/\mu) (a + jm)^2 = jp + v(a + jm)$, which is identical with the above.

Now take the case of self-induction. $Z_1 \infty ; Z_2 \quad L_2(d/dt)$.

Case 1.
$$\frac{L_2}{\mu} \cdot \frac{d}{dt} \cdot \frac{d^2\Phi}{dx^2} = \frac{d\Phi}{dt} + v\frac{d\Phi}{dx}$$
$$(L_2 jp/\mu) (a + jm)^2 = jp + v (a + jm).$$

Case 2.
$$\frac{L_2}{\mu} \cdot \frac{d}{dt} \cdot \frac{d^2\Phi}{dx^2} = \frac{d\Phi}{dt}$$
$$(L_2/\mu) [jp + v (a + jm)] (a + jm)^2 = [jp + v (a + jm)]$$
or $$(L_2/\mu) (a + jm)^2 = 1.$$

It will be seen that the equations obtained from Case 1 and Case 2 are entirely different. The reason for this is as follows—

Case 1 ($d/dt = jp$) implicitly assumes that the self-induction L.

always operates on a constant frequency, while Case 2 permits the frequency to vary with the speed. The difference between these two cases may be illustrated by reference to the ordinary polyphase induction motor.

Suppose the rotor of such a motor to contain no resistance, but merely self-induction. As the speed rises from standstill the rotor E.M.F. falls in proportion to the frequency. The rotor reactance also falls in exact proportion to the frequency, so that the rotor current remains absolutely constant and independent of the speed. If, however, the said rotor is fitted with a commutator, and the brushes resting on this are closed through a reactance, the case is quite altered. Since the frequency of the current through the reactance is always equal to the line frequency, there is no change in the reactance with the speed, and since the rotor E.M.F. falls with the speed as before, the rotor current now falls with the rotor E.M.F. instead of remaining constant. This case is merely a simple example of that illustrated by the two differential equations, and serves to explain the difference between them quite clearly.

Case 2, therefore, is that in which the self-induction is connected direct to the rotor and revolves with it. It may be shown independently that this equation applies even in the limiting case of continuous current. Case 1 refers to a machine fitted with a commutator, the self-induction being outside the commutator and fed with currents at full frequency. Case 2 is, therefore, correct for our purpose.

Now take the case of capacity. We have

$$Z_1 = \infty, \text{ and } Z_2 = 1/KD.$$

Case 1 gives
$$\frac{1}{\mu} \cdot \frac{d^2\Phi}{dx^2} = K\frac{d}{dt}\left(\frac{d\Phi}{dt} + v\frac{d\Phi}{dx}\right)$$

or substituting,

$$[1/(K\mu)]\,(a + jm)^2 = jp[jp + v(a + jm)].$$

Case 2 gives
$$(1/\mu)\,(d^2\Phi/dx^2) = K(d^2\Phi/dt^2),$$

or
$$[1/(K\mu)]\,(a + jm)^2 = [jp + v(a + jm)]^2,$$

two results which again are quite different.

Confining ourselves to Case 2 we get the following three equations—

(a) *Resistance.* $(1/\mu)\,(a + jm)^2 = [jp + v(a + jm)]\,(1/r_2).$

(b) *Self-induction.* $(1/\mu)\,(a + jm)^2 = \dfrac{jp + v(a + jm)}{L_2[jp + v(a + jm)]}$

(c) *Capacity.* $(1/\mu)\,(a + jm)^2 = \dfrac{jp + v(a + jm)}{1/K[jp + v)a + jm)]}$

The various cases may thus be obtained from the general equation

$$\frac{1}{\mu} \cdot \frac{d^2\Phi}{dx^2} = \frac{d\Phi}{dt} \Big/ \left[r + L\frac{d}{dt} + \frac{1}{K(d/dt)} \right]$$

by putting $d/dt = jp + v(a + jm)$, and $d/dx = (a + jm)$

Case 1 gives

$$\frac{1}{\mu} \cdot \frac{d^2\Phi}{dx^2} = \left(\frac{d\Phi}{dt} + \frac{vd\Phi}{dx} \right) \Big/ \left[r + L\frac{d}{dt} + \frac{1}{K(d/dt)} \right]$$

with $d/dt = jp$ and $d/dx = a + jm$.

If now we put $Z_2 = \infty$, or, in other words, open circuit the rotor, our differential equations reduce to two instead of three, since all our windings are now on the same member.

Our differential equations now are $(1/\mu)(d^2\Phi/dx^2) = X$, and $E = d\Phi/dt$, and we shall make use of Case 1 (primary stationary).

(a) Putting $Z_1 = r_1$, we get $(1/\mu)(d^2\Phi/dx^2) = (1/r_1)(d\Phi/dt)$, on substituting, $(r_1/\mu)(a + jm)^2 = jp$.

(b) Putting $Z_1 = L(d/dt) = jLp$, we get $(L/\mu)(a + jm)^2 = 1$ on substituting.

(c) Putting $Z_1 = \dfrac{1}{K(d/dt)} = \dfrac{1}{Kjp}$, we get $\dfrac{1}{K\mu} \cdot \dfrac{d^2\Phi}{dx^2} = \dfrac{d^2\Phi}{dt^2}$, or $S^2/(K\mu) = (jp)^2$.

To recapitulate, we have now derived the following equations, where $S = (a + jm)$ if there are no windings except the primary on the stator.

(a) $(r_2/\mu)S^2 = jp + vS.$

(b) $(L_2/\mu)(jp + vS)S^2 = (jp + vS).$

(c) $\dfrac{S^2}{K\mu(jp + vS)} = jp + vS.$

It will be noted at once that all these equations are quadratics in S, the quadratic arising, of course, from the fact that the differential equations are of the second order. It will be useful, then, to attempt a solution of the general quadratic in complex quantities.

In the general equation, $aS^2 + bS + c = 0$, we can always multiply throughout by any complex number in such a way that the coefficient b becomes a pure real. We shall, therefore, for the future assume that b is a real or scalar, while a and c are any two arbitrary vectors.

Let $OA = aS^2 = ar^2 \text{ cis } 2\theta \text{ cis } \alpha$ (Fig. 6)

$AB = bS = br \text{ cis } \theta$

$BO = c = c \text{ cis } \gamma$ (see Fig. 6)

$OA' = a \text{ cis } \alpha$

We are here using cis α as an abbreviation for cos $\alpha + J$ sin α $= e^{j\alpha}$.

Draw $OA' = a$ cis α, $OB = -c$ cis γ, an arbitrary vector $OP = r$ cis θ, and another $OA = ar^2$ cis $(2\theta + \alpha)$. Draw through B a line BP parallel to OA.

Then if $\delta =$ angle OBP and $\psi =$ angle POB

$$\delta = 2\theta + \alpha - \gamma, \text{ and } \psi = \gamma - \theta$$
$$2\psi + \delta = 2\gamma - 2\theta + 2\theta + \alpha - \gamma$$
$$= \gamma + \alpha, \text{ that is, a constant.}$$

If, therefore, we draw a line OC making with OB an angle $COB = 2\psi$, it follows that the angle BCO is constant, and the point C,
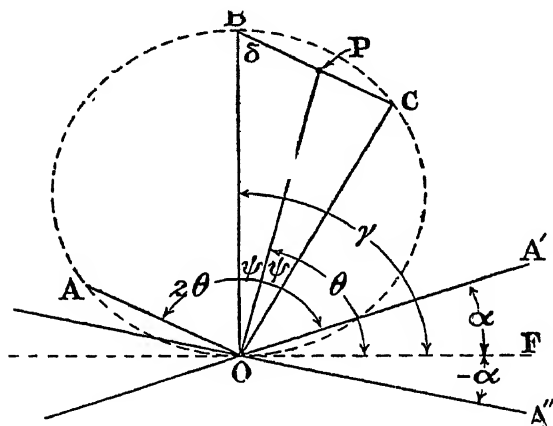
FIG. 6

therefore, moves in a circle passing through O and B. If we now draw a line OA'' making an angle $\gamma + \alpha$ with OB, it follows that the circle must touch this line, which gives us a construction whereby we may draw it conveniently.

By means of this diagram we may plot the locus of P (where $OP = bS$) for every value of θ from 0° to 360°.

If, therefore, we have some further means of ascertaining which value of θ satisfies the equation, the above diagram will give the value of r corresponding to it.

The value of θ may be found as follows : Start again from the equation $aS^2 + bS + c = 0$, with b a pure scalar. This may be written $S^2 + S(b/a) + c/a = 0$,

or
$$S^2 + S(b/a) + (b/2a)^2 + c/a - (b/2a)^2 = 0,$$

or
$$(S + b/2a)^2 = c/a - (b/2a)^2 ;$$
$$\therefore (S + b/2a) = \sqrt{[c/a - (b/2a)^2]}$$
$$\therefore bS + b^2/2a = b\sqrt{[c/a - (b/2a)^2]}$$

We shall endeavour to construct the vector $[c/a - (b/2a)^2]$.
Remembering that b is a pure scalar we have (see Fig. 7)
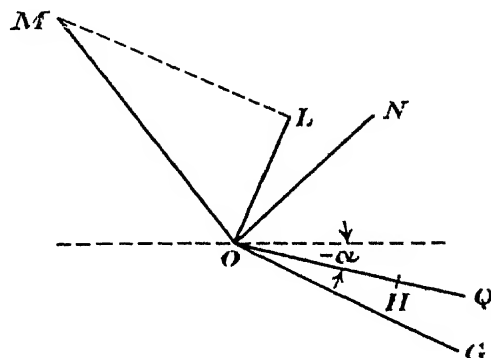
FIG. 7

$$b^2/2a = OQ \; ; \quad b/2a = OH \; ; \quad c/a = OL.$$

$$(b/2a)^2 = OG \; ; \quad c/a - (b/2a)^2 = OM \; ; \quad \sqrt{[c/a - (b/2a)^2]} \quad ON.$$

Now in the previous diagram we had $OP = bS$, and since b is a pure scalar the direction of b times ON is the same as that of ON.
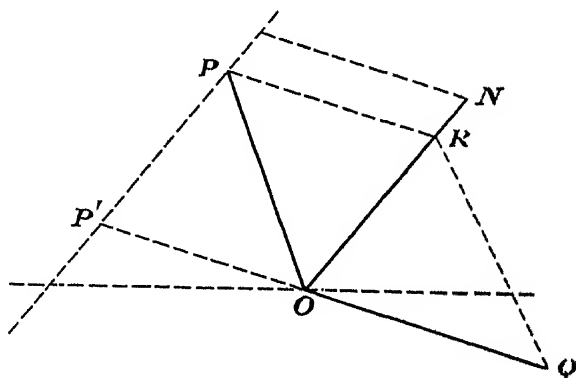
FIG. 8

In terms of the vectors in our diagram $bS + b^2/2a = b\sqrt{[c/a - (b/a)^2]}$ now reads $OP + OQ = bON = OR$ (see Fig. 8), and hence since OQ is constant the locus of P must be the straight line PP''.

Combining this result with the diagram previously obtained based on the circle, we obtain the final graphical solution of our equation (Fig. 9).

OP must satisfy two conditions—

1. It must lie on a straight line parallel to *ON*.
2. It must lie on the locus which may be traced by means of the circular construction.
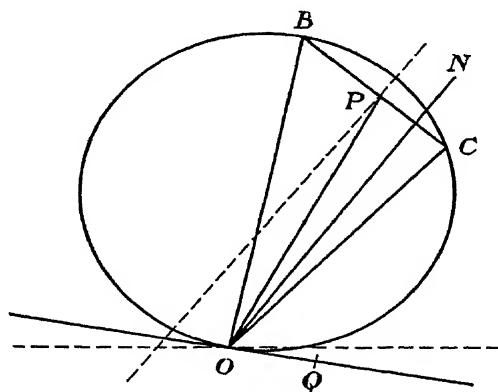
FIG. 9

The straight line cuts the locus in two points and hence we obtain our two roots.

CHAPTER V

In the present section we shall endeavour to obtain and interpret the solutions of the equations formulated above. At the end of Chapter IV we obtained a general solution of the quadratic equation in complex quantities by means of a graphical construction. In the present section, however, we shall usually adopt a method which consists essentially in eliminating v from the equations by some convenient means.

Starting with the equation $(r_2/\mu)(a + jm)^2 = jp + v(a + jm)$, which relates to the case of rotor resistance only, we resolve it into real and imaginary components—

$$(r_2/\mu)(a^2 - m^2) = va \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

$$(r_2/\mu) 2am = p + vm \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

Put $a = S \cos \theta$ and $m = S \sin \theta$, and we get

$$v = 2 (r_2/\mu) S \cos \theta - p/(S \sin \theta) \; ; \quad va = 2 (r_2/\mu) S^2 \cos^2\theta - p \cot \theta$$

Substituting in (1) we get

$$(r_2/\mu) S^2 (\cos^2\theta - \sin^2\theta) = 2 (r_2/\mu) S^2 \cos^2\theta - p \cot \theta \; ;$$

or $$S^2 = (p\mu/r_2) \cot \theta.$$

Another important conclusion is the following

$$-p/m = v_0, \text{ the synchronous speed of the wave.}$$

Hence equation (2) can be written (dividing by m)

$$(2r_2/\mu)a = - (- p/m - v) = - (v_0 - v) = - v_1$$

where v_1 is the slip velocity. In words this equation may be interpreted—

The decrement is proportional to the slip velocity.

Another deduction from the same equation is as follows

$$(2r_2/\mu) [a - v\mu/(2r_2)]m = p.$$

$$\therefore [a - v\mu/(2r_2)] m = p\mu/(2r_2).$$

If we put $x = a - v\mu/(2r_2)$ and $m = y$ we get $xy = p\mu/(2r_2)$.

If we put $r_2/\mu = c$, we get $xy = p/(2c)$, that is, the equation of an hyperbola.

Moreover, $x = (p/m)[\mu/(2r_2)] = - v_0/(2c)$. Hence, x is proportional to the synchronous speed.

Now plot the two equations

$$S^2 = (p/c) \cot \theta \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

and $$xy = p/2c \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

on the same sheet. If we do this the abscissa of curve (1) will be $YP = a$, and that of curve (2) will be

$$YP_1 = a - v\mu/(2r_2) = a - v/(2c).$$
$$\therefore\ YP - YP_1 = - v\mu/(2r_2) = - v/(2c) = PP_1,$$

or PP_1 is proportional to the speed.

Summing up these results we have

Fig. 10. $YP_1 = - v_0/(2c) = (-$ synchronous speed$)$.

,, $PP_1 = - v/(2c) =$ actual speed.

,, $YP = YP_1 - P_1P = - (v_0 - v)/(2c)$
$$= - v_1/(2c) = \text{slip velocity}.$$

$OP =$ current ; OP^2 or $UT =$ torque.

Since the torque is proportional to B^2 it is proportional to OP^2 $= S^2 = (p/c) \cot \theta$. Cot θ is proportional to UT.

Since throughout these investigations we have taken x as parallel

FIG 10

to the real axis, we have $E = jp^\Phi$ parallel to OY, and hence the angle $POY = \phi$, the angle of phase difference.

These investigations relate to the occurrences on one side only of the primary coil. Both sides must be considered.

In this case we may consider that on one side of the exciting coil the speed is $+ v$, and on the other $- v$.

In this case the total current in the exciting coil $= OP + OP_2$. We have

$$PP_1 = P_1P_2.\ \ OP_1 = OP + PP_1 = OP_2 - P_2P_1.$$
$$OP + OP_2 = 2OP_1 \text{ (see Fig. 10)}.$$

and since OP_1 moves on an hyperbola, so does the current in the exciting coil.

It may be of interest to prove that at the maximum point of the curve $\theta = 45°$.

The shape of the curve is as shown in the figure. Transforming the equation

$$S^2 = k \cot \theta \text{ to Cartesian co-ordinates,}$$

it becomes $x^2 + y^2 = k (S \cos \theta)/(S \sin \theta) = kx/y$,

or $\qquad x^2 y - kx + y^3 = 0$, that is, a quadratic in x.

The value of y for which the two roots of this quadratic are equal is clearly the maximum value.

$$x = \frac{k \pm \sqrt{(k^2 - 4y^4)}}{2y}$$

When $2y^2 = k$ or $y = \sqrt{(k/2)}$ the two roots are equal.

,, $\theta = 45°$, $\cot \theta = 1$ and $x = y$.

The equation then becomes $S^2 = x^2 + y^2 = 2y^2 = k$, the result just obtained. Hence y is a maximum when $\theta = 45°$.

Besides the construction given above for the speed, another may
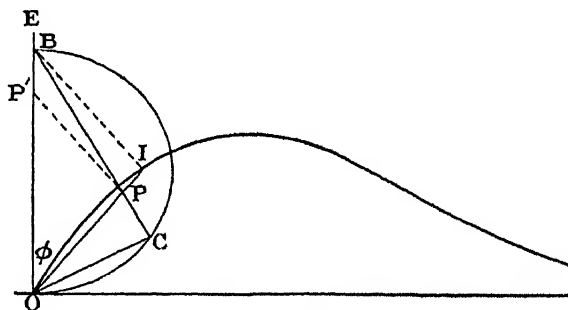
Fig. 11

be very easily deduced from the E.M.F. triangle, which, as it is of very general application, it may be of interest to give.

We gave a general solution of the quadratic equation in complex quantities for any values of the coefficients. We have now to consider what are the values of the coefficients in the case we are considering.

The equation which results from the differential equation in the case of rotor resistance is

$$(r_2/\mu) S^2 - vS - jp = 0.$$

Hence $c = -jp$; $a = r/\mu$; $b = -v$ (see page 455) ; and following the construction we must draw our circle passing through the extremity of $OB = jp$ (see Fig. 11) and touching the real axis (since $a = 0$). Since OB is at right angles to the real axis it is a

diameter of the circle. We saw in general that $OP = bS = -vS$ in this case. Hence our speed construction will be as follows—

Draw the locus of the current vector $S^2 = (p/c) \cot \theta$ as before, and also the circle as above. Corresponding to any current vector OI draw a line OC to cut the circle making an angle 2ϕ with OE. Join BC ; P will be the point where BC cuts OI. Then $OP/OI = v$.

In order to read off v on a fixed line all we have to do is to join I to some fixed point such as B and draw through P a line PP' parallel to IB. Then $OP'/OB = OP/OI$, and, consequently, OP' is a measure of the speed to such a scale that $OB = 1$.

This speed construction is perfectly general and applicable to any case in which the coefficient of S is proportional to v.

We must now consider the effect of stator resistance, inductance, and capacity, combined with rotor resistance.

Taking the case of inductance first and supposing the primary to be stationary, we have

$$(1/\mu) (d^2\Phi/dx^2) = E_1/(jLp) + E_2/r_2 ;$$

$$E_1 = d\Phi/dt - jp\Phi ; \quad E_2 = d\Phi/dt + v (d\Phi/dx) = jp\Phi + v (a + jm)$$

or $\qquad (r_2/\mu) S^2 = r_2/L + jp + vS.$

In the case of stator capacity we have

$$(1/\mu (d^2\Phi/dx^2) = E_1 Kp + E_2/r_2, \text{ or } (r_2/\mu) S^2 = - r_2 Kp^2 + jp + vS.$$

And in the case of stator resistance

$$(1/\mu) (d^2\Phi/dx^2) = E_1/r_1 + E_2/r_2,$$

$$\therefore (r_2/\mu) S^2 = jpr_2/r_1 + jp + v(a + jm) = jp(1 + r_2/r_1) + v(a + jm)$$

The sole influence, therefore, of stator resistance, inductance, and capacity is to vary the absolute term of the quadratic either in magnitude only or in both magnitude and direction.

Taking the case of resistance first, it is clear that since we have merely multiplied the absolute term by $(1 + r_2/r_1)$ the resultant locus will be unchanged in shape, but merely multiplied by the same quantity, viz. $(1 + r_2/r_1)$.

Hence the locus of the current vector will now be—

$$S^2 = (pr_2/\mu) (1 + r_2/r_1) \cot \theta \text{ instead of}$$

$$S^2 = (pr_2/\mu) \cot 0 \text{ as it was before.}$$

Hence the size of the diagram is increased, but its shape is unchanged.

Returning to the cases of self-induction and capacity the equations we have just arrived at are

(a) $\qquad (r_2/\mu) S^2 - vS - (jp + r_2/L) = 0.$

(b) $\qquad \text{and } (r_2/\mu) S^2 - vS - (jp - r_2 Kp^2) = 0.$

In these two equations we have

(i) $c = -(jp + r_2/L)$; b $-v$; a r_2/μ.

(ii) $c = -jp - r_2Kp^2$; b $-v$; a r_2/μ.

Hence the diagrams are as follows· ·

The real axis is still a tangent to the circle in both cases ; OB is, however, no longer a diameter, but lags or leads relatively to its former position. This is illustrated in Fig. 12 for the case of self-induction, and Fig. 13 for capacity.

We may, of course, discuss the matter by eliminating v as we

FIG. 12   FIG. 13

did before. Referring back to the equation given above, we see that we now have in the case of self-induction

$$(r_2/\mu)\ (a^2 - m^2) = va + r_2/L, \text{ and } (r_2/\mu)\ 2am \quad p \mid vm.$$

The second of these equations remains the same in all cases.

Put $a = S \cos \theta$, and $m = S \sin \theta$. We then get

$$v = 2\ (r_2/\mu)\ S \cos \theta \cdot p/(S \sin \theta),$$

and $$va = 2\ (r_2/\mu)\ S^2 \cos^2\theta - p \cot \theta,$$

thus $(r_2/\mu)\ S^2\ (\cos^2\theta - \sin^2\theta) = 2\ (r_2/\mu)\ S^2 \cos^2\theta - p \cot \theta \mid r_2/L.$

$$\therefore\ S^2 = (p\mu/r_2)\ \cot \theta \mid r_2/L.$$

In the case of capacity we merely have $-r_2Kp^2$ instead of r_2/L, and we, therefore, get

$$S^2 = (p\mu/r_2)\ \cot \theta - r_2Kp^2.$$

In the former case, if $S = 0$ we have $\cot \theta_1 = r_2^2/(L.p\mu)$. S now becomes zero for some definite value of θ different from $90°$, and the diagram has the shape shown in Fig. 14(a) when we use self-induction in the stator. If condensers are used we obtain the shape shown in Fig. 14(b), and a, the decrement, vanishes for some finite value of the current. The torque is still proportional to S^2.

Since the decrement is still proportional to the slip, it follows that at the point of unity power factor the machine is running in synchronism. Beyond that point it is running above synchronism, and there will be an increment instead of a decrement. If we complete the discussion of this case by considering a second primary coil at $x = c$ we shall find that the machine on the whole is now a generator.

We saw that stator self-induction or capacity produced no effect on the equation determining the slip. This equation, viz. $(r/\mu)2am = p - vm$, may be written $y[x - v/(2c)] = p/(2c)$, which represents an hyperbola. Hence the speed construction remains the same in both these cases. In any case, no matter what the angle of lag
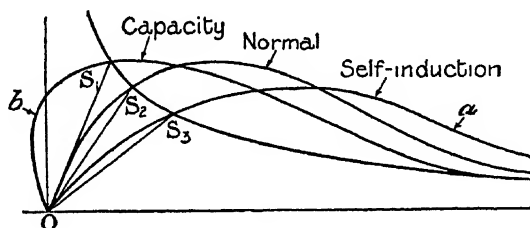

Fɪɢ. 14

at starting, the hyperbola passes through the point of the curve corresponding to $v = 0$. As the hyperbola is independent of the stator self-induction or capacity, it is the locus of the starting current when we vary the latter. Since the hyperbola is the locus of the starting current it is clear that the angle of lag at starting is affected by the use of condensers (S_1) or self-induction (S_3). In the former case it is reduced and in the latter increased.

Having now considered the case of rotor resistance, either alone or combined with resistance, self-induction, or capacity in the stator, we may now consider the cases of rotor self-induction and capacity.

The case of rotor self-induction is very simple. We have already derived the equation $(L_2/\mu)S^2 = 1$ for this case; whence it follows that $S = a = \sqrt{(\mu/L_2)}$ at all speeds. The apparatus will, therefore, absorb a constant current lagging 90° behind its electromotive force at every speed, there being no " phase splitting " and, hence, no torque.

Such an apparatus is thus only suitable for purposes of neutralization and not for power. Hence, even if we supply a machine having merely self-induction on the rotor with a stator winding having resistance or capacity to produce phase splitting or even a uniform polyphase winding, no torque can result. Thus this simple investigation disposes of a whole class of cases.

In the case of condensers, the equation we derived was $1/(K\mu)S^2$ $= (jp + vS)^2$. Putting $1/(K\mu) = v^2{}_o$ and transposing, we get

$$(v_o^2 - v^2) S^2 - 2vjpS - (jp)^2 = 0.$$

Solving this quadratic in S we get

$$S = jm = \frac{vjp \pm \sqrt{[-v^2 p^2 - p^2 (v^2{}_o - v^2)}}{v_o^2 - v^2}$$

or

$$S = jp\left(\frac{v \pm v^o}{v_o^2 - v^2}\right) = \frac{jp}{v_o \mp v}$$

At zero speed we have a current $OS_o = jp/v_o$. As the speed rises the current falls if the rotor revolves in the same direction as the field, and rises if the rotor revolves in the opposite direction. At the same time m the number of poles per unit length varies proportionally to the current. This is the Doppler effect as it appears in our case.

We have seen that the case of rotor condensers gives rise to an interesting result, viz. an analogue of the Doppler effect. Let us, therefore, now attempt to investigate the case of rotor condensers and stator resistance, inductance, or capacity.

1. *Stator self-induction.* Putting $Z_1 = L(d/dt)$ and $Z_2 = 1/(KD)$, we get

$$\frac{d}{dt} \cdot \frac{1}{\mu} \cdot \frac{d^2\Phi}{dx^2} = K\frac{d^3\Phi}{dt^3} + \left(\frac{d\Phi}{dt} - v\frac{d\Phi}{dx}\right)\frac{1}{L}$$

Remembering that $d/dt = jp + vS$, in Case 2 above we get on substituting

$$1/(K\mu) (jp + vS) S^2 = (jp + vS)^3 + jp/(L_1 K).$$

2. *Stator resistance*

$$\frac{r_1}{\mu} \cdot \frac{d^2\Phi}{dx^2} = Kr_1\frac{d^2\Phi}{dt^2} + \left(\frac{d\Phi}{dt} - v\frac{d\Phi}{dx}\right)$$

$$\therefore \frac{r_1}{K\mu} S^2 = (jp + vS)^2 + jp/K$$

3. *Stator capacity*

$$\frac{1}{\mu} \cdot \frac{d^2\Phi}{dx^2} = K_1\frac{d^2\Phi}{dt^2} + K_1\frac{d}{dt}\left(\frac{d\Phi}{dt} - v\frac{d\Phi}{dx}\right)$$

$$\therefore \frac{1}{K_1\mu}S^2 = (jp + vS)^2 + \frac{K_1}{K_2}(jp) (jp + vS)$$

Expanding the three equations they may be written as follows, putting $1/(K\mu) = v_o^2$.

1. *Resistance*

$$(v_o^2 - v^2) S^2 - 2jpvS + p^2 [1 + 1/(jKpr_1)] = 0.$$

2. *Capacity*

$$(v_o^2 - v^2) S^2 - jpvS (2 + K_1/K_2) + p^2 (1 + K_1/K_2) = 0.$$

3. *Inductance*

$$(v_o^2 - v^2) vS^3 + (v_o - 3v^2) jpS^2 - - 3jp^2vS - jp (1/LK - p^2) = 0.$$

The first two of the above equations may be treated by the same method as that applied in the case of rotor capacity only, viz. by solving the quadratic in S. By this means we get the following results—

1. $S = jp \dfrac{v \pm \sqrt{\{v^2 + (v_o^2 - v^2) [1 + 1/(jKpr_1)]\}}}{v^{o2} - v^2}$

2. $S = jp \dfrac{v(1 + K_1/K_2) \pm \sqrt{\{v^2(1 + K_1/K_2) + (v_o^2 - v^2)(1 + K_1/K_2)\}}}{v^2 - v^2}$

CHAPTER VI

SOME MECHANICAL ANALOGIES

FROM the mathematical discussions in the preceding chapters it is very difficult to form a clear physical idea of the phenomena actually taking place—at any rate the author finds it so. On the other hand it is inadvisable, from the point of view of clearness, to burden a mathematical investigation with long physical explanations. It has, therefore, been decided to take the subject up afresh from the physical standpoint, and to endeavour to give a clear account of it as free from mathematics as possible, referring the reader for proofs to the preceding chapters where necessary.

Let us first consider under what conditions it is possible for a set of conductors on an electric machine to experience a steady force.
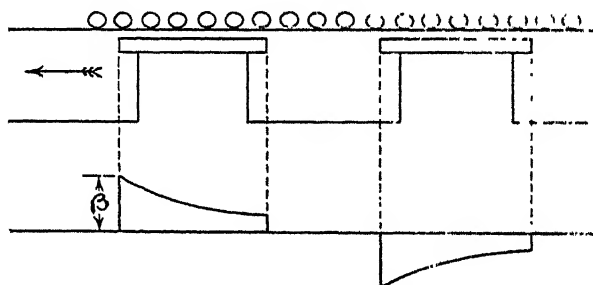
FIG. 15

There is first of all the ordinary case occurring in " neutralized " machines in which conductors that lie opposite to one another on the two members carry equal and opposite currents except for a small component which serves for magnetizing. In this case the force on each member is $\int B \, \Delta dx$, and B and Δ are entirely independent of one another.

In certain types of machines, such as continuous-current and synchronous machines, B and Δ are not independent, it being possible to express Δ as a function of B. In these machines there is no neutralizing winding, the winding on one member the field being concentrated in one large slot per pole. An attempt is usually made to reduce the distortion or influence of Δ on the distribution of flux density B as much as possible by exaggerating the field ampere-turns relatively to those of the armature, and by employing long air-gaps, high saturations, etc. In this manner these cases are made to correspond approximately to those in which a

neutralizing winding is employed. An entirely different treatment, however, will be found below, in which the force is expressed in terms of the distribution of the field.

As an illustration of this we may apply the investigations of Part I to the only case in which windings concentrated in a single slot are widely used in practice, viz. that of the synchronous and continuous-current machines mentioned above. To the case of the synchronous alternator these investigations are directly applicable if we consider a machine having as many phases as there are slots per pole, each phase being closed through a resistance load. We then have the exact case discussed in Part I, of a concentrated " primary " winding carrying continuous current in this case and " secondaries " short-circuited through resistance only. If, therefore, we put $p = 0$ in our previous equations (page 18) we shall get a solution applicable to this case, viz. $a = v\mu/(2r_2)$; $p = m = 0$; giving $B = B_0 e^{v\mu x/(2r_2)}$. Thus the flux density in the pole face is distorted by the action of the armature currents, and is distributed (assuming an infinite number of slots) in accordance with an exponential curve with an increment in the direction of motion and whose exponent is proportional to the speed. We shall see from the principles developed below, that when there is an increment in the direction of motion the torque produced is opposed to the direction of motion, as, of course, is the case in the apparatus we are discussing. In fact, since B is a function of Δ the torque of the machine can be expressed purely in terms of B, and the methods of regarding the question which are discussed below apply in their entirety.

To understand this clearly it will be best to start by considering a single conductor carrying a current and lying in a magnetic field.

Let this resultant field, for instance, be compounded of an original field and the field due to the current. The field will be strengthened on one side of the current—that on which the magnetomotive force H due to the current assists the original field—and weakened on the other where they are opposed (Fig. 16). The net result is a force moving the conductor in the direction of the weaker field. The amount of this force is equal to the difference of magnetic energy density $\frac{1}{2}\mu H^2$ on opposite sides of the conductor. A more usual expression for this force is $F = B\Delta$. Since, as we saw above, $\Delta = dH/dx = (1/\mu)(dB/dx)$ in the dynamo-electric machine, we have—

Force on a system of conductors extending from $x = A$ to $x = B$

$$= \frac{1}{\mu}\int_A^B B\,(dB/dx)\,dx = \frac{1}{2\mu}(B_B^2 - B_A^2) = \frac{1}{2}\mu(H_B^2 - H_A^2)$$

the same as before, since $B = \mu H$.

The expression $F = \frac{1}{2\mu}(B_B^2 - B_A^2)$ may also be written in the form $F = B\Delta$, for

$$\frac{1}{2\mu}(B_B^2 - B_A^2) = [(B_B - B_A)/\mu] \cdot \frac{1}{2}(B_B + B_A)$$

= current between B and A multiplied by the mean density.

If, therefore, we can by any means whatever set up a difference of magnetic densities on opposite sides of a current carrying conductor there will be a force acting on the conductor. Since the force is
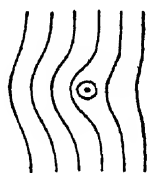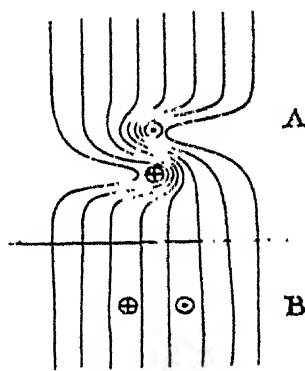
FIG. 16   FIG. 17

proportional to the square (and not to the first power) of the density the direction of the lines of force is immaterial — i.e. whether upwards or downwards in Fig. 16.

Now suppose a number of conductors to be arranged on the periphery of an electric machine, and consider the torque on the whole set of conductors in various instances. If the distribution of B is periodic there can be no resultant torque, for if the torque on conductors (1) and (2), say, has a certain direction, that on conductors (3) and (4) will be in the opposite direction.

If B, however, starting from a certain point continually gets less and less following an asymptotic curve, the force on every conductor will have the same direction. If this curve, however, crosses the zero line, the intensity of B increasing in the opposite direction, the force will reverse at the point where the curve crosses the zero line.

Some possible distributions of density are shown in the table below, together with a note of the force given by them.

1. *Asymptotic curve.* The force on every conductor is in the same direction.

2. *Sine wave with decrement.* While the force on every conductor

is not in the same direction, the average force over a complete period always has the same direction.

3. *Sine wave or other periodic curve.* There is no resultant force over a complete number of periods.

4. *Straight line.* The force reverses where B reverses. The direction of the resultant force depends on whether the positive or negative flux is the greater.

It is important to notice in cases in which we have two conductors close together, as on the rotor and stator of a dynamo-electric machine, that the force referred to is the force on both conductors considered as a single system. For instance, in the case given in Fig. 21, in which the stator and rotor currents are equal and opposite, there may be a uniform field and, consequently, no resultant force, and yet large relative forces between the rotor and stator conductors.

This, however, is an essentially different case, though it is the principle on which the torques of almost all existing machines depend. For instance, if we have two equal and opposite currents in a magnetic field the distribution of flux which they will produce is shown in Fig. 17, if the line joining them is parallel to the original direction of the flux. A couple will then be exerted, tending to turn the conductors into such a position that the lines threading them will be as few as possible. If, however, they are mechanically constrained so that they cannot move in the manner indicated, one of the conductors will continue to be urged to the right and the other to the left. If we have a succession of such pairs as shown in Fig. 21, the lines will take the form shown, and equal and opposite forces will be exerted on the two sets.

In the case in which B is a function of the time, and thus admits of a mean value, i.e. is a periodic function, we have

$$\text{Mean of } (B_B^2 - B_A^2) = \text{Mean of } B_B^2 - \text{Mean of } B_A^2.$$

Hence, in this case

$$\text{Mean force} = \frac{1}{2\mu} \text{ (difference in mean square densities)}.$$

Hence, in order that the mean force on every conductor shall be in the same direction, it is only necessary that the mean distribution of density shall be asymptotic. For instance, if the density is distributed as a travelling wave with decrement, that is, $B = B_0 e^{ax} \sin(pt - mx)$, then for any value of x such that $mx = a$, the mean value of $B^2 = \frac{1}{2} B_0^2 e^{2ax}$, since the mean of $\sin^2(pt - a) = \frac{1}{2}$, independently of the value of a. Thus, such a travelling wave would give a mean force having the same direction for every conductor.

The theorem that the force on a body is equal to the difference of energy densities on either side of it is in no way peculiar to

electricity, but is a general theorem of mechanics. For instance, the average force per square inch on a partition between two fluids of different levels is proportional to the mean pressure, i.e. to the mean density of potential energy on the two sides. In a vibrating cord or chain the force between adjacent links is proportional to the difference between the kinetic energies of the links.

We have now to consider what means may be adopted in the various electric, hydraulic, etc., cases to realize a distribution of magnetic, potential, or kinetic energy, etc., adapted to give a steady force.

We have seen that if we can by any means produce a travelling wave moving in the direction in which we wish the force to be, we can obtain such a force if the wave has a decrement in the direction of travel, but not otherwise. If there is no decrement there
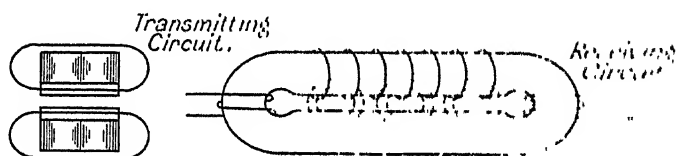
Fig. 18

will be no mean force, while if there is an increment there will be a force opposed to the direction of travel. Our inquiry then resolves itself into a study of the means of production of such waves.

Two distinct types of electromagnetic transmission exist whereby energy may be transmitted from one place to another along a line—

1. The type so much studied in connection with cables, etc., in which the current flows along the axis of x and the magnetic lines form closed rings round it. The electrostatic lines are radial near the cable, and if it is sufficiently long both ends rest on different parts of it, these parts being a half wavelength apart. Energy flows along the cable in the form of waves at a definite speed, as has been very thoroughly discussed by Kelvin, Heaviside, Pupin, and many others.

2. It has not hitherto been realized that there is another type of power-transmission line possible in which the functions of the electrostatic and magnetic lines in the above example are interchanged.

Consider a closed iron core (Fig. 18), the two limbs of which, A and B, are separated by a certain air-gap and carrying at least two electric circuits, a transmitting and a receiving circuit. Magnetic lines circulate round the closed core, and, if they vary with the time, electrostatic lines encircle it in the manner shown on the right. The effects of these lines may be made evident by causing

a number of conductors (in addition to the transmitting and receiving circuits) to encircle the iron core, whose circuits are completed through resistance, inductance, or capacity. If the air-gap between the two limbs A and B is sufficiently small, an electromagnetic wave will be formed as shown in Fig. 18 in the case of resistance and capacity, whereby energy is conveyed from the transmitting to the receiving circuit. The wave of flux cuts the receiving circuit, inducing an electromotive force therein, and if it is closed a current will also flow of such magnitude that the resultant magnetomotive force summed round the entire circuit and including that required to overcome the reluctance of the iron is zero.

If these intermediate conductors be omitted, and the air-gap between the two limbs be made large, our apparatus reduces to the ordinary transformer, which we may compare with an ordinary
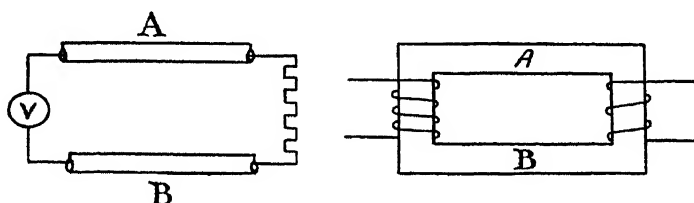
Fig. 19

electric circuit, too short to permit of the development of a wave (see Fig. 19). In the electric circuit a difference of electric potential is set up by a generator of some sort between the limbs A and B, while in the transformer a difference of magnetic potential is set up by the primary or transmitting coil.

In the electric case (Case 1) this electric difference of potential appears almost unchanged at the terminals of the receiving circuit, being diminished only by the drop of pressure due to resistance in the line. In the transformer the magnetic difference of potential set up by the primary also appears almost unaltered at the extremities of the secondary, reduced only by the pressure drop due to reluctance in the iron. The electric potential difference of Case 1 is finally balanced by the potential difference across the receiving circuit, while in Case 2 the magnetic potential difference is balanced by that due to the secondary or receiving coil. Just as any electric circuit has some self-induction, though this may be very small, owing to the fact that some magnetic lines interlink it, so any magnetic circuit has some capacity owing to the fact that some electrostatic lines interlink it. This is so small in an ordinary transformer that even its theoretical existence is scarcely recognized ; but it becomes evident at once if we surround the limb of our magnetic circuit by a conductor, or conductors, whose circuits are closed by condensers, say. The effect of this is analogous

to surrounding an electric circuit with an iron core. In this case, in spite of the formation of a travelling wave, the whole of the energy entering the primary is received by the secondary.

If the conductors of Fig. 18 are closed through a resistance, however, a wave will still be formed (see Chap. V), but it will be a wave of decreasing amplitude whose energy is gradually absorbed by the resistance of the conductor and transformed into heat, so that but little of the energy entering the primary coil may reach the secondary or receiving coil.

Such a wave satisfies the conditions laid down above as being necessary, in order that mean force may exist on the conductors
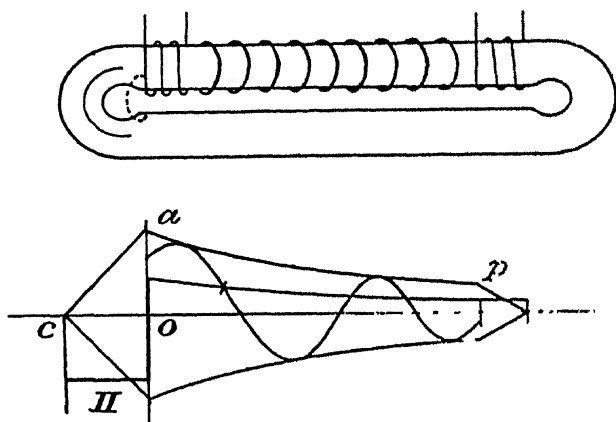
Fig. 20

and that the force may always have the same direction. It is shown in Part I, in fact, that the air-gap density B is denoted by the equation $B = B_0 e^{ax} \sin (pt - mx)$, a, B_0, m, and p having values determined by the circumstances.

There will, therefore, in such a case be a force on all the short-circuited secondary conductors urging them away from the primary, that is, the secondary will be repelled from the primary and the primary from the secondary. This statement may be interpreted to include the receiving coil among the secondary circuits if desired, or we may consider that practically the whole of the energy of the wave is absorbed before it reaches the receiving coil, which, in this case, may be neglected. This force is usually said to be due to the Elihu Thomson repulsion effect.

Let us consider the root-mean-square distribution of current and energy density or B^2 in such a transmission, including the transmitting and receiving circuits.

The air-gap density on the outer side of the transmitting coil at C is zero, as there is no reason why the lines of force should cross the air-gap in the manner shown dotted rather than pass wholly

through iron as shown in the full lines. The same applies to the outer side of the receiving coil, also shown dotted. We may, therefore, conceive the function of the primary coil as being that of increasing the air-gap density from zero at *C* to *OA* (Fig. 20).

The primary current density (amperes per cm. length) which is constant is shown in curve *II*. When the density has reached the value *Oa* at *O*, due to the action of the primary coil, it begins to be reduced by the action of the short-circuited coils as we proceed from *O* to the right towards *p*. The current shown in curve *II* is now reversed, and, instead of building up the density to still higher values, begins to reduce it again. When we reach *p* the remainder of the density is absorbed by the receiving coil, on the further side of which it is again zero. By a fundamental law, as we have seen, $(1/\mu) (dB/dx) = \Delta$, the number of amperes per cm., and,
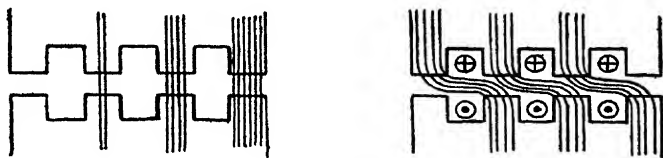
Fig. 21

hence, the current curve is the slope of the curve of densities and changes sign when this slope changes sign. This is clearly shown in the curve marked *II*.

If we compare two cases : (1) that in which there is a number of conductors arranged in slots and all carrying the same current, and (2) that in which there is a number of pairs of conductors in corresponding slots carrying equal and opposite currents (see Fig. 21), we see that in Case 1 the density continually becomes greater and greater, while in Case 2 the density is constant in all the teeth.

The function of the second set of conductors, therefore, may be said to be to restore the energy density or B^2 in those parts of the system where it has been weakened by the first set, and to weaken it where it has been increased beyond the average. For instance, in the figure the effect of the second set of conductors is to cause two lines to leave tooth 3 and traverse tooth 2, and two lines to leave tooth 4 and traverse tooth 3. Hence, if one of these sets of conductors be primary and the other secondary conductors, we may say that the primary conductors continually restore the densities which would be reduced by the action of the secondary conductors.

In the case of the transmission previously discussed, however, the density is built up to its maximum value in one part of the system, and reduced again by the action of the secondary in another part—an essentially different arrangement.

It is not necessary that the two limbs of our system should be made in one piece, but there may be an air-gap between them, as in Fig. 22. In this case let the primary be held fixed and attached to the lower limb *B*, while the receiving coil is either fixed and attached to *B* or movable and attached to *A*, and the short circuited conductors *c, c, c* are attached to *A*. Then there will be a force on
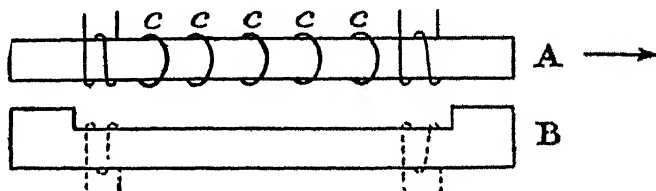
FIG. 22

the limb *A* relative to the limb *B*, which will tend to move *A* in the direction of the arrow when *B* is fixed. If the motion produced by this force is to be continuous, provision must be made for replacing the conductors *c, c, c* by others as they move away from the primary.

It is also unnecessary that the primary coil should surround the limb *A*, but it may surround the limb *B* instead, as shown dotted in Fig. 22. All that is necessary now in order to get continuous rotation is that the limb *A* shall be made in annular form, as in Fig. 23,
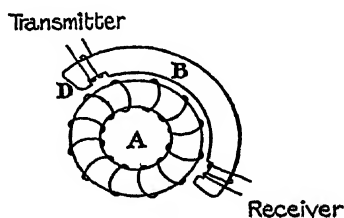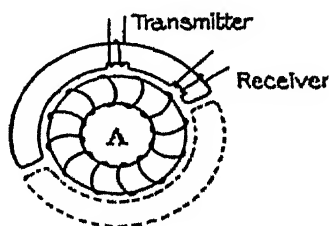
FIG. 23   FIG. 24

where the transmitting and receiving windings are shown wound into large slots in a segmental stator which terminates at *D*. It is essential in this construction that it should terminate at *D*, as if it did not the primary would transmit waves equally in both directions and there would be no resultant torque.

Another method whereby the transmission of waves from the primary equally in both directions may be prevented, is by placing the receiving coil so close to the primary that the wave is absorbed by it before it has decreased much in amplitude, the wave transmitted by the primary in the other direction travelling quite freely.

We may suppose that a multiplicity of such separate systems, as shown dotted in Fig. 24, may be mounted so as to interact with

a single rotor, or we may suppose that one system extends all round. In either case we have to remember that by proceeding sufficiently far round the core in a given direction we return to the same point.

Bearing this carefully in mind we may regard our system as cut at *A* (Fig. 25) and developed into a flat surface *ABH*, in which case the two ends really represent the same point. In this figure the
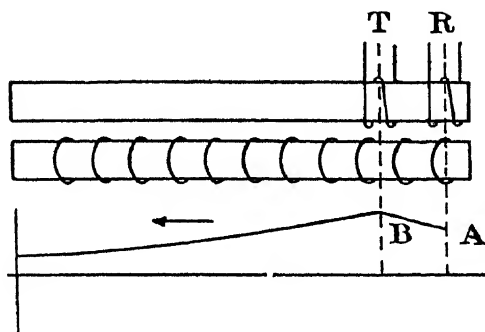
Fig. 25

wave is propagated from the transmitting or primary coil in the direction of the arrow completely round the periphery until it reaches the receiving coil, where it is absorbed. Having been absorbed, either fresh energy is imparted to it by the transmitting coil and it again passes round the periphery, or a new wave is generated by the transmitting coil. Which of these two cases will occur depends entirely on the arrangement of the windings between *T* and *R*. In whatever way they are arranged, however, their ultimate object is the absorption of a wave of small energy (the less the better) and the emission of a wave of much greater energy.

Before proceeding much farther with the electrical case of our theorem, it will be well to develop and apply thereto the well-known mechanical case of a vibratory cord.

CHAPTER VII

A MECHANICAL ANALOGY TO THE VIBRATIONS OF THE ELECTROMAGNETIC SYSTEM

BEFORE proceeding any farther we shall explain more fully the well-known analogy between the vibrations of our electrical system and those of a cord, which we shall suppose to be submitted to a total tension P and of such length that the displacement y of the cord perpendicular to its length, i.e. to the X-axis, is always very small relative to the total length. Now the tension of the cord is everywhere the same in amount and parallel to the tangent to the cord. It, therefore, varies in direction owing to the curvature of the cord when in vibration. Now the force tending to move any element of the cord is equal to the difference of tension on the two ends. These tensions, as we have seen, do not differ in absolute magnitude, but only in direction, and for such small lengths of cord as we may assume to be circular arcs the difference of direction is proportional to the length of the element ds. Hence, if dT be the difference of the tensions, we have

$$dT/ds = \text{force tending to move } ds.$$

This force is necessarily perpendicular to the cord, since the components of the tension on either end of the cord taken parallel to it are equal and opposite.

Now since the tension of the cord is constant in amount and varies in direction only, dT/ds is proportional to the curvature of the cord at the point considered.

On the assumption mentioned above that the displacement is small relative to the length, and that the slope of the curve is, therefore, never very large, we may replace s measured along the curve by x measured along the axis, and thus obtain another expression for the curvature, viz. d^2y/x^2.

We may now write—

$$P \, (dy^2/dx^2) = F,$$

where P = initial tension on the cord,
and F = force perpendicular to x.

This force must be equated to the mass × acceleration of the element perpendicular to x. If the mass per unit length be ρ this gives $\rho \, (d^2y/dt^2) = P \, (d^2y/dx^2)$ as the equation of motion.

Let us now compare these equations and the quantities contained therein with those of our electromagnetic system.

Electric System	*Mechanical System*
Φ = total flux at a point.	y = displacement.
$d\Phi/dt$ = time rate of change of ϕ.	dy/dt = time rate of change of y.
$d\Phi/dt = E$, electromotive force.	$dy/dt = V$, velocity.
$d\Phi/dx = B$, magnetic density.	$dy/dx = y'$, slope of the curve.
$d^2\Phi/dx^2$ = rate of change of density.	d^2y/dx^2 = curvature.

We have seen that the following equations hold

$(1/\mu)(d^2\Phi/dx^2) = \Delta$, current per unit length.

If we assume that

$$\Delta = K\,(dE/dt)$$

our circuit being closed through a condenser, we get, substituting for E,

$$(1/\mu)(d^2\Phi/dx^2) = K(d^2\Phi/dt^2)$$

two perfectly analogous equations. Also

$E\Delta$ = power at point x.

$P\,(d^2y/dx^2) = F$, force perpendicular to x per unit length.

Putting $F = \rho\,(dV/dt)$ = rate of change of momentum, we get, substituting for V,

$$P\,(d^2y/dx^2) = \rho\,(d^2y/dt^2).$$

FV = force $\times$ velocity = power at point x.

Thus there is a complete formal analogy between the two cases which we shall find helpful as we go on. For instance, no abrupt change or discontinuity can take place in the flux Φ owing to the fact that magnetic lines are always closed, and similarity since we suppose our cord is continuous, there can be no abrupt change in the displacement y.

We have seen that whether we consider the mechanical or the electromagnetic case, we arrive at the equation $d^2y/dx^2 = c(d^2y/dt^2)$.

The solution of this equation, subject to the condition that when $x = c$, y is a pure sine function of t, is

$$y = A \sin\,(mx + pt) + B \sin\,(mx - pt),$$

A and B being arbitrary constants.

Two terminal conditions are necessary, and two only, completely to determine the constants A and B. These may conveniently be values of y at two different values of x. It follows, therefore, that a knowledge of the values of y for two given values of x completely determines y for all other values between the two given values.

In the case of the vibrating cord this is easily seen from the physical standpoint. If we know the motion of the two ends of the cords, the differential equation determines the motion of any intermediate point.

Similarly, if we know the flux entering the two ends of our apparatus we can tell how it is distributed at intermediate points.

If, however, we know the motion of more than two points of our cord we must divide it into a number of sections, according to the number of points whose motion is known. For instance, if we have a cord whose motion is known at the points A, B, C, D, E, F, we must regard the sections AB, BC, CD, DE, and EF as separate cords vibrating independently (see Fig. 26).

If we suppose that at the point B the cord is clamped to some
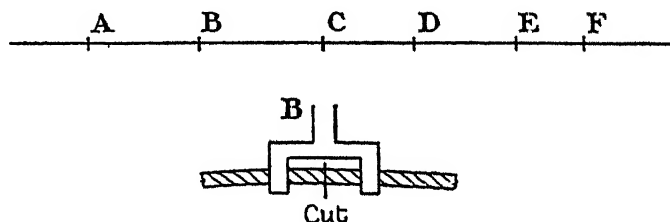
Fig. 26

support which vibrates with a simple harmonic motion or otherwise, as in the figure, it clearly makes no difference, so long as the cord is clamped at the given point and submitted to a forced oscillation whether we cut it or not. If, therefore, we regard the motions of B and C, say, as known, and regard the cord as cut at these points, the motion of the remaining parts of the cord makes no difference whatever to its motion between B and C. In fact, we can remove the remainder altogether without affecting the part between B and C.

The above is the physical view of the fact which appears mathematically in the statement that two terminal conditions completely determine a particular solution of our equation.

The net result of the above discussion, then, is the following statement.

If at any two points a cord (or electromagnetic system) is subjected to any forced motions whatever, the motion of the cord is completely determined between the two points, and is absolutely independent of the motion of any other part of the cord.

Since, in practice, our electrical system will usually have its conductors disposed in a number of separate slots, it will be convenient in some cases to imitate this by considering a chain having a number of links of definite length instead of a flexible cord. The

chain or cord is supposed to vibrate in a resisting medium producing a decrement.

The analogue of our electrical system is then the following.

The cord or chain is supposed to vibrate between two sliders, whereby the ends are held and a certain tension imparted. No
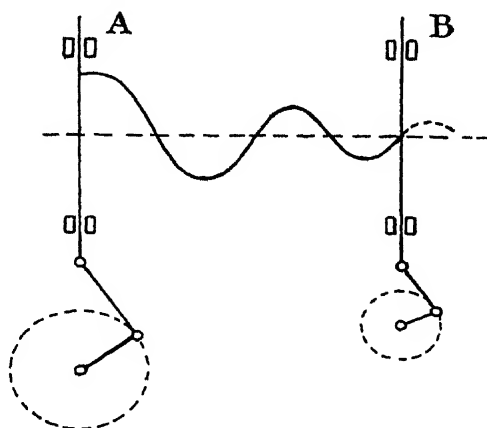
FIG. 27

matter how great this tension the sliders are supposed to be without friction or mass, unless otherwise stated. If the slider at one end is set in simple harmonic vibration, say, by a crank and connecting rod mechanism as in Fig. 27, while the motion of the cord is damped
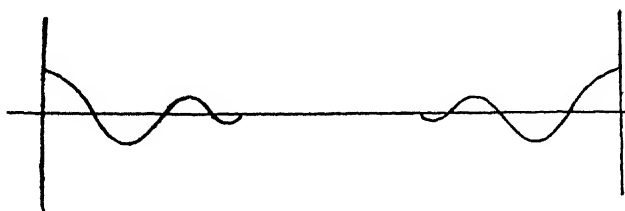
FIG 28

by friction, a decreasing wave will travel from the end in forced motion to the other end.

In the general case both ends may be in forced motion, and in this case the left-hand oscillator produces a wave flowing to the right, and the right-hand oscillator a wave flowing to the left.

This is readily seen in cases where the damping is so great that each wave dies away before the other commences (Fig. 28), but even if this is not so the two waves will exist superimposed on one another. The amplitudes of the two waves will be such that the sum of their amplitude at the two ends is equal to the known amplitude of the forced oscillation at every instant.

Suppose, for instance, that the left-hand oscillator A (Fig. 27) produces a wave travelling to the right, and that the cord does not terminate at B but extends as shown dotted to an unlimited distance. Then the slider B will be set in oscillation in a certain definite manner, depending on the amplitude and frequency of A's oscillation and the mass of the cord together with the amount of damping.

Suppose now the slider B to be kept in a forced state of motion, by means of crank mechanism or the like, which is exactly the same in frequency, amplitude, and phase as that which it had before when it was free, while that portion of the cord shown dotted on the right is removed. The motion of the cord between A and B will be absolutely unchanged, the crank mechanism now supplying the force previously supplied by the reaction of the part of the cord shown dotted on the right.

Under these circumstances, the wave generated by the oscillator A is said to be completely absorbed at B. Complete absorption can only take place for a single particular amplitude and phase of the oscillation at B, viz. that determined by the wave motion due to A. If the motion of B differs from this, a second wave as mentioned above will start from B, travelling towards the left and having an amplitude at B at every instant equal to the difference between that of the wave determined by A at B and the forced amplitude of B. This differential wave is usually spoken of as a " reflected wave " when regarded with reference to A, while the wave travelling towards B and away from A is called the " incident wave."

Two particular cases of reflected waves are of special interest

1. The case in which B is permanently fixed and its amplitude is zero. The wave due to A is then said to be completely reflected at B. In this case the forced amplitude of B is zero, and the magnitude of the reflected wave according to the above rule is equal to the difference between zero and that of the wave determined by A. In other words, the incident and reflected waves are equal and opposite at B, their resultant being, therefore, zero.

2. The case in which the slider at B is quite free, the cord being supposed to terminate there. This must be carefully distinguished from the case discussed above where the slider was quite free, but the cord (shown dotted in Fig. 27) was supposed to extend to infinity. As mentioned above, the forces due to the reaction of the right-hand or dotted section of the cord serve to modify the motion of the cord between A and B, which is, therefore, different from that of a cord terminating at B, at which point, therefore, the force acting on it is zero.

We saw above that the force on our elastic cord was proportional to its curvature, and it is easy to see that the curvature of an elastic cord naturally straight will be zero at a free end, but not

zero at all times at an intermediate point of an indefinitely long cord.

The following considerations will help us very much to obtain clear ideas of the motion of a cord having a free end—

The waves we have hitherto been discussing have been waves of displacement from a mean position which the cord would have if there were no motion. Since the force acting on the cord at any point is proportional to the curvature of the cord, such waves of displacement must be accompanied by waves of force of the same general shape, but differing in magnitude and phase. Such a curve of force distribution is plotted in Fig. 29. and has a fixed phase relation to the wave of displacement so that it moves at the same speed.

Instead of considering waves of displacement, therefore, and deducing therefrom the values of the force at any point, we may
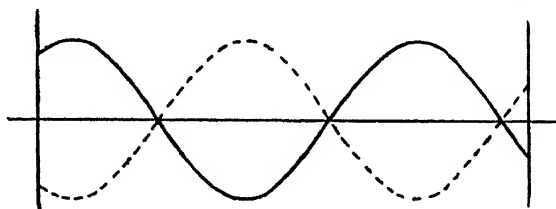
Fig. 29

consider waves of force directly and deduce therefrom the displacement at any point. If we do this, the present case in which the slider *B* is free is reduced to the former one, since the force at *B* is now zero, whereas before, the displacement was zero. Hence, everything said about Case 1 applies to Case 2, if we interchange the two words " displacement " and " force." In other words, we may state the following theorem-

The two cases of a cord carrying wave motion and fixed at one end, and that of a cord carrying wave motion and free at one end, are reciprocal to one another, the force in the second case corresponding to the displacement in the first.

Hence, if the end of the cord is free at *B*, and the force, therefore, zero, we can decompose the force wave into incident and reflected waves which are equal and opposite at *B*, their resultant being zero. Thus, in Case 1 the displacement is completely reflected at *B*, and in Case 2 the force is completely reflected. Similarly, if the force used to drive the slider at *B* be equal to that exerted by the incident wave, due to *A* and *B*, we have a case where the force wave is completely absorbed.

Since, as we saw above, force corresponds to current and velocity to electromotive force in the case of our transmission, or, conversely, force to electromotive force and current to velocity in the

cable, we may compare the cord having a free end with a cable having its end earthed where the electromotive force is zero but the current free, while the cord with the fixed end we may compare with the cable whose end is insulated, so that the current (velocity in the mechanical case) is zero, but the electromotive force free.

We have now carried the mechanical case of our general physical theorem to a point where we may usefully employ it further to elucidate the electromagnetic case.

We may compare the primary or transmitting coil to the oscillator *A*, and the receiving coil to the oscillator *B*, which absorbs
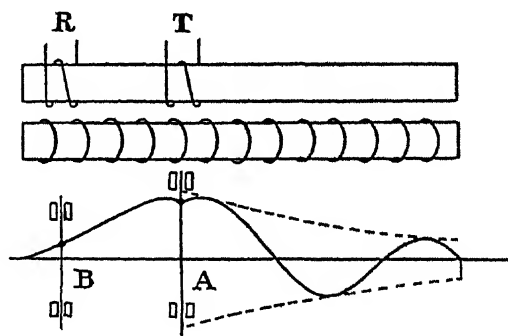
FIG. 30

wholly or partially the oscillations transmitted from *A*. If work is done by *A* on the cord in setting up the oscillation, an amount of work may be given out by *B* equal to that put in, less the work done by the cord on the resisting medium. If the work done by the cord is equal to the amount put in at *A*, then the wave dies away before it can reach *B* and *B* can do no work. Exactly the same remarks apply to the electrical case.

We saw that it was necessary to consider the flux at the two ends of our transmission to be the same on account of its annular nature. To parallel this we must consider the displacement of the two ends of our cord to be the same, the fact that all magnetic lines are necessarily closed being parallel by considering that our cord is unbroken and forms a loop.

Let us now parallel Fig. 25 by Fig. 30, showing the exact relation between the cord and the electromagnetic case. The transmitting coil or primary is now compared with *A*, a slider which is vibrated with a large amplitude, generating a wave which travels to the right around the loop, and reappears on the left of *B*. It is there absorbed by *B*, an oscillator vibrating with small amplitude, and passes on to *A* again to be re-transmitted round the loop. The distribution of energy density in both cases is exactly the same, the object of the two oscillators being, as before, the absorption of

a wave of small energy and the emission of a wave of much greater energy.

In both the above diagrams we have shown the two oscillators as being each concentrated at a single point of the system, the motion of the wave being of the same general type (i.e. consisting of an incident and a reflected wave) in both the two intervals between A and B. If, however, B is adjusted to absorb the wave travelling from A towards the right, it cannot absorb that travelling towards the left as well, and, hence, this wave will be largely reflected.
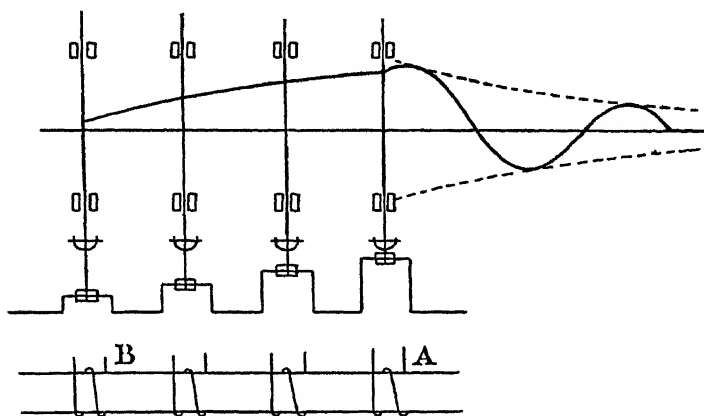
Fig. 31

Another arrangement which we may adopt in our cord and which has useful electromagnetic analogies is the following—

Between A and B we have a number of intermediate oscillators of varying amplitude and phase, typified by the multi-crank mechanism of Fig. 31, which serve to guide the cord and fix its oscillation at every point between B and A. This converts the wave between B and A into a forced wave, while it still remains free in the other interval between A and B.

In this case it is perhaps better to think of the cord as a chain fixed to the sliders at the junction between two rigid links.

In the electric case we think of the member carrying the transmitting and receiving coils as being slotted, and carrying a number of intermediate coils excited by electromotive forces of differing amplitude and phase—carrying, in fact, an unbalanced or elliptical polyphase winding.

It is clear in the case of the cord that A and B must not be placed too close together owing to their different amplitudes of movement, or there will not be sufficient cord to allow them to execute their vibrations independently.

Similarly in the magnetic case, we must not put A and B too close together or the density between them (slope of the cord) will become too great. Subject to these conditions, however, they should be placed as close together as possible.

ROTATION

We have now to consider what occurs when the apparatus is in motion. As the result of the investigations given above, we find that when the secondary is in motion relatively to the primary the wave is carried along in the direction of motion. Thus, if the motion of the wave is in the direction of motion of the secondary, its decrement is reduced by the motion, since when the secondary is in motion the wave is carried a greater distance while decreasing by a given amount. If the direction of the wave is opposed to that of the motion, its decrement is increased since the wave cannot travel so far while decreasing by a given amount. The decrement in fact is proportional to the slip between the motion of the wave and that of the rotor.

We also find that in the absence of condensers or commutators, i.e. of means for self-excitation, the motion tends to smooth out the ripples in the wave, i.e. to increase its wavelength, and this to an equal degree whether travelling with or against the motion.

Fig. 10, on page 19, summarizes the characteristics of the wave motion in a similar manner to the circle diagram that is in such common use for polyphase induction motors.

It will be remembered that the equation of this curve is

$$S^2 = a^2 + m^2 = (p\mu/r_2) \cot \theta \qquad (p\mu/r_2)(a/m),$$

if the flux wave be represented by

$$\Phi = \Phi_0 e^{((a + jm)x + jpt)}, \text{ giving } B = (a + jm) \Phi.$$

We saw above that the primary ampere-turns are equal to the sum of all those in the closed secondary circuits, supposed to extend to infinity in one direction only, say, to the right, from the primary at $x = 0$. We shall not at first take account of conductors on the left of the primary, but, having dealt with those on the right in the first place, shall find no difficulty in taking account of the others later on.

Since $\Delta = (1/\mu)(dB/dx)$,

$$\text{total secondary ampere-turns} = \int_0^\infty \Delta dx = \frac{1}{\mu} \int_0^\infty \frac{dB}{dx} dx$$

= the value of B when $x = 0$, since B is zero at infinity.

Hence the value of B at the origin is proportional to the current in the primary coil, and may be taken to represent it both in magnitude and phase. We have just seen that $B = (a + jm) \Phi$ and

$E = jp\Phi$. Hence, if we lay out Φ_0, the flux threading the primary coil, as a horizontal vector in the above diagram, E will be represented by a vertical vector, and OP or $a + jm$ will represent the current in the primary coil.

This diagram not only informs us as to the values of a and m at different speeds, but it also gives the current and power factor in the primary coil. It is, in fact, a complete epitome of the wave motion at all speeds.

It will be remembered that we arrived at the following constructions of the principal quantities involved—

If OP is a vector to the curve, $S^2 = (p\mu/r_2)\cot\theta$ and SP_1 is the hyperbola, $xy = p\mu/(2r_2)$ intersecting it in S. Then YP_1 is the speed of the wave; PP_1 is the speed of the rotor to the same scale; $YP = a$ is the rate of decrement and also the slip of the rotor relatively to the wave; OP is the primary current; $OY = m$ is proportional to the number of poles per unit length; UT is the torque due to the wave.

It should be mentioned that the speed v is taken positive when in the side we are considering the rotor is moving away from the primary coil, and negative if it moves towards it. The wave always flows away from the coil.

By way of an example we may elucidate the application of such a diagram by considering how with its aid we may calculate the wave motion, due to a single ring-wound primary co-operating with a short-circuited secondary, and wound on a core of such diameter and constants that the wave motions in both directions die away before they interfere with one another.

First of all, from the dimensions of the apparatus, the cross-section of bars, core length, and air-gap we calculate the two constants r_2 and μ, and thence $p\mu/r_2$. These known, we proceed to plot the two curves mentioned above. We have next to determine the scale to which YP_1 represents the synchronous speed of the wave $v_0 = -p/m$. Since $OY = m$, we can readily calculate v_0 for any value of OP and the corresponding length of YP_1, and so determine the scale.

If we suppose the rotor (Fig. 5) to revolve in a clockwise direction, then on the right of the primary it will be moving away from it and on the left towards it. Hence, OP corresponding to v positive, or moving with the wave, is correct for the disturbance on the right, and OP_2 (v negative) for that on the left. The first thing which we notice is that the ordinate m determining the speed of the wave is the same in both cases, and, hence, that the wave flows away from the primary coil at the same speed in both directions. But the decrements vary very much, being, as stated above, proportional to the slip between the wave and the rotor, and, therefore, being very great when v is negative, and very small when it is positive.

The vector OP, as has also been mentioned above, represents the total current required in the primary to supply the wave motion on the right-hand side of it from the primary coil to infinity, while OP_2 represents the current required for the motion on the left-hand side. The total current in the primary is, therefore, proportional to the vector sum $OP + OP_2 = 2OP_1$, and, therefore, to another scale the hyperbola $xy = p\mu/(2r_2)$ is the locus of the primary current as we vary the speed.

It has also been shown (Fig. 10) that the torque due to such a wave motion is proportional to the intercept between the point U and the point T, where OP (or OP produced) cuts a horizontal line US. In the case which we are studying, the resultant torque will be the difference between the "motor" torque UT, due to the wave which flows in the same direction as the rotor, and the much larger back torque UT', where T' is the point where OP produced cuts US, due to the wave flowing in the opposite direction. The resultant torque is always opposed to the direction of motion, and the apparatus is a brake absorbing large lagging currents in the primary, the RI^2 losses being supplied by generator action within the machine.

The effects of various modifications of the constants of the apparatus on the diagram are fully investigated above, such as the effect of including inductance or capacity in the rotor circuits, or of winding the stator with auxiliary coils in addition to the exciting coil, closed through inductance or capacity. However, as the case we have discussed is the most important one, it is unnecessary to recapitulate the others.

Having discussed fully the effect of a single ring-wound exciting coil on a short-circuited rotor, we can readily proceed to consider two such coils spaced a certain distance apart and carrying equal and opposite currents. These two coils are equivalent to a single drum-wound coil. Having discussed one drum-wound coil we can then proceed by the method of super-position to build up the effects of a number of drum-wound coils, thus arriving finally at the exact occurrences which take place in a slotted stator of the ordinary type when used with a squirrel-cage rotor. These will often be found to differ quite widely from those which would be expected from the ordinary sine-wave theory.

We may again illustrate these facts by reference to a vibrating cord. As one of the most important points we have to study is the results they lead to when limited by certain terminal conditions, we shall first suppose our cord to be fixed at both ends.

The question we have to discuss is : What will be the exact character of the motion if the decrement is so small that very little of the wave energy is absorbed before it reaches the reflector, subject to the condition that the decrement of the reflected wave is

much greater than that of the incident wave, while their wavelengths are the same ?

If the oscillations are so slow that the eye can follow them, or if they are observed by stroboscopic means, we shall see a wave starting from the oscillator at a definite rate of travel towards the right. As it proceeds this rate gets slower owing to the effect of the reflected wave, and finally, when it reaches the reflector, it becomes stationary (see Fig. 32). On the left the reflector or fixed point may be placed so close to the oscillator that the standing portion of the wave extends practically up to the oscillator, or better still, we have a forced wave between reflector and oscillator, whereupon we get practically the condition shown in Fig. 23.

In Fig. 32 is shown an electromagnetic case of such a system in which the incident wave has a small decrement, and the reflected wave a very great one. This, as we have stated, is characteristic of the case where there is relative motion between the primary with its reflector and the secondary.

We see that in this case the wave proceeds much as before till it reaches the immediate neighbourhood of the reflector,

Fig. 32

beyond which it cannot pass, and since all the lines of force must be closed they are crushed into a very small space adjacent to the reflector, giving rise to very high densities indeed at that point. In practice, saturation would somewhat relieve these densities.

Flux and density curves are shown above and below, in the upper curve the small decrement of the incident wave and the great decrement of the reflected wave (shown dotted) being clearly seen. The resultant wave is the sum of these at any point.

It is easily seen by the aid of the general physical theorem with which we started that an increase of density in the direction of
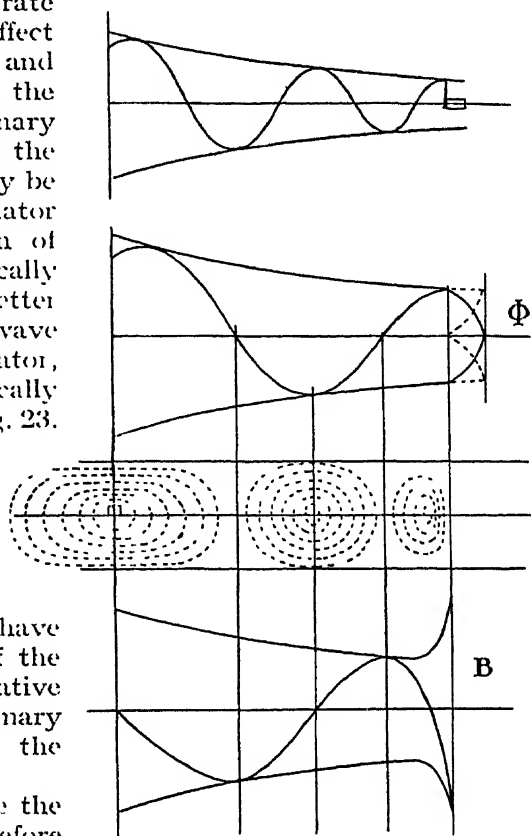
motion gives rise to a retarding torque, which in a motor should be avoided.

We shall now endeavour to get a clear physical idea of the method of generation of wave systems, both in the electric and the mechanical case. We shall take the case of a cord or iron core of indefinite length, oscillated at one point. In the mechanical case we may suppose the cord gripped at one point, and a simple harmonic oscillation impressed on it by a crank and connecting rod mechanism as shown. In the electromagnetic case a concentrated exciting coil is supposed wound round one point of the core, and this is excited by a simple harmonic electromotive force (see Fig. 33).

In Fig. 34 are shown diagrams of five phases, illustrating the generation of two travelling waves, mirror images of one another,
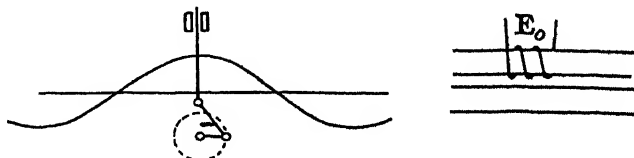
FIG. 33

by this method of setting up oscillations, whether in the electric or mechanical case.

The upper curve shows flux Φ or displacement y, the second magnetic density B or slope y', and the lower diagram shows the distribution of lines of force corresponding to the flux and density above them.

We see that at the origin or point of oscillation there is a cusp in the Φ curve, though Φ does not change in value, since all magnetic lines are closed, and an abrupt break in the B curve where B changes abruptly from positive to negative. Let us confine ourselves to the electromagnetic case for the moment. We saw in Chapter II that $(1/\mu)\,(dB/dx) = \Delta$, the current density, and, therefore

$$\int_a^b \Delta\,dx = \frac{1}{\mu}\int_a^b \frac{dB}{dx}\,dx = \frac{1}{\mu}\,(B_b - B_a)$$

was the total current between $x = a$ and $x = b$. In general, as a and b approach one another B_b approaches B_a, and thus the total current gets less and less.

Now let us take a and b on opposite sides of the discontinuity of Fig. 34. As a and b approach one another $B_b - B_a$ no longer approaches zero, but tends to assume a value equal to the total length CD of the break. Hence, we conclude that

Total current at the point of discontinuity $= (1/\mu)\,CD$, or

The discontinuity in the curve of densities measures the current in the exciting coil.

In the mechanical case we find—

The discontinuity in the curve of slopes measures the force supplied by the oscillator.

As we saw above at points of discontinuity such as A, we may suppose that one section of the cord terminates and another begins. In that case AC is the force necessary to balance the reaction of
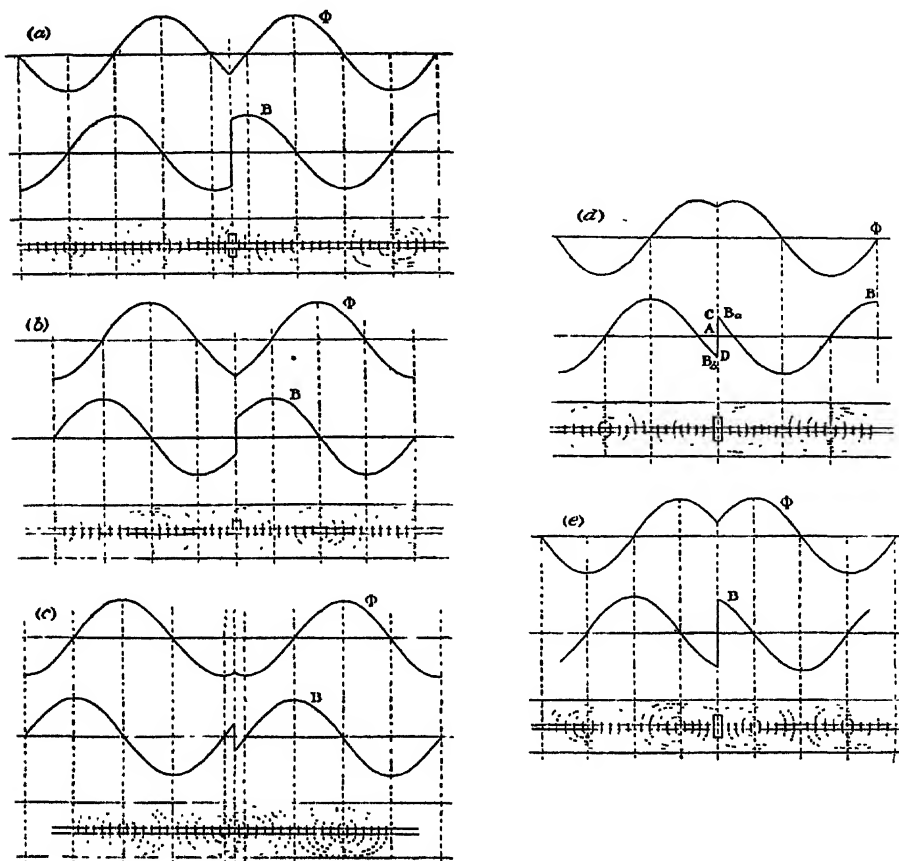
FIG. 34

the rope on the right of A, and AD that necessary to balance the reaction on the left of A. In the electric case AC is the current necessary to excite the disturbance on the right, and AD that necessary to excite it on the left.

Returning to the curves of Fig. 34, we may note a number of interesting points—

1. Starting with Fig. 34(a) we see the lines of force which

form the curve start with a small nucleus surrounding the exciting coil.

In Fig. 34(*b*) the nucleus has grown almost to its full proportions, while in Fig. 34(*c*) the single nucleus we have hitherto been dealing with begins to become double, some of the lines of force on either side forming a closed circuit which does not enclose the exciting coil. In Fig. 34(*d*) this action has proceeded much farther, while in Fig. 34(*e*) our original nucleus has almost divided into two entirely distinct waves. Coming back to Fig. 34(*a*) again, a fresh nucleus starts which is now of opposite sign, there being two to each complete period, one positive and one negative.

2. It is instructive to see that when the flux interlinking the exciting coil is greatest in Fig. 34(*c*), the primary current (*I*) is least. This current is in quadrature with the flux, and, therefore, in phase with the electromotive force.

3. It is also very instructive to note how the direction of the lines of force reverses when the flux crosses the zero line, so that the portion of the flux curve between two crossings of the zero line forms an isolated portion or nucleus of the wave.

Let us now study a case of a wave motion reflected at a point. A point of reflection is a point such that the displacement which is represented by a wave is necessarily zero at the point. If we have a travelling wave going towards a point it will be reflected at the point, and another travelling wave superposed on it of such magnitude that the two are at all instances equal and opposite at the point of reflection. The resultant vibration of the cord is a standing wave having its node at the point of reflection, and extending from the point of reflection where the displacement or flux is zero to the point of oscillation but not beyond.

In the case of a vibratory cord, a point of reflection is formed at any point of the cord which is clamped so as to be incapable of movement, *y* being necessarily zero at all times.

In the electromagnetic case, a point of reflection is formed by a short-circuited coil surrounding the iron core, or by a termination thereof.

In Fig. 35 are shown five stages in the generation of a wave train by an oscillator fitted with a reflector. For the sake of variety we have shown the generation of a wave with logarithmic decrement in this case.

Starting, for instance, with Fig. 35(*a*) we see the flux at its maximum surrounding the exciting coil.

In Fig. 35(*b*) we see that the wave has begun to travel towards the right, and some of the lines of force which before surrounded the exciting coil are now able to close without doing so. In the portion of the core between the reflector and the oscillator, however, the density is simply reduced without any change in distribution.

In Fig. 35(c) we see that this action has gone farther, the greater part of the lines now closing without surrounding the exciting coil.

In Fig. 35(d) the whole of the flux is one side of the exciting coil, there being none whatever between oscillator and reflector; while in Fig. 35(e) we see how the nucleus of a new wave grows up surrounding the exciting coil again, and when it has grown to its full
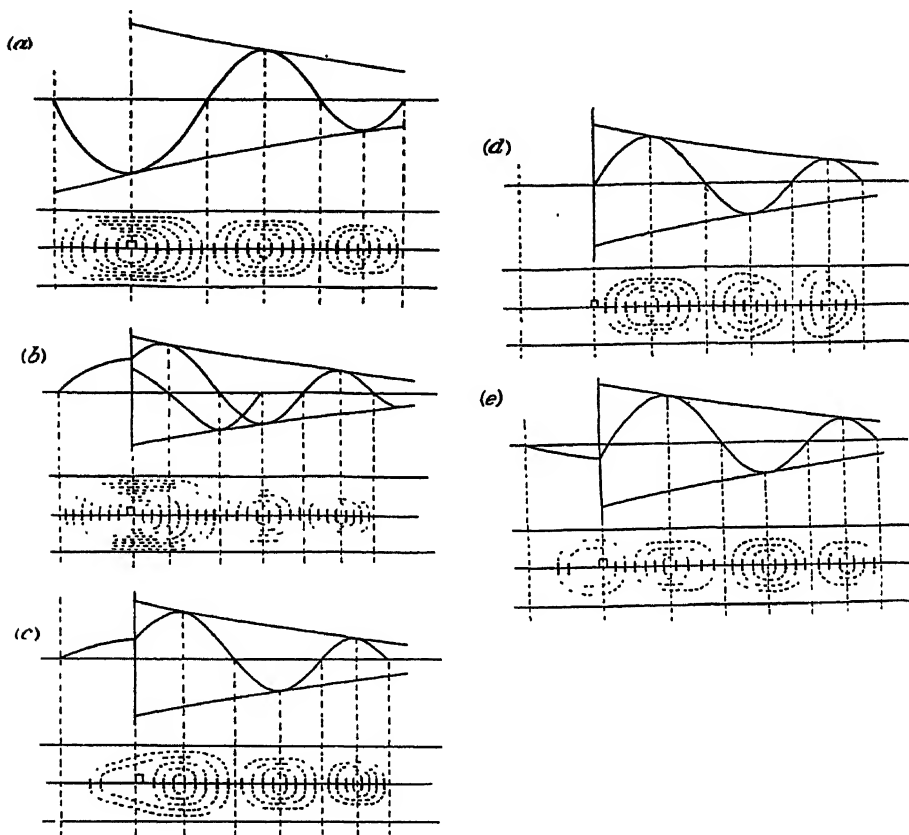
Fig. 35

dimensions in Fig. 35(a) again, it commences to move off to the right, the lines of force of which it is composed cutting the exciting coil as they do so.

Another important point to note in the case of a wave with decrement is that its maximum, which is the point where the density is zero, is now no longer midway between the two points at which it crosses the zero line.

The effect of this is to make the density on the rear side of the nuclei of which the wave consists greater than that at the front,

and this generates a drag on the conductors in the direction of motion of the wave. The power corresponding to the product of this force and the speed of the wave is stored up as magnetic energy in the wave and supplied by the decrease of the flux as it proceeds.

We have hitherto considered wave trains proceeding from an oscillator whose oscillations were of constant amplitude, or in the
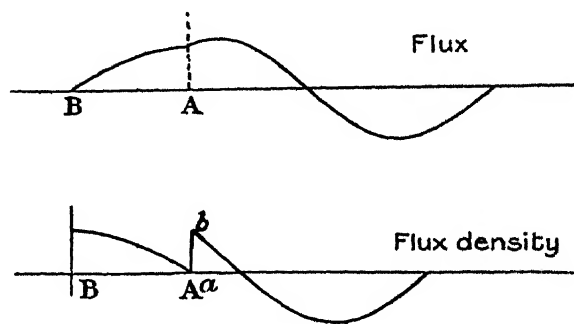
Fig. 36

electric case we have considered cases where a constant flux or electromotive force was applied to the oscillator, and from this assumption we have determined the force (or current).

We shall now consider the case where the force (or current) is
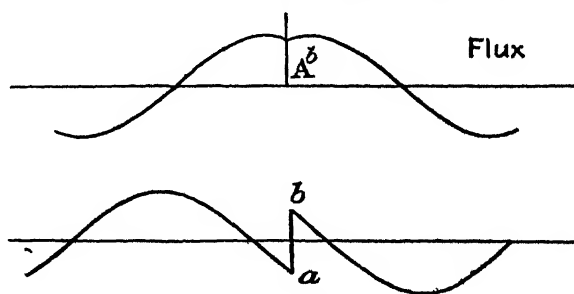
Fig. 37

regarded as fixed, and endeavour to determine the amplitude (or flux in the electric case).

Let us first take the case of an oscillator at A and a reflector at B a quarter wavelength away from A, and compare it with the case in which no reflector exists. The effect of the reflector as we saw above was to reduce the wave train between A and B to a stationary wave having its node at B. If we draw curves showing the flux density, we note that the flux density at A due to the standing wave is always zero (see Fig. 36). If now we draw corresponding curves for the case where there is no reflector, we see that the density at A due to the wave train on the left is no longer zero, but **equal and**

opposite to that due to the wave train on the right (see Fig. 37). Now on the hypothesis of constant current the distance *ab*, in the two figures, must be equal, and, hence, the amplitude of the right-hand wave train must be twice as great where there is a reflector as where there is none.

Thus a given force applied to the oscillator (or current flowing in the primary coil) gives rise to a wave of displacement (flux) having twice the amplitude in the case of a reflected wave of that obtained when we have two wave trains, mirror images of one another. In other words, it takes twice as much power to generate two wave trains as it does to generate one of the same amplitude.

In the case just considered we may regard the oscillator as generating two wave trains in the normal manner, the left-hand one of which is reflected at *B* and, returning on its path, passes the original oscillator and becomes superposed on the right-hand train, thereby doubling its amplitude. We have here a case in which the occurrences on the right-hand side of the oscillator *A* are affected by what occurs in other portions of the cord. In other words, the different sections into which the system is divided by the oscillator and reflector are no longer independent of one another.

In fact, the above instance is only a particular case of an important general theorem.

If we know the displacement (flux or E.M.F.) at two points of our system, the disturbance between those points is entirely independent of what goes on beyond them. If, on the other hand, we assume the forces or currents to be known, the various oscillators in the system are quite independent, their effects being merely superposed.

It is this theorem that permits us to build up the flux distribution in a slotted stator from a knowledge of that due to one drum-wound coil.

CHAPTER VIII

IN order that our investigations may have the requisite degree of generality, and their results may be worthy to be called a general treatise on " Electric Mechanism," it is absolutely necessary that we should consider the theory of the electrostatic machine, the more especially as, with the exception of a very meagre discussion in Maxwell, I cannot find that these machines have ever received theoretical treatment at all.

Before doing so, however, it is first of all essential that we form clear ideas on the nature of electrostatic induction. We have in the electrostatic machine two plates carrying conductors revolving in close proximity to one another with only a short air-gap inter vening, so it may be thought that electrostatic induction was to be
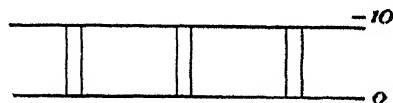
FIG. 38   FIG. 39

explained by the cutting of the electrostatic lines by the revolving conductors. This opinion is entirely erroneous, and our first task will be to explain as far as necessary the nature of electrostatic induction.

The fundamental conception with which we have to deal in the present chapter is no longer the magnetic line of force, but the electrostatic tube of induction.

We conceive a tube of induction (Fig. 28) as always terminating on two oppositely charged conductors, while at the one end a unit charge of positive electricity exists, and at the other end a unit charge of negative electricity. The number of tubes impinging on any conductor, therefore, is a measure of the charge on it. The direction of the tube at any point is the direction of the potential gradient at the point, i.e. it is the direction in which the difference of potentials between two points, near the given point and a given distance apart, is greatest. Thus, by plotting out the distribution of the electrostatic tubes of force, we may determine the potential distribution throughout the field.

Another method of defining the direction of the tubes of force is by saying that they are everywhere perpendicular to the equi-potential surfaces. These equi-potential surfaces are surfaces drawn in space, such that at every point of them the potential is the same. Thus, for instance, in the case of an electrified sphere all concentric spheres surrounding it are equi-potential surfaces.

54

Both electrostatic tubes of induction and equi-potential surfaces are fully discussed in the regular treatises on electrostatics, and so need not be more fully described here.

Imagine two condenser plates A and D, say, 10 cm. apart (Fig. 40), having a density of $D = 100$ tubes per sq. cm. throughout their area. Let them both be insulated and maintained at a constant difference of potential. Let there be a third plate C capable of reciprocating between them touching the plate D by means of the knob D, and being connected to earth by means of the knob E. We shall suppose that when C touches D it becomes completely discharged.

First suppose it connected to earth by E. It will now receive a charge DA equal to that of the plate A. Now suppose it to be moved towards the plate D carrying its charge with it, mechanical work being done by stretching the tubes of force, proportional to the distance L between the plates. When C touches D it is supposed to be completely discharged, and all the tubes of force which previously ended on it now end on D. If the positive ends of any number of tubes of force rest on it an equal number of negative ends must also rest on it, since it is now

FIG. 40

uncharged. Thus, the tubes of force may be supposed to pass completely through it, as shown in Fig. 40. The movable plate now moves back till it touches E again and is put to earth, whereupon those tubes of force which extended from its right-hand side to D move off to the walls of the room, say, and only those between A and C remain. The moving plate C is now charged again as it was initially, and we have completed the first cycle of operations. Now the work done in a cycle $= DA \times L$, in suitable units, and if there are N cycles per second we have—

Work done per second $= DLAN$.

This formula reminds us irresistibly of the common steam-engine formula,

$$\frac{PLAN}{33,000} = \text{H.P.}$$

where P = cylinder pressure
L = length stroke
A = area of piston
N = strokes per minute
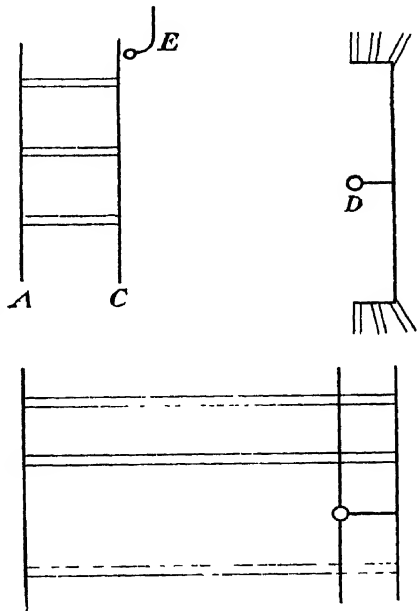
We must note that our electrostatic formula was deduced on the assumption that the potential difference between A and D was maintained constant, while the steam-engine formula depends on the cylinder pressure remaining constant.

From a more purely electrical point of view, we may interpret the formula $DLAN$ as representing the current flowing away from the plate A. For DA is the charge on the reciprocating plate which is supposed to be communicated to D in each cycle, while N is the number of cycles per second, L being the distance moved through per cycle. LN is the distance moved through per second; therefore,
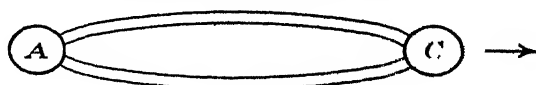
FIG. 41

$DLAN$ is proportional to the charge per second or current. It is clear that if the potential of D is to remain constant this current must be permitted to flow away by some means.

Without pursuing the analogy with the steam-engine any farther, we may note that electrostatic induction is due to the *stretching* and not the cutting of the tubes of force, and requires motion *along* and not at right angles to them. This is the fundamental difference between electrostatic and electromagnetic induction. Moreover, in the electrostatic field we have

$$\text{Rate of stretching} = \text{Induced current}$$

the potential of the conductor depending on surrounding circumstances, while in the magnetic case we have

$$\text{Rate of cutting} = \text{Induced E.M.F.}$$

the current depending on surrounding circumstances.

As an example of electrostatic induction of current, take the case in which a charged body C carrying, say, 2 units of charge moves with a speed of 10 cm. per second. Then the two tubes that join C to some fixed body A (Fig. 41) are stretched at the rate of 10 cm. per second, and the moving charge constitutes a current whose magnitude $=$ charge $\times$ velocity $=$ 2 units $\times$ 10 cm. per second, and, hence, the above equation is justified. It is, of course, a special version of Maxwell's equation of electric displacement adapted to the case we wish to consider. According to the electron theory all conductor currents are of the nature of moving charges, and, hence, this equation is very general.

Now the local current flowing *between* two sectors or carriers C_1 and C_2 (Fig. 42) will be the difference between the currents J_1 and J_2 flowing into each. If we had $J_1 = J_2$, for instance, the same current would flow into both carriers and the local current, which is

the current that flows into one and out of the other, would be zero.

In the limit we have

<p style="text-align:center">Current between two adjacent sectors $= dJ$.</p>

Now since C_1 and C_2 are insulated and do not move any current which flows in at J_1 and out at J_2, must be due to variation of the charge of C_1 and C_2 considered as a condenser.

Now the charge of C_1 and C_2 is the number, D, of tubes of induction, opposite ends of which terminate on both C_1 and C_2, and we

FIG. 42

have just seen that the current is due to the rate of change of this quantity or $\dfrac{dD}{dt}$. Hence, we have

$$dJ = \frac{dD}{dt}dx$$

The dx is used in the above equation because, for infinitesimal intervals, the difference between J_1 and J_2 will be proportional to the distance dx. Hence, finally, we have

$$\frac{dJ}{dx} = \frac{dD}{dt} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

for all cases where there is no motion, and the plate is a perfect insulator.

The general form which the electrostatic machine assumes in practice is that of a revolving plate of insulating or partially conducting material, usually bearing a number of carriers of metal on which the charges are induced. On this revolving plate bear certain brushes for the purpose of equalizing the potentials of different parts of the plate, collecting the charge, etc. This revolving plate is placed in the neighbourhood of certain charged bodies or inductors, whereby the charge is induced in it.

We shall consider first a carrier plate fitted with a number of carriers C, C, C, and with brushes b, b, b, resting on each carrier. In practice only a few brushes will be used, but, as at present we are aiming at complete generality, we must not distinguish some carriers as being connected to brushes and others not, but must suppose that all are fitted with brushes. Chapter X describes similar electromagnetic machines.

Suppose each carrier to be of unit area, and let J^1 be the conductor current flowing into any carrier. If, however, the plate is in motion, a current will flow between adjacent brushes, due to the actual mechanical transportation or convection of charge. If ρ be the charge per sq. cm. and v the speed, this current will be equal to $v\rho$. Adding this term to the equation we had before, we get

$$\frac{dJ}{dx} = \frac{dD}{dt} + v\rho \qquad . \qquad . \qquad . \qquad . \qquad . \qquad .. \qquad (2)$$

If also we suppose the carrier plate to have a certain finite resistance r, we must add a further term, getting

$$\frac{dJ}{dx} = \frac{dD}{dt} + v\rho + \frac{E}{r} \qquad . \qquad . \qquad . \qquad (3)$$

where E is the P.D. between adjacent brushes. This last term is important in explaining the action of that type of machine which is not provided with carriers.

We have now to investigate the relation between D and ρ. D is the resultant number of tubes of induction extending in the positive direction from one carrier to the next, while ρ is the charge on the carrier.

If in Fig. 42 the number of tubes extending between C_1 and C_2 and between C_2 and C_3 were equal, there would be no charge on C_2. Hence we see that

Charge on $C_2 = \rho = D_1 - D_2$, or in the limit,

$$\frac{dD}{dx} = \rho \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (4)$$

This we shall call the equation of Charging

Everything we have hitherto said has been applicable to the case of a single plate only. We shall now suppose that our revolving plate is surrounded by two fixed plates, one on either side, all fixed carriers which lie opposite one another being connected together as shown dotted (Fig. 43). D is now the number of tubes crossing any cross-section AA of the machine in the positive direction. Hence, tubes which merely pass from a stator carrier to the rotor carrier opposite to it do not count.

We still have the equation $\frac{dD}{dx} = \rho$, but it is now clear that the tubes of induction need no longer pass solely from one carrier on the rotor to another. They may now pass from a stator carrier to a rotor carrier, and this fact enables the machine to stretch the electrostatic lines and to do or absorb electrical work. Equations (1) and (2) still hold good, and may now be said to apply to the stator and rotor respectively.

Collecting up our results, therefore, we obtain the following equations—

$$\frac{dD}{dx} = \rho \qquad\qquad \frac{dH}{dx} = \Delta_o$$

$$\frac{dJ_1}{dx} = \frac{dD}{dt} + \frac{E_1}{r} \qquad\qquad \frac{dE_1}{dx} = \frac{d\beta}{dt}$$

$$\frac{dJ_2}{dx} = \frac{dD}{dt} + v\rho + \frac{E_2}{r} \qquad\qquad \frac{dE_2}{dx} = \frac{d\beta}{dt} + v\frac{d\beta}{dx}$$

which bear a most striking analogy to the equations of the electro-magnetic machine which are shown in parallel columns. The term $\frac{E}{r}$ in the electrostatic equations has no analogy in the magnetic case, since there is no such thing as " magnetic conduction," hence,
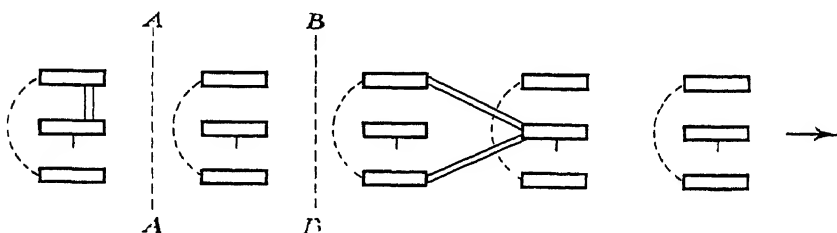
<p style="text-align:center">Fig. 43</p>

the class of machines in which it is important, viz. the " sectorless Holtz and Wimshurst machines," have no analogue in the magnetic case. They constitute a solution of the problem of direct current machines without commutators which, unfortunately, gives a voltage as much too high for practical purposes as that given by the homopolar machine is too low. Since electrostatic induction takes place in the direction of motion, and not at right angles thereto, it follows that electrostatic homopolar machines are also impossible. Making these two exceptions, however, we may enunciate the following fundamentally important proposition—

" To every multipolar electromagnetic machine there corresponds an electrostatic machine, and to every electrostatic machine with non-conducting dielectric corresponds an electromagnetic machine." Thus we may design electrostatic synchronous motors and generators, induction motors and generators, repulsion motors, and other A.C. commutating machines, and, in fact, a treatise on " Electrostatic Mechanism " might be written by merely applying the analogy which we have developed in the present chapter consistently to all the apparatus described in this book. Though the applications

of this type of machine are not at present sufficiently numerous to justify the time spent in so complete a development, and the writer has, therefore, not attempted it, yet it is to be hoped that it may soon be carried out, as it is relatively easy, and such an advance in theoretical knowledge could not fail to stimulate practical application.

PART II

CHAPTER IX

THE CLASSIFICATION OF THE DYNAMO-ELECTRIC MACHINE

In the following chapters an attempt is made to arrive at a natural classification of the electromagnetic machine. One of the principal uses of such a classification is to bring out the common points of different types which are usually regarded as totally distinct, even though they are almost identical from the constructional standpoint, and to reduce their differences to proper relative proportions. We may preface the detail matter by a few general remarks.

1. No discussion will be attempted of homopolar machines, which form a class entirely apart from all others.

2. While almost all the machines described may be used as generator or motor, they will, in practice, be used only as motors, since the requirements of a generator are so uniform that there is no scope for a large variety of types, while the infinite variety of industrial work gives rise to a corresponding variety of types of motors. In generators the tendency is to uniformity, and in motors to variety, and, hence, our subject might almost as well be called " The Classification of Electric Motors."

3. For a similar reason, viz. the powerful tendency towards standardization, and the necessity for one system of distribution catering to a vast variety of applications, together with the urgent necessity of economy due to the great length of transmission lines, we may dismiss from our minds the likelihood of new and more complicated systems of transmission arising, although several such are imaginable, and confine our attention to machines operating from standard direct-current, single, or polyphase systems.

4. Of the two distinct constructional forms in use, viz. the type having uniformly slotted stator and rotor and a constant air-gap all round, and the salient pole type, we shall consider the former as universal and ordinary, and the latter as a special case applicable to certain types only.

5. Throughout this chapter, unless a statement to the contrary is made, we shall treat our machines as " ideal," i.e. entirely devoid of all losses and leakages.

6. Theories come and go, while the machines we wish to classify remain the same. Our methods of classification should be of a fundamental nature, and should not owe their validity to the acceptance of a particular theory, nor should they be unintelligible

to those who have not gone through a particular course of study. They should be based either on constructional features, where these can be proved to have a general significance and not to be accidental, or on mechanical considerations in connection with torque, power, and energy. These considerations apply to nomenclature also, and should render us very conservative in inventing new names for old apparatus.

A well-understood name, even though based on an obsolete theory, is usually better than a new name based on a theory which may become obsolete in its turn.

Having cleared the ground by the above considerations, we may proceed to our main subject.

We shall regard the induction machine or " general alternating-current transformer," as Steinmetz calls it, as fundamental, and shall proceed to derive all other types from it.

Our general dynamo-electric machine, then, consists of two concentric magnetic elements, separated by an air-gap, and capable of relative rotation. Each of these elements bears a number of conductors disposed next the air-gap in slots parallel to the shaft. The conductors of one or both elements are interconnected usually in such a way as to give rise to a definite number of poles, conductors being led off from the stationary element direct to the line, save in a few cases where it is short-circuited.

The rotor may be of three distinct types : (*a*) short-circuited, (*b*) fitted with collector rings, (*c*) fitted with commutator. Every type of dynamo-electric machine whatsoever, with the exception mentioned above, may be built in a form covered by the above description, and the differences between different types consist merely of differences in the method of connection to the line and of connection between rotor and stator.

We have next to classify these types of connection and show why they are required. In order to do this we must first consider our electric system as a machine. From the mechanical point of view, then, the generator is a gearing which transmits the motion of the prime mover to the polyphase line, changing the power given out by the prime mover from the mechanical form to that of a rotary system of electrical stresses revolving at a definite speed, say, of 25, 50, or 60 cycles (or revolutions) per second. The polyphase line we may compare to a mechanical line shaft transmitting the power to a distance and the motor to another gearing, retransforming the power into a mechanical torque operating at a certain speed.

What will this speed be ? To determine this, consider the following relation which it is well known exists between the speed and primary and secondary frequencies of the induction machine. Speed (rev, per sec.) × number of pole pairs = difference between primary and secondary frequencies (cycles per sec.), or mechanical

speed of rotor equals speed of wave relative to stator, minus speed of wave relative to rotor.

or $$S = W(f_1 - f_2) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

where W is twice the polar pitch (cm.).

$\quad S$ = peripheral speed of rotor (cm. per sec.).

$\quad f_1$ = primary frequency.

$\quad f_2$ = secondary frequency.

These equations show that we cannot tell the mechanical speed of the rotor unless we know the rotor as well as the stator frequency. Multiplying both sides of this equation by the torque T, we get T multiplied by mechanical speed equals T multiplied by speed of wave relative to stator minus T multiplied by speed of wave relative to rotor, or, mechanical output equals primary electrical input minus secondary electrical output $\qquad . \qquad . \qquad (2)$

This equation shows that besides the mechanical power and the electrical input into the primary, there is a certain amount of power generated in the secondary proportional to the secondary frequency. The above simple considerations bring us to our first principle of classification. It has been emphasized above that the motor is a device for changing electrical power incoming from the line into mechanical power, and, hence, the existence of this secondary power raises a serious problem—what to do with it. We shall classify our machines first according to means adopted for disposing of the secondary power. A substantially identical principle for adjustable speed motors may be deduced from the equation (1) above.

This equation shows that the speed can only vary

1. If the primary frequency varies,
2. If the secondary frequency varies,
3. If the number of poles varies,

and we may classify adjustable speed motors according to which of these means is adopted to vary the speed.

Before proceeding further we shall develop a mechanical analogy between our induction machines and certain types of epicyclic or differential gears which is capable of being carried into considerable detail, and throws a great deal of light on the real nature of many arrangements of machines. This analogy enables us to abstract entirely from the electrical features of the problem, and consider power and torque alone. We shall show also, that if we compare the relative speed of the two elements of our machine to the speed in revolutions per second of one shaft of a differential gear, and the frequencies in the two members to the speed of the other two shafts, the above relation between speed and frequencies corresponds to

that between the speeds of the three independent shafts of the differential gear. Such a gear is shown in Fig. 45.

If the shafts A, B, and C' go at speeds of A rev. per sec., B rev. per sec., and C' rev. per sec., respectively, it is well known that C' will be the mean of A and B, or $C' = \frac{1}{2}(A + B)$.

If, therefore, we gear another shaft C to the shaft C', so that the speed of C is twice that of C', or $C' = \frac{1}{2}C$, we shall have

$$C = A + B$$
$$B = C - A$$
$$A = C - B$$

Or, in order to introduce a constant corresponding to the " number of pole pairs " in the electric machine, we may suppose B or A geared to another shaft by gearing of any desired velocity ratio.
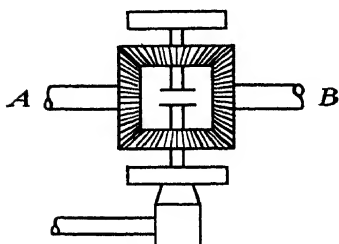
Fig 45

The point of particular interest to us about this gear is the fact that the equation regulating the speeds contains three potentially independent variables, so that it is necessary to know two of them before we can determine the third. For instance, if we know A we cannot find B unless we also know C.

In the electric case, if we know the primary frequency of an induction motor, for instance, we cannot tell the speed until we know the secondary frequency also. It will be useful to develop the analogy between a differential gear and an induction motor somewhat further. Let us put

Primary frequency $= A =$ speed of driver shaft of differential.

Mechanical speed $= B =$ speed of driven shaft of differential.

Secondary frequency $= C =$ speed of intermediate shaft of differential.

We then have $B = C - A$ in both cases, if we suppose our induction motor has two poles.

So long as C, the intermediate speed or secondary frequency, remains unsettled, we cannot tell what B will be.

We may make a number of suppositions relating to this which are tabulated below—

Electric Case	Mechanical Case
Secondary open-circuited.	*Shaft C free.*
Machine can deliver no torque, but can run at any speed without opposition, if driven.	B can deliver no torque, but can run at any speed without opposition, if driven.

Electric Case	Mechanical Case
Secondary short-circuited through zero resistance.	*C fixed.*
Machine can run at same speed as *A*, and deliver same amount of power as flows into primary.	*B* runs at the same speed as *A*, and delivers same amount of power taken in at *A*.
Secondary, neither open nor short-circuited but closed through a fixed resistance R, there being no secondary leakage.	*C, neither fixed nor free, but its power output consumed by a brake giving a torque proportional to the speed at which it is driven, and consuming power, therefore, proportional to the square of the speed.*

Electric Case:

Torque is proportional to i, the secondary current which is equal to the secondary E.M.F., divided by the resistance.

Secondary E.M.F. and, therefore, current is proportional to secondary frequency, or, say, $T_o = KC$ as in the mechanical case.

Thus we get

$$B = \frac{T_o}{K} - A$$

exactly the same torque speed characteristic as in the mechanical case.

Mechanical Case:

The torque of each of the three shafts must be equal, for, multiply the equation $B = C - A$ by the torque T_o required by the load, and we get

$$T_oB = T_oC - T_oA$$

This equation can only be consistent with the conservation of energy, if T_o is also the torque of the other two shafts.

In this case

T_oB = output of driven shaft.

T_oC = output of intermediate shaft.

T_oA = input of driver and the equation becomes true.

If we now put
Torque of intermediate shaft $= T_o = KC$ in accordance with the above assumption, we clearly determine the speeds of all three shafts, for we now have

$$B = \frac{T_o}{K} - A$$

This is the torque speed characteristic of our gear.

According to this analogy, therefore, we may compare any electric machine having a simple harmonic wave of flux to a differential gear, the primary input corresponding to the input of the driver shaft, the mechanical output corresponding to the output of the driven shaft, and the secondary output corresponding to the output of the intermediate shaft.

Hence, a set consisting of motor and generator may be compared to two such differential gears coupled together.

Now it is well known that no combination of gearing, however arranged, can give us a smooth and gradual speed change, the uttermost possible being a number of steps. Yet certain electric
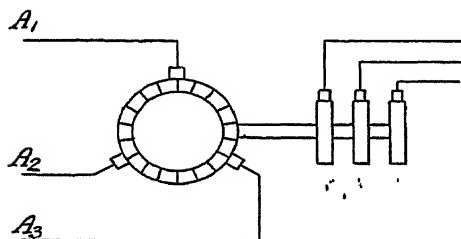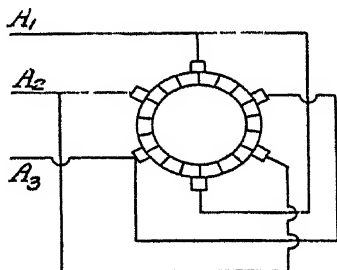
FIG. 46   FIG. 47

machines do give this gradual speed change, since they embody a device which we have not yet discussed, viz. a commutator. The commutator may be regarded as the only practical gradually adjustable gear in existence (suitable for large powers, at any rate).

Consider a commutator fitted with a polyphase arrangement of brushes, as in Fig. 46. If polyphase currents of frequency f_1 cycles per sec. are fed in through the terminals A_1, A_2, A_3, and the commutator revolves with speed S revs. per sec., the slip rings B_1, B_2, B_3, which are supposed to be equal in number to the commutator segments, one being joined to each, will deliver polyphase currents of frequency $f_2 = f_1 \pm S$, according to whether the commutator revolves with or against the direction of rotation of the inflowing current.

Hence, for the case of a " two-pole " arrangement of the brushes on the commutator, we have

$$S = \pm (f_1 - f_2)$$

This is particularly obvious if $f_1 = 0$. If, however, the brushes are connected " four-pole," for instance, as shown in Fig. 47, so that the slip ring which is connected to a certain segment, say, goes through a complete cycle while the commutator turns through 180° instead of 360°, the difference between the commutator and slip-ring frequencies will be twice as great as before, or $2S = \pm (f_1 - f_2)$.

In general, if $\dfrac{P}{2}$ be the number of pole pairs for which the commutator brushes are joined

$$S \times \frac{P}{2} = \pm (f_1 - f_2)$$

exactly the same equation as we had for the induction machine.

For a two-pole machine $\dfrac{P}{2} = 1$, and for a four-pole machine $\dfrac{P}{2} = 2$.

In the commutator frequency convertor the whole of the power flowing into the commutator flows out of the slip rings, notwithstanding any change of frequency, whereas in the electric machine, the power flowing into the primary divides into two parts, one part proportional to S appearing in mechanical form, and the other proportional to f_2 appearing as electrical power in the secondary. From considerations of power, this is the outstanding difference between the two cases.

We may now summarize the results we have already obtained.

1. That motor and generator form together with the transmission line a mechanism in the ordinary sense of the word, that is, a means of modifying motion and force, or in more general language, of changing the flow of energy.

2. That the electrical and mechanical speed torque characteristics of the machine are those of a differential gear, there being a definite relation, $S \times \dfrac{P}{2} = (f_1 - f_2)$, between the stator and rotor frequency and the mechanical speed, which is identical with that in a differential gear. The relation between the mechanical and electrical speeds, S, f_1, and f_2, can only be changed by changing the number of poles.

3. The primary function of the commutator is that of a frequency changer, which can change frequencies arbitrarily without change of voltage or power.

4. The same equation, $S \times \dfrac{P}{2} = f_1 - f_2$, is also characteristic of the commutator, which is a purely electrical differential gear inherently incapable of transmitting a torque.

Hence, the different types of electrical machine and systems of electrical transmission may be compared with a number of differential gears, with or without means for varying the number of poles, which corresponds to the velocity ratio of mechanical gear and with or without commutators, which may be regarded as almost the only practical form of gradually adjustable gear in existence. Other differences which may be noted to exist are as follows—

5. Systems otherwise identical differ in their method of magnetization.

6. Several revolving fields may be superposed in the same structure as in the "internal cascade" machine, the single-phase or elliptic-field machine, or the split-pole convertor. Thus the above considerations give us three principles of classification –

(*a*) According to the disposition we make of the secondary power output or output from the "intermediate shaft" of the differential gear. A parallel principle leading to approximately the same result is according to which of the quantities I^2d, f_1, or f_2 is regarded as variable in adjustable speed machines.

(*b*) According to the manner in which we employ the commutator in our system.

(*c*) According to the method of magnetization. In addition to this, the machines may be either generators, motors, convertors, double-current generators, phase advancers, etc., but this really introduces no new feature, as most machines are capable of all or most of these functions. If our classification is exhaustive, we shall find that when we have assigned our machines a place in each of our classes it is completely defined, and nothing further remains to be specified, apart from constructional details.

We shall avoid circumlocution if we define the primary as the element into which power flows from the line.

First Principle—Methods of Disposing of the Secondary Power

The secondary power may be made zero by making the secondary frequency zero or as low as practicable. This is done in almost all the standard types of machine in wide commercial use. Examples of this are the following—

1. Ordinary synchronous machines of all kinds where the secondary frequency is made exactly zero.

2. Induction machines with short-circuited rotor, where the secondary frequency is made very low, and the small amount of secondary power generated is dissipated as heat.

3. Direct-current machines where, according to the above definition, the armature must be considered as the primary. From our present point of view they only differ from synchronous machines, in that the alternating currents which flow in the windings are produced from direct current by the commutator instead of being supplied by the line.

4. Repulsion machines (ordinary and inverted) and, in fact, all types in which all the windings on one member are short-circuited. How the secondary power in elliptic field machines may be reduced to zero without confining the machine to synchronous speed will be shown below.

SECONDARY POWER MAY BE ENTIRELY WATTLESS, as in the single-phase series machine.

SECONDARY POWER USED IN ANOTHER APPARATUS (cascade systems).

In this case the power flowing out of the secondary is supplied to a second machine, which is either independent or mechanically coupled to the first. Examples of this are induction machines cascaded with other induction machines, synchronous machines, or alternating-current commutator machines.

SECONDARY POWER TRANSFORMED TO PRIMARY FREQUENCY BY A COMMUTATOR AND RETURNED TO THE LINE.

Examples of this are certain types of alternating-current commutator machines when running at speeds differing from synchronism.

SECOND PRINCIPLE—ACCORDING TO THE MANNER OF USING THE COMMUTATOR

If we confine ourselves as we are doing to a single machine, the only two possible methods of using a commutator are—

1. On the primary, as in the direct-current machine.

2. On the secondary, as in the alternating-current commutator machines mentioned above. If, however, as may readily be done, we extend our survey to groups of machines working in conjunction, as in cascade sets, for instance, a large variety of further interesting methods of application are revealed. A third way of applying it, however, is to confine currents to a definite axis in space, as in the repulsion motor, etc. This method only applies to elliptic field machines.

THIRD PRINCIPLE—METHODS OF MAGNETIZATION

Two distinct methods of magnetization may be distinguished—

1. The primary may be the seat of the magnetizing currents, which may be led in direct from the line as in the normal induction machine and several others. This may be called high-frequency magnetization, and involves the appearance of a considerable amount of reactive power proportional to the frequency in the line circuit.

2. The secondary may be the seat of the magnetizing currents, which may be led in direct from the line through a frequency-changing commutator or supplied by an exciter. This latter method may be called low-frequency magnetization, and does not give rise to reactive power in the line. In order to understand the various methods of magnetization better we shall devote further space to the matter later on.

Circular or constant intensity field machines. To make up a complete circular field dynamo-electric machine we have four elements : the line, the stator, the rotor, and the commutator. By making various combinations of these, we obtain different types of machine having different characteristics and adapted to different purposes.
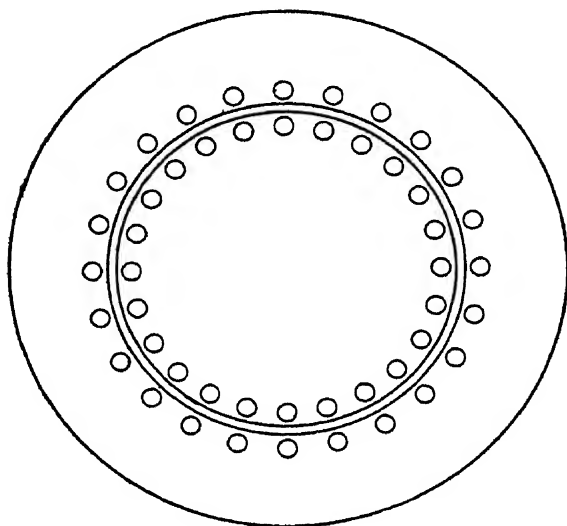
Different types of winding exist, each capable of being adapted for connection to direct current, single or polyphase lines, or to a commutator. Each of these is capable of being built for many different polarities. Hence when we say, "connect the line to the primary," we, therefore, assume that it is provided with an appropriate winding.

The general dynamo-electric machine, as mentioned above, consists of two concentric magnetic elements, separated by an air-gap, and capable of relative rotation (see Fig. 48). Each of these elements bears a number of conductors disposed next the air-gap in slots parallel to the shaft. The conductors of one or both elements are interconnected in such a way as to give rise to a definite number of poles, conductors being led off from the stationary element direct to the line, save in a few cases where it is short-circuited.

FIG. 48

So far nothing has been said about the interconnection of the face conductors. It is seldom realized that the actual types of winding employed are in reality adopted in order to adapt the machine to take electrical energy in a form which is convenient for transmission. For transmission purposes high voltages are desirable, and, hence, windings permitting a large number of conductors per slot are used, notwithstanding the fact that this means a great

reduction in the amount of copper which can be got into a slot of a given area.

Again, the number of transmission lines must be kept down to 2, 3, or 4 at the outside, and, hence, windings must be adapted to use direct-current or single, two- or three-phase alternating current. It is in order to adapt the machine to these requirements that the face conductors are interconnected in the way we actually find, but if all that we wish is to study the combination of generator and motor as a machine in the manner referred to above, it is unnecessary to complicate the subject by these considerations. We shall, therefore, assume that each bar on the generator is connected to a corresponding bar on the motor, either directly or through a commutator.

The free ends we shall suppose joined to a common or star point by a short-circuiting ring. Four different types of apparatus may be built up in this way, which illustrate the different possible methods of connecting the dynamo-electric machine. These will be described in turn.

In Fig. 49 is shown the combination of generator and motor, which is perhaps the easiest to understand. The conductors on one member of both generator and motor are grouped so as to produce two distinct magnetic poles, marked N and S in the figure. The face conductors on the other members are distributed in slots, as shown in Fig. 49, and short-circuited together at one end by a ring, while each conductor on the generator is connected to a corresponding conductor on the motor. The motor and generator, therefore, are identical in construction, and connected together in symmetrical manner.

A fundamental principle of any such generator-motor combination must be that the sum of the E.M.F.'s in each bar from end-ring to end-ring shall be zero ($E_1 + E_2 = 0$). This will be brought about if opposite the north pole on the generator we have a south pole on the motor, and the two travel together in the same direction at the same speed. This forms the simplest possible type of synchronous generator and motor combination.

In the lower part of Fig. 49 is shown the distribution of flux and ampere conductors in the core, the dotted lines being intended to indicate the lines of force, and the two curves showing the generator and motor E.M.F.'s. The generator E.M.F. is referred to as E_1, and the motor E.M.F. is referred to as E_2. They are, of course, equal and opposite. They are shown in the diagram as being distributed in accordance with a sine curve, and, in order to produce this wave-shape the poles used are rounded off in the manner shown. There will be a slight difference between E_1 and E_2 sufficient to drive the current through the resistance of the bar, and this difference will be proportional to the load and vary with the

instantaneous value of E_1 and E_2 as we go round the circumference. Thus the current will be in phase with these E.M.F.'s, and will travel round the machine at the same rate as the poles. Opposite
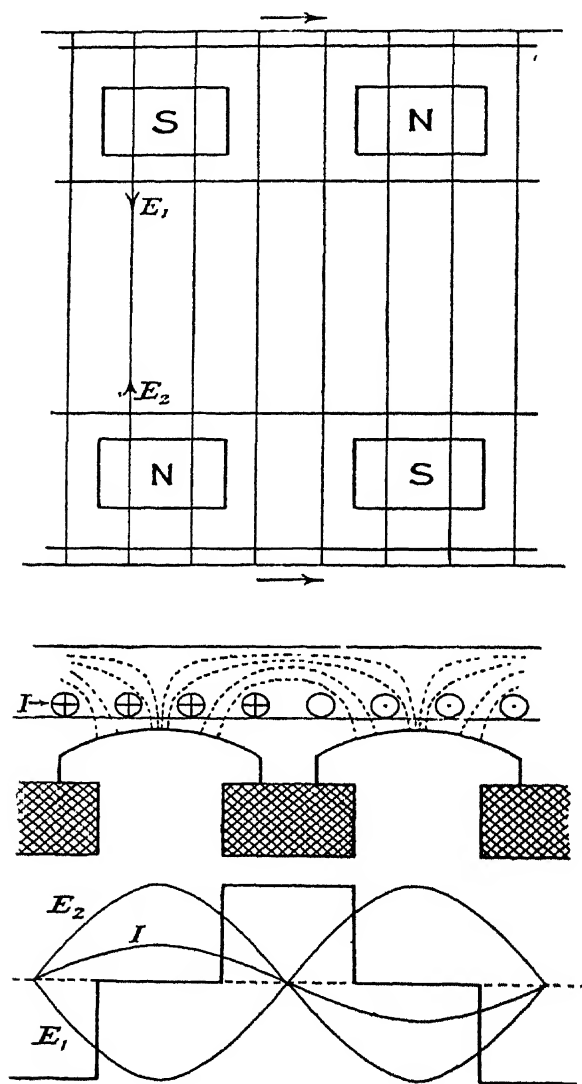
FIG. 49

the north pole, for instance, the current flows in one direction, and opposite the south in the other direction. In the same figure is shown a curve of the distribution of rotor ampere conductors, which is zero over the space occupied by the material of the pole, and has

a constant positive or negative value in the space occupied by the coil.

By fundamental electric laws $E_2 = vB$, where v is the linear speed, say, in centimetres per second. By another fundamental law, $F = \dfrac{B \times I}{10}$, where F is the force in dynes acting on the conductor, I the current in it (amperes) and B the magnetic density of the field in which it is placed (lines per sq. cm.).

Another combination. In Fig. 50 is shown another combination illustrating a different principle. This is known as the "synchronous generator induction motor combination." In all these combinations the generator will be supposed to be of the same synchronous type as that in Fig. 49. In Fig. 50 the stationary portion of the motor remains as before, while the rotor, which previously was of a polar form wound for direct current, is replaced by a rotor consisting of a number of bars, distributed round the circumference, as shown in Fig. 50, and short-circuited at both ends by two end-rings.

The magnetic flux of the motor is now produced in an entirely different way from that of Fig. 49. As before, the E.M.F.'s in each bar, including the generator and motor portion, must sum to zero, and, hence, the motor flux must be exactly equal and opposite to the generator flux, and must revolve with it at exactly the same speed.

In order to understand correctly how the motor fluxes are produced in this case, we shall have to study the subject of magnetization from a somewhat broader point of view.

Wherever we have magnetic fluxes we have a store of energy, and wherever we have varying fluxes we have a flow of energy, a maximum of energy being stored when the fluxes are greatest, and none when they are zero. If we have an alternating flux, the energy surges to and fro, according to the variation of the flux. A magnetic flux, in fact, produces an E.M.F. in the circuit which excites it, which is *opposed* to the current on rising flux when energy is being absorbed, and assists the current on falling flux, when it is being given out. The product of this E.M.F. into the magnetizing current at any instant is the power input or output at that instant, or the rate of flow of energy.

If alternating fluxes are to be maintained, therefore, means must be supplied to store the energy released during falling flux, and to supply the energy required during rising flux.

The two chief forms of storage of magnetic energy are as the electrostatic energy of condensers and the kinetic energy of revolving (or oscillating) masses. The law which regulates them in general, of course, is that the power output of the inductance must be equal to the power input of the storage device at every instant, and vice versa, or that the storage device shall absorb the energy at the instant at which it is liberated by the flux.
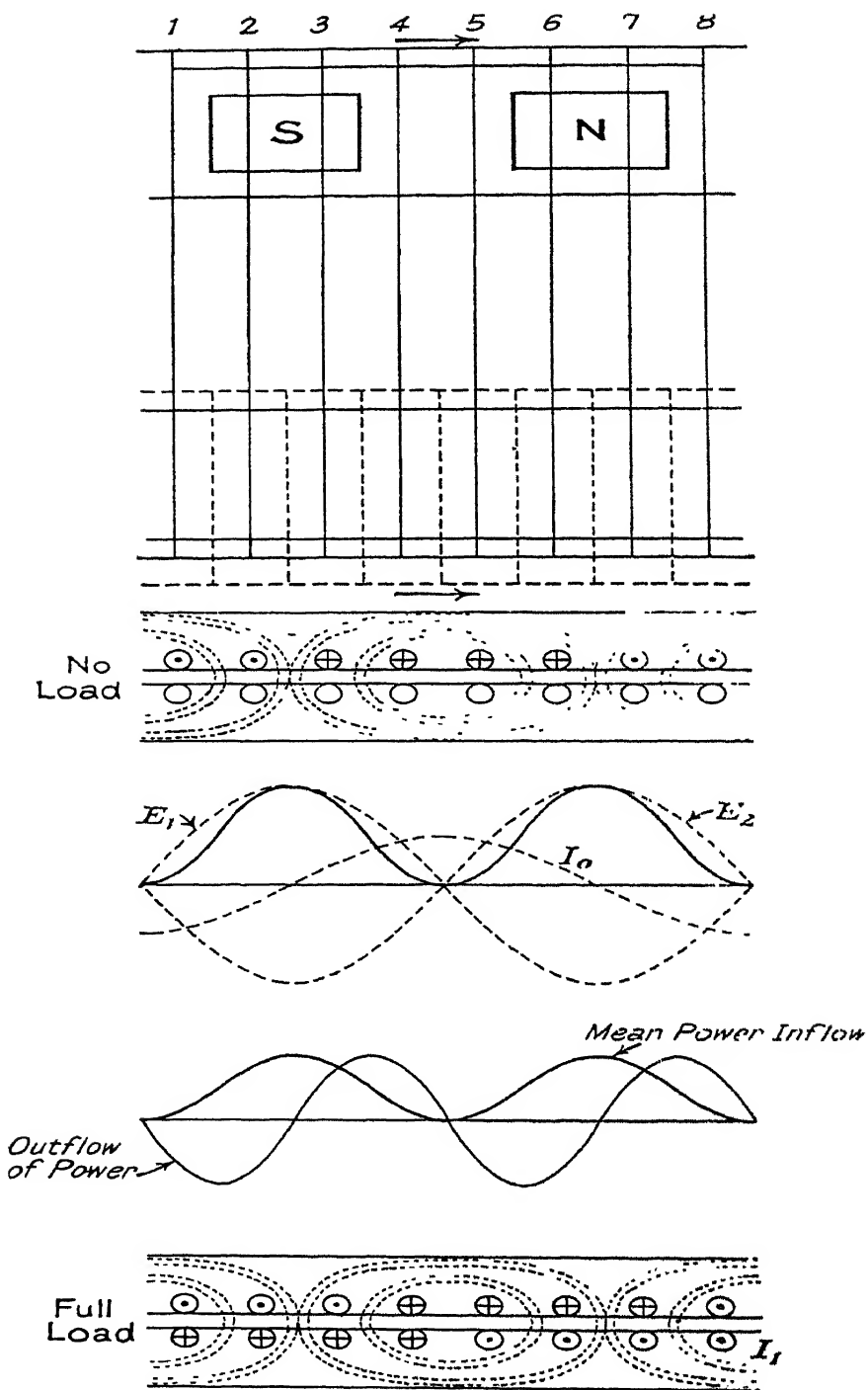
FIG. 50

The apparatus used to store this surging energy of the magnetic fluxes consists usually either of a synchronous machine (motor or generator) or of some type of alternating-current commutating apparatus.

In order to explain in what manner these apparatus can periodically store and give out energy, take first a single-phase synchronous generator on a purely inductive load.

Three positions of the poles relative to the armature are shown in Fig. 51.

It will be seen that during rising current the torque is, say, negative and the poles are being retarded, the machine, therefore, giving out energy. At maximum current the torque is zero, because the flux from a given pole cuts as many conductors in which the current flows one way as it does those in which it flows the other. With falling current the torque is reversed, i.e. the machine is being accelerated and absorbs energy, which, of course, is being given out by the falling flux in the inductance with

Rising Current

Falling Current

Max Current

Fig. 1

which it is loaded. Other types of synchronous machine exist in which both members are excited by alternating current of the same or different frequencies. These may also be used to store energy in the manner described above.
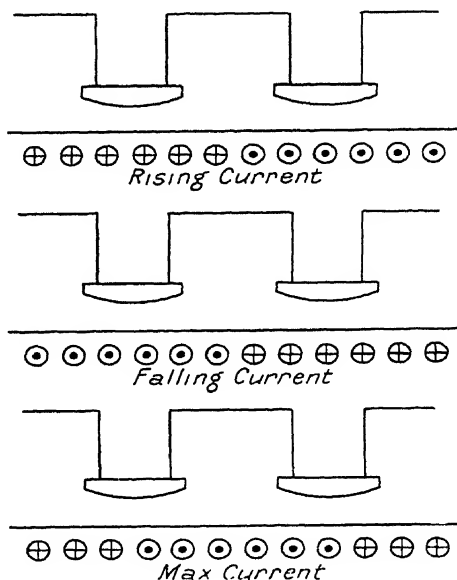
In a polyphase inductive load fed from a polyphase synchronous machine, the flux does not actually vary but merely rotates. Nevertheless, it requires reactive power to keep it in rotation, and this is supplied by the synchronous machine. As this machine is not called upon to store energy, its rotor is not periodically accelerated and retarded while feeding such a polyphase inductance.

Another method whereby the energy of a magnetic flux may be stored is by means of alternating-current commutating machinery.

Consider an alternating flux β cutting a commutator armature carrying a current i. If the flux is such that it induces an E.M.F. in the armature *leading* 90° on the current i the torque will be negative, i.e. the armature retarded or delivering energy on rising, and positive or accelerated and absorbing energy on falling current.

Thus such an arrangement is also capable of supplying the power needed by an alternating flux, and innumerable different arrangements based on this principle may be designed.

These arrangements will be discussed in later sections of this work, the present chapter being devoted to general principles. We have already discussed an elementary case of single-phase magnetization. We shall now discuss the propagation of a wave of flux along the gap surface of our electric machine.

The general law of magnetization, as stated above, is that energy flows out of an electric circuit threaded by a falling flux, and into a circuit threaded by a rising one.

Consider a flux $\beta = \beta_0 \sin (mx - pt)$, travelling along the axis in our machine accompanied by its magnetizing current distribution, $\Delta = \dfrac{dH}{dx} = m\beta_0 \cos (mx - pt)$.

The E.M.F. induced in the conductors is

$$E_1 = v_1\beta \text{ as explained above.}$$

FIG. 52

It is clear that the waves F_1 and Δ are in quadrature with one another. Their product is the flow of energy or power. It is clearly negative for the rear portion of each half wave where energy is flowing out, and positive for the forward portion where it is flowing in.

Another method of arriving at the same result is the following-

Energy density due to a magnetic density β

$$= W = \tfrac{1}{2}\beta H = \tfrac{1}{2}\frac{\beta^2}{\mu} = \frac{1}{2\mu} \times \beta^2 \sin^2 (mx - pt)$$

$$= \frac{1}{4\mu}\beta_o^2 (1 - \cos 2 (mx - pt))$$

$$\frac{dW}{dt} = \frac{m}{2\mu} \sin 2 (mx - pt) = \text{energy flow.}$$

Consider a flux wave β travelling along the x axis in the machine shown in Fig. 50, accompanied by its magnetizing current distribution I_o, the E.M.F. induced in the conductors is $E_2 = v\beta$ as explained above.

The waves E_o or $v\beta$ and I_o are shown in quadrature with one another for a reason which will appear shortly. Their product is the flow of energy or power.

In the diagram the curves E_1 and E_2, and I_o the magnetizing current, are shown. It has already been pointed out that E_2 is proportional to and in phase with β, the curve of magnetic density,
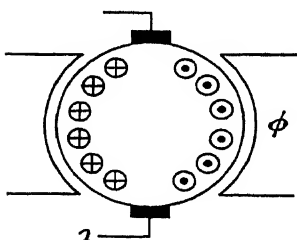
and, therefore, the curve marked E_0 in the figure may also represent β to a different scale.

The density of magnetic energy in the air-gap at any point where there is a flux density β is equal to $\frac{\beta^2}{2\mu}$, μ being the permeance of 1 sq. cm. of the gap area. The curve of energetic density obtained from this figure is already shown in the same figure, and, of course, reaches its maximum when β is a maximum, and is zero when β is zero. It is, of course, the same whether β is positive or negative.

In the lower curve is shown the rate of change of this energy density which gives the inflow and the outflow of the magnetizing power. This rate of change is zero wherever the curve of density passes through a maximum or minimum point, and is represented by a sine curve having half the wavelength of the curve of magnetic density.

This curve shows that power is flowing out of the motor along the conductors 1 and 2, 5 and 6, and is flowing into the motor along the conductors 3 and 4, 7 and 8, the former conductor being on the rear side of the wave, and the latter conductors on the wave front.

Power is flowing out of the motor when the E.M.F. E_2 and the magnetizing current I_0 are flowing in the same direction. It is flowing into the motor when they are flowing in opposite directions, the total inflow and outflow of power summed over all the conductors being zero. Referring to the upper curve, it will be seen that I_0 and E_2 are flowing in the same direction in conductors 1 and 2, 5 and 6, and in opposite directions in the remaining conductors. Hence, if I_0 is in quadrature with E_2, as shown, the conditions required by the general principle of magnetization are met. This proves that the magnetizing current must be in quadrature with both the applied and counter E.M.F. of the machine.

Process of magnetization. The process of magnetization is the following—

Power flows out of the rear side of the motor flux wave to the generator and tends to accelerate it. Meanwhile, power flows out of the generator at another point into the front of the motor wave, thus propagating it and producing a retardation which cancels the above acceleration. This is the mechanism of magnetization, and as the generator may be miles away from the motor its unsatisfactory and uneconomical nature is quite clear. The magnetizing currents have to traverse twice the distance between generator and motor in order to move forward one wavelength—perhaps a foot. Since the rotor winding is completely short-circuited, the E.M.F.'s in each bar must be either zero, or only such as can be consumed by the resistance drop in the bar and end-ring. The only way in which this E.M.F. can become zero is if the rotor revolves at the same speed as the poles of the generator, that is, the same speed as the

motor flux, which, therefore, does not cut it and produces no E.M.F. in it.

If the rotor lags behind the revolving flux by a small amount known as the " slip " S, a small E.M.F. will be induced in it which will produce a current $\dfrac{SE_2}{R}$ in phase with itself. This is shown in the lower portion of Fig. 50, in which it will be seen that the rotor current is in phase with the curve of flux density, and is a maximum at the point of maximum flux density.

We have already discussed the current which flows into the stator on no-load, and serves to produce the magnetic flux. This magnetic flux remains practically the same on full load since, as we have seen, E_2 can only balance E_1 in virtue of the existence in the motor of a flux equal to that in the generator, which latter, of course, does not vary. Hence, to prevent the rotor currents exerting any magnetic action, a further set of currents will flow in the stator winding equal and opposite to those in the rotor winding. These are known as the stator load currents. Compounding with the stator magnetizing current we see that the resultant stator current is nearly in opposition to the total rotor current, but lags somewhat from that position, due to the influence of the magnetizing current. This is shown at the bottom of Fig. 50. It is impossible in this type of machine to cause the load current and magnetizing current to flow in different circuits, but this may be done in machines such as will be described later.

A description of the nature and function of the commutator has been given above.

In Fig. 53 a third combination is shown, in which the flux of both motor and generator is produced by a definite polar winding, as in Fig. 49. It is most convenient to suppose in this case that these poles are stationary, and that the distributed face conductors are in rotation. Each of these conductors must now be supposed connected to a bar of a commutator, both on the motor and generator, which are identical as in Fig. 49, and if each generator bar is still to be placed in series with a corresponding motor bar, we must suppose a separate brush rests on every segment of both generator and motor, each generator brush being connected to a corresponding motor brush. Here the magnetization of the motor, as just mentioned, is supplied independently.

It is still, of course, necessary that we should have E_2 equal to E_1, but now it is possible for this to be brought about in a different manner. In the cases of both the synchronous and the induction machines, this condition rendered it necessary for the two fluxes to be exactly equal and opposite. Here, however, since the bar which is connected to a given brush always lies in a particular part of the field whose density is constant, the potential of that

brush will no longer be alternating but constant. Thus, if the fields are no longer equal, the motor field being, say, weaker than the generator field, so that the equation $E_1 = E_2$, takes the form of $v_1 \times B_1 = v_2 \times B_2$, v_1 being the speed of the generator and v_2 that of the motor, while B_1 is the generator density, and B_2 the motor density.

As before, there will be a slight difference between E_1 and E_2
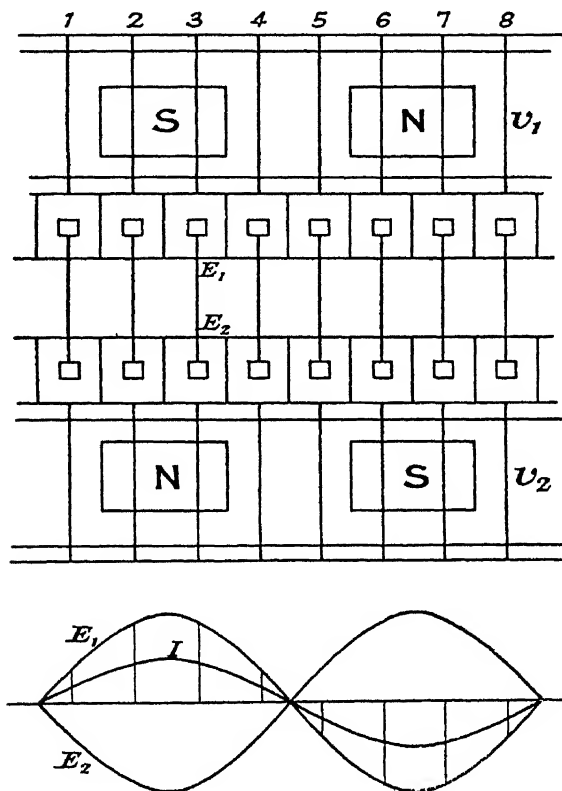
FIG. 53

necessary to drive the current against the resistance of the circuit. Since we still suppose the distribution of the magnetic density to be in accordance with a sine wave, the currents in the different line wires joining the brushes will now be different, being also distributed in a sine wave, thus they will be greater in bars 2 and 3 than in bars 1 and 4, and the currents in bars 5, 6, 7, and 8 will be reversed in respect to those in bars 1, 2, 3, and 4.

In actual direct-current machines, types of winding are adopted which dispense with the necessity of one brush for each bar, and require a number of brushes equal to the number of poles only.

But, for our present purpose, these types of winding need not detain us.

In Fig. 54 a fourth method of interconnecting the face conductors is shown. In this there are two distinct stator windings, one
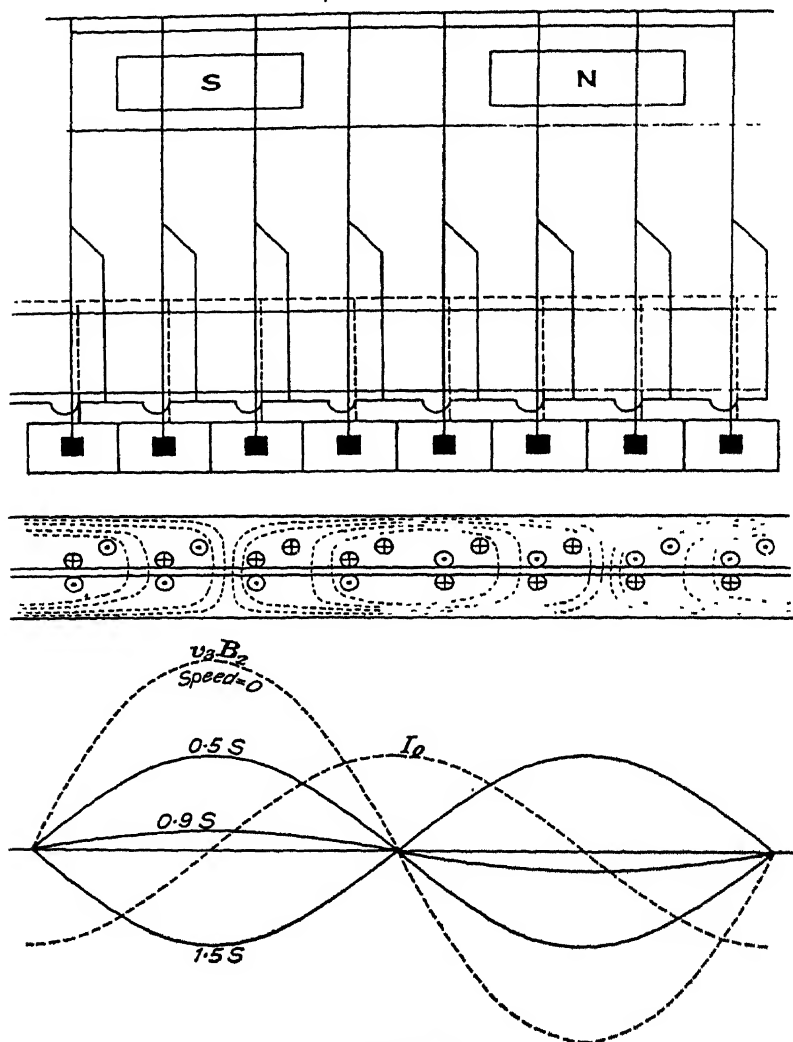
FIG. 54

for carrying the load current, as discussed in connection with Fig. 50. The rotor, however, is no longer short-circuited as in Fig. 50, but is constructed with a commutator in the manner shown in Fig. 54. The free end of the stator winding is not short-circuited by means of an end-ring as in Fig. 50, but connected to the brushes on the

commutator of the rotor, the connection being as shown in Fig. 55. Hence, any current flowing through the stator bar from *L—R*, say, is conducted into the rotor and flows back again from *R—L* along the rotor bar to the star point. Thus the magnetic effect of the stator bar is exactly cancelled by that of the rotor bar, since they carry exactly equal and oppositely flowing currents.

Comparing this with Fig. 50, we see that the equal and opposite stator and rotor load currents which are found to exist in the induction motor are here brought about in a different way by conductive rather than inductive means, but so far no provision whatever has been made to produce the magnetic flux necessary to induce a C.E.M.F. to balance the generator E.M.F.

This is produced by a separate winding, shown dotted, whose bars are connected in parallel with the line bars. These bars are short-circuited by means of an end-ring as before, and, therefore,
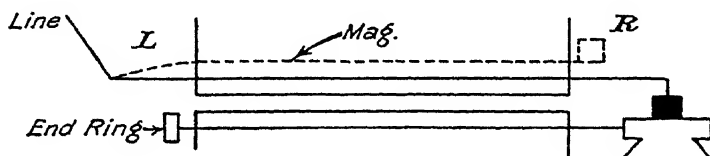
FIG. 55

there are no corresponding currents in the rotor to neutralize their effect, and, consequently, they supply the magnetizing current of the machine. If the rotor of this machine runs at synchronous speed, as in the case of Fig. 50, the E.M.F. in the rotor bar will be zero, as before, and the current and the voltage distribution will be absolutely identical with that in the induction motor. But owing to the presence of the commutator it is possible by running above or below synchronism to add to the voltage in each bar of the stator, a voltage in the rotor bar in series with it, which will be either in phase or opposite in phase according to whether we run above or below synchronism.

The currents and voltages in this machine are shown in the figure below, the voltages being indicated by dotted lines. At zero speed the voltages in the rotor bars will be equal and opposite to those in the stator bars, since they are connected in opposite directions, and hence, any rotor bar together with any stator bar with which it is in series will be, as a whole, non-inductive. As the machine speeds up, in the same direction as the field, the voltage in the rotor bar is reduced. At half synchronous speed the voltage will be reduced to half, as shown in the curve. At $\frac{9}{10}$ of synchronous speed, it will be reduced to $\frac{1}{10}$, while at exact synchronism it vanishes. At 1·5 synchronous speed it will be equal and opposite to its value at half synchronous.

At synchronous speed, therefore, the voltage induced by the flux in the stator bar is no longer balanced by any such voltage in the rotor bar.

For the same reason, as in the case of the direct-current machine, the equation $E_1 = E_2$ no longer involves equality of flux in the motor and generator, the equation $E_1 = E_2$ being now replaced as before by $v_1 \times B_1 = (v_2 - v_3) B_2$ where v_2 is the speed of the flux wave relative to the rotor. At standstill, $v_3 = 0$, and, if the motor is running at k times synchronous speed, $v_3 = (1 - k)v_2$. Substituting this in the above equation we get $v_1 B_1 = k.v_2.B_2$. Of course, v_2 being the speed of the flux wave which is determined by the frequency of supply it is invariable, and hence, if B_2 is weakened, a rise in the speed of the machine is necessary, as in the direct-current case previously considered, in order that the equation may remain true. In fact, the direct-current machine is simply the limiting case of the type of machine now under discussion when $v_2 = 0$.
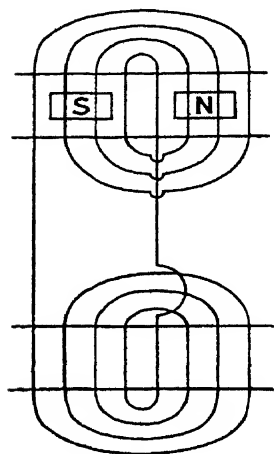
FIG. 56

It will be of interest now to study the amount of power appearing at the commutator of this machine. Since the rotor is directly in series with the stator, the currents in the two must necessarily be equal. Hence the ratio of the amounts of power appearing in the two members will simply be the ratio of the E.M.F.'s in these members. At synchronism the rotor voltage is zero, and hence, the power appearing at the commutator will also be zero.

At half synchronism the rotor voltage is half the stator voltage, and, therefore, the rotor power half the stator power. Since the current flows in opposition to the E.M.F. in the stator, and in the same direction as the E.M.F. in the rotor, the stator power is opposite in sign to the rotor power. Below synchronism, therefore, power is flowing out of the rotor ; above synchronism the E.M.F.'s in stator and rotor coincide in direction relative to the current (both being in opposition to it), and hence there is an inflow of power into the rotor.

In general, whenever the secondary member (that is, the member which is not connected directly to the line) differs from synchronous speed, it is necessary to provide for an inflow or outflow of electrical power from it. In the majority of types, for instance, the synchronous, the induction, or direct-current machines, we do not depart from synchronous speed. But in other cases, such as the present, the disposal of this secondary power forms a serious

problem. In this case, below synchronism, the secondary power cancels a portion of the power represented by the product of stator E.M.F. and current, and is thus returned to the line through the agency of the commutator. In other cases, other means of disposing of this secondary power will be discussed.

In all the types of machine so far discussed the flux is of constant intensity and revolves at a uniform speed, but not all machines have this type of flux. The homopolar machine, of course, has a flux density which is constant all round the periphery, and is not made up of a number of successive north and south poles. This machine excepted, all known types of field can be analysed into a multiplicity of such constant intensity fluxes revolving at uniform speeds, either relative to each other or to one or both of the members.

For instance, one large class of machines are the elliptic flux machines, where the resultant flux may be considered to consist of two opposite rotating fluxes of the type described above, each having the same number of poles. A machine of this type is shown in Fig. 57, which is similar to Fig. 50), save for the nature of the impressed wave of E.M.F. This is no longer a revolving

Fig. 57

polyphase E.M.F., in which each conductor in turn has the same E.M.F. induced in it, but consists of a standing wave in which certain conductors have a maximum E.M.F. induced and others a minimum, or none at all in the single-phase case.

Clearly, in Fig. 57, a north pole of one set crosses a south pole of the other at a point such as A, and here, since they are moving
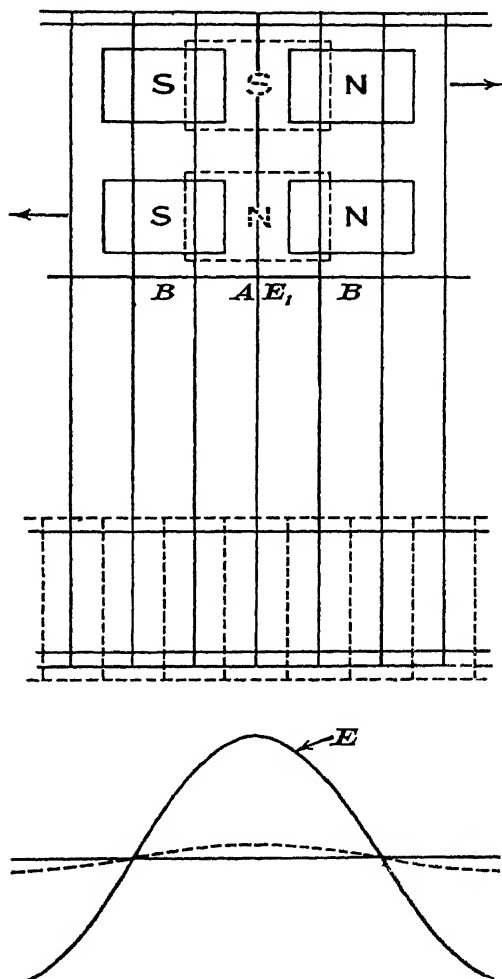
in opposite directions, the E.M.F. induced in the bar will be a maximum. A quarter of a wavelength away from this, for instance, at *BB*, the north pole of the two sets will cross one another, and the E.M.F. induced in the bar will be zero. Thus, the E.M.F. impressed on the motor will be a standing wave, as shown at the bottom of the figure. This case should be very carefully distinguished from the case shown in Fig. 56, in which a single set of revolving poles is assumed to cut a winding wound as a single-phase concentric
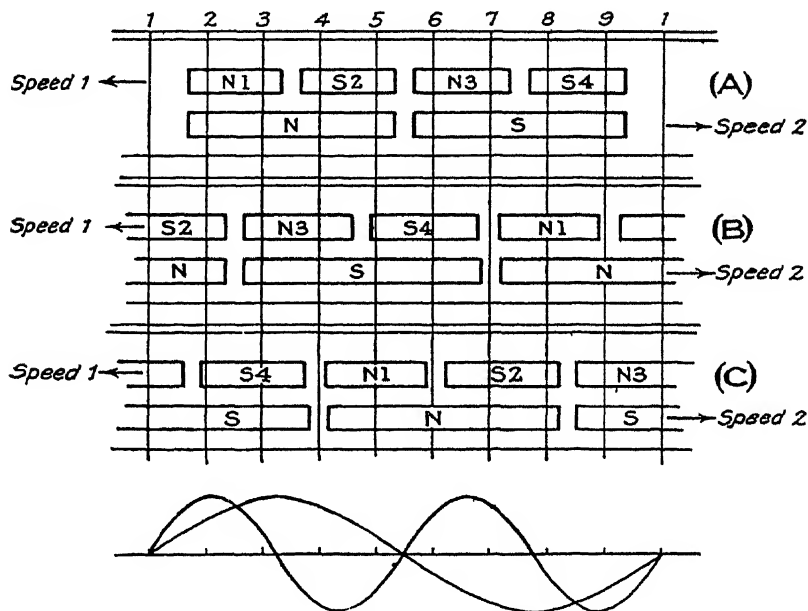
FIG. 58

coil. Here, owing to the method of connection, a standing wave of current only is impressed on the motor, that is to say, the current distribution in the motor will be the same as that in the generator.

Clearly, the E.M.F. impressed on the motor, due to the revolving field cutting the single phase generator winding, can be balanced by a flux in the motor exactly equal and opposite to the generator flux, that is, a revolving field exactly resembling the generator field. Hence, while in Fig. 57 the motor field cannot be of the simple revolving type, but must consist of two oppositely rotating fields, in Fig. 56 a revolving field is quite suitable, and hence the distinction between the two is very great. Later on it will be necessary to develop the full meaning of this difference in detail.

Still another type of field is that known as the multiple polarity field, in which the two fields may still revolve in opposite directions, but are of different pole numbers instead of the same. Here it will

still be found that the E.M.F.'s induced in certain bars are greater than those induced in others. Consider the case of two and four poles revolving in opposite directions, the four-pole moving to the left at half the speed of the two-pole, which travels towards the right. This is shown in Fig. 58. We have here to draw attention to a very important characteristic of such combination, the consequence of which will be developed fully later on. In Fig. 58(A) we show the first position of the two- and four-pole combination. It will be seen that poles N_1 and S_2 are opposite N, and N_3 and S_4 are opposite pole S.

In Fig. 58(B) we suppose that the two poles are moved through two-thirds of the circumference towards the right, and that the four poles have moved over one-third of the circumference towards the left It will be seen that as a result of this motion the relative position of the poles is exactly the same as before, namely, N_1 and S_2 opposite pole N, and N_3 and S_4 opposite pole S. In Fig. 58(C) the poles N and S are

FIG. 58(D)

supposed to have moved through a further two-thirds of the circumference to the right, while the four poles have moved through one-third towards the left. And it will be seen that the relative position of the poles is still the same as before. Thus, with a two- and four-pole combination moving at speeds inversely proportional to the number of poles, the relative position of the poles at a particular instant is exactly repeated at two further instances before the poles return to their original position. The reason for this is perhaps seen more clearly from Fig. 58(D), in which we suppose two vectors starting from A moving in opposite directions, one of them revolving clockwise at a particular speed, and the other counter-clockwise at twice that speed.
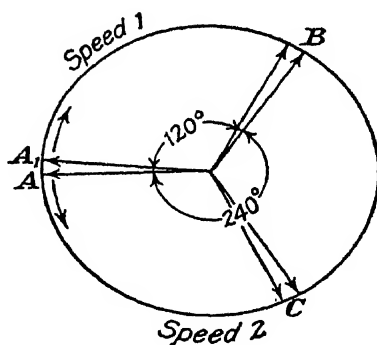
It is clear that when the clockwise revolving vector has passed through 120° the counter-clockwise revolving vector will have passed through 240°, and they will, therefore, again coincide at the point B. Moreover, when the clockwise revolving vector has passed through 240° the counter-clockwise vector will have passed through 480° = 360° + 120° measured in the counter-clockwise direction. Hence the two vectors coincide a third time at the point C.

To sum up, of all the possible permutations, only four distinct types of machine emerge—those briefly described above, all of which may be derived from one another by simple mechanical

processes of inversion, etc. Thus, starting from the direct-current machine we may (1) replace the commutator by a polyphase set of collector rings, and we have a synchronous machine ; (2) replace the field winding by a squirrel cage, and we have the induction type ; (3) invert, making the secondary the rotor, replace the squirrel cage by a commutator carrying a polyphase set of short-circuited brushes, and reconnect the brushes in the manner described above, and we have the polyphase commutator type. These constructional modifications have one of two objects—

(*a*) Adaptation to different kinds of current, as when we change a direct-current to a polyphase machine.

(*b*) To obtain characteristics adapted to some particular variety of industrial work, as when we change the induction to the commutator type. It should be recognized, however, that in spite of all variations, there is but one dynamo-electric machine retaining its main features unchanged throughout. It is interesting to note that machines with series characteristics invariably have commutators.

Convertors. Various types of convertors exist calling for a place in our classification. The function of a convertor is essentially to produce a change in frequency, and, hence, when it consists only of a single machine it is invariably of the commutator type. Such an apparatus (in the ideal case) is not called upon to develop torque, and, hence, no secondary power exists. Certain types of frequency convertor are fitted with field windings whose sole purpose is to produce a flux. In this case the secondary power is wattless or reactive, the only possible case of reactive secondary power in a circular field machine. Convertors to operate on constant voltage must necessarily be shunt machines.

CHAPTER XI

ELLIPTIC FIELD MACHINES

WHEN we come to elliptic field machines an entirely new set of possibilities presents itself. The secondary power in such a machine may be reduced to zero, though neither the secondary E.M.F. nor current is zero. Moreover, in certain cases it may become entirely wattless, a case which cannot occur in circular field machines. An elliptic field machine must necessarily have a commutator, since any machine having a short-circuited or separately-excited secondary must necessarily give rise to a circular field. (*Note*: The case of continuous dissipation of power, as in the single-phase induction motor operating below synchronism, is considered to be excluded by the assumption that we are dealing with "ideal" machines.)

Owing to the presence of this commutator, the secondary power, when it exists, appears at line frequency, and there is, therefore, no difficulty in returning it to the line or deriving it therefrom. In dealing with circular field machines, we found it useful to draw a distinction between the cases in which the commutator was on the primary or on the secondary, and we shall still find it useful to draw the same distinction. It enables us to divide our machines into two classes, containing pairs of machines which may be shown to be exactly reciprocal to one another. For instance, dealing with single-phase series type motors, we may divide them into a number of pairs.

Diagrams of these machines are shown in Figs. 59, 60, 61, 62, 63, and 64. The last machine, which I have provisionally called the compensated inverted repulsion motor, is shown in Fig. 64. It is quite as practical a type as any of the others, and may even have some advantages. Let us now consider in what manner the secondary power is disposed of in some of these pairs of machines.

In the repulsion motors, ordinary and inverted, the secondary circuits are completely short-circuited, and, hence, the secondary power zero. In the neutralized series and compensated repulsion motors, as also in the last pair, the secondary power is used entirely for magnetizing and is, therefore, wattless. If the magnetization is of the low frequency type, as in the compensated repulsion motor, the secondary power (*Note*: It should perhaps be mentioned that by "secondary power" we mean the actual power measurable by a wattmeter, and not any fictitious quantity such as some theories accustom us to consider) vanishes completely at a particular speed (synchronism). The distinction which we have drawn between

machines having the commutator on the primary and on the secondary merely symbolizes other important differences. Machines with commutator on the primary are " pulsating field machines " having a field distribution of the well-known type characteristic of the neutralized series motor, viz. having a definite axis fixed in space. Such machines may conveniently be built with salient poles. Machines with commutator on the secondary are " revolving field machines," in which the field revolves elliptically at all speeds,
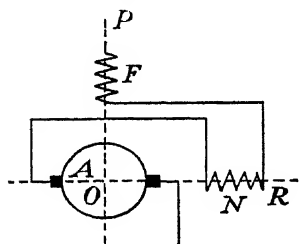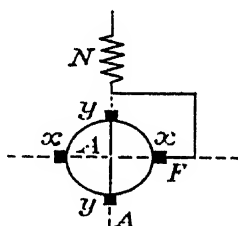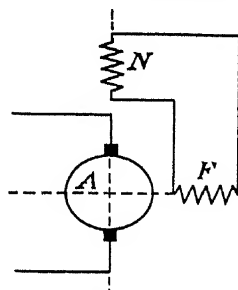
FIG. 59   FIG. 60   FIG. 61
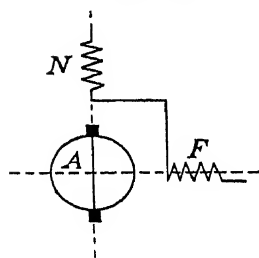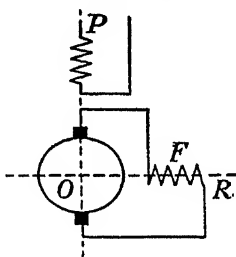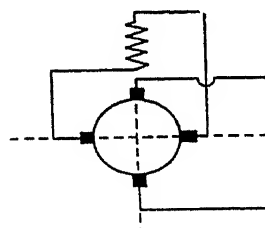
FIG. 62   FIG. 63   FIG. 64

becoming circular, as a rule, at synchronism. Such machines require an induction motor type of stator with uniform air-gap all around. There are, moreover, important differences between the commutation of the two types. In the " revolving field type," since the flux revolves with the armature at synchronism, the commutating conditions are identical at this speed with those in direct-current machines. In the pulsating field type, of course, no such conditions exist, hence the above-mentioned distinction symbolizes a large number of practical differences independent of any theory. Moreover, the same distinction has been shown to be significant with reference to all standard types of machine also. In order to discuss the elliptic field machines more fully, it is necessary to enter upon their theory to a certain extent. The writer has accustomed himself to think in terms of a certain theory which is described in a treatise entitled *Single-phase Commutator Motors*,[1] published by

[1] Constable.

him in 1913. Although this method of studying elliptic field motors is not at present very widely known, it presents considerable advantages, and it will accordingly be made use of here. The purpose in the following discussion, however, is purely that of explaining the two reciprocal classes of single-phase series type motor which have been mentioned above and, as it adds nothing further to our results, may be omitted by the reader who is unfamiliar with the methods used, and who is willing to take for granted the reciprocity of the two classes.

Power calculations in elliptic field machines. The secondary power is now the product of an elliptically distributed E.M.F., and an independent elliptically distributed current which may have quite different axes from that of the E.M.F. The multiplication of such ellipses has been discussed in the writer's *Single-phase*
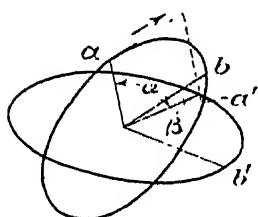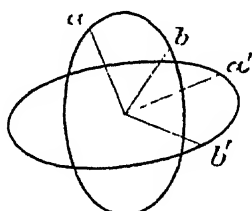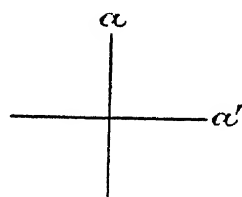
FIG. 65   FIG. 66   FIG. 67

Commutator Motors, Appendix, pages 108–109, where two products, the " sine " and the " cosine " product, are defined as follows : Suppose we wish to multiply any two vector ellipses, we must resolve each into a pair of conjugate diameters which will be in quadrature with one another. These are shown in Fig. 65 as ab and $a'b'$, and they must be so chosen that a is in phase with a', and b with b'. If α is the angle between a and a', and β that between b and b', we define the cosine product of the two ellipses as aa' $\cos \alpha + bb' \cos \beta$. If one ellipse represents a current distribution and the other an E.M.F. distribution, then the " cosine " product represents the maximum power flowing due to the two, and $\frac{1}{2} (aa'$ $\cos \alpha + bb' \cos \beta)$ is the mean power. In the case in which both ellipses reduce to circles, we have $a = b$, $a' = b'$, $\alpha = \beta$, so that the mean power will be $a' \cos \alpha$, which is the usual expression. Since a is in quadrature with b', and b with a', we have not considered the products of these terms, as they will give rise to purely wattless or reactive terms. It is quite clear that the above expression may be reduced to zero in many ways without either of the factors being zero, and, hence, the secondary power of our elliptic field machines may be zero or purely wattless, without either the secondary current or E.M.F. being zero. Hence, such machines are not restricted to synchronous speed in the absence of some means

of disposing of the secondary power, as are machines of the circular field type. We shall now discuss some of the principal cases in which such a product may be zero without either of the factors vanishing.

1. Ellipses similar and of similar phase and at right angles to one another (see Fig. 66). In this case $\alpha = \beta = 90°$, and the product is wattless since neither ab' nor $a'b$ is at right angles.

2. Each ellipse reduces to a straight line, both being at right angles (see Fig. 67). In this case the product is accurately zero, not wattless, and this is the only case in which it is accurately zero, except with a circular field at synchronism

$$aa' \cos \alpha = -bb' \cos \beta$$

In this case the two ellipses rotate in opposite directions. We shall not come across any instance of this. Let us now consider what distributions of current and E.M.F. occur in some of the best known types of single-phase motor having series characteristics. Consider first the case in which the commutator is on the primary. This case includes two well-known types, the neutralized series motor and the inverted repulsion motor. Both of these involve a stator element which is, or permissibly may be, short-circuited.

(a) *The Inverted Repulsion Motor.* In the stator or secondary element the current flows entirely in the short-circuited coil, and will be represented by a straight line ellipse parallel to the axis of that coil. Since the coil is short-circuited the flux cannot inter-link it, and must be, therefore, of the " pulsating " type represented by a straight line ellipse perpendicular to the axis of the coil. The same ellipse may also represent the E.M.F. to a suitable scale if we displace the phase by 90°. Hence in this case, the two ellipses of E.M.F. and current reduce to two straight lines at right angles, and the " cosine " product is, therefore, accurately zero (see Fig. 68). The ampere-turns necessary to excite the flux are, of course, supplied by the primary.

(b) *The Neutralized Series Motor* also contains a short-circuited element on its stator or secondary, viz. the neutralizing coil. For the same reasons as stated above, the current I_n in the neutralizing coil and the flux and E.M.F. will be represented by two straight line ellipses at right angles (see Fig. 69). In the present machine, however, the magnetizing ampere-turns necessary to excite the flux are on the secondary, being supplied by the field winding. The field currents will be represented by another straight line ellipse I_f, parallel to α and the resultant stator current by I. Thus, since I and E are no longer at right angles their product will not be accurately zero, but since they are still in quadrature, it will be wattless. We shall now turn to the machines of the " rotating field " type

which are reciprocal to the two just discussed, viz. the ordinary and compensated repulsion motors.

3. *The Repulsion Motor.* In the rotor or secondary element the current flows entirely in the short-circuited circuit, and will be represented by a straight line ellipse parallel to the axis of that circuit. The E.M.F. across any short-circuited circuit is obviously zero, and the E.M.F. ellipse therefore reduces to a straight line perpendicular to the first, though, of course, it no longer follows that the flux is of the same form. Thus the rotor E.M.F. and current ellipses of the ordinary repulsion motor are identical with the stator E.M.F. and current ellipses of the inverted repulsion motor (see Fig. 70).

4. *The Compensated Repulsion Motor* also contains a short-circuited element on its rotor or secondary. For the same reasons as
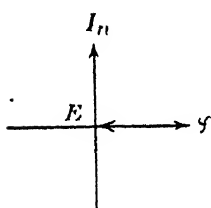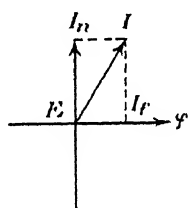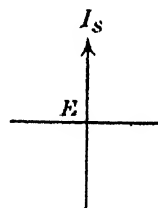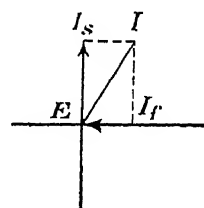
Fig. 68   Fig. 68   Fig. 70   Fig. 71

in the ordinary repulsion motor, the current in the short circuit I and the E.M.F. will be represented by two straight line ellipses at right angles (see Fig. 71). In this machine, however, the magnetizing ampere-turns necessary to excite the flux are on the secondary, being supplied by the circuit through the field brushes. The field currents will be represented by another straight line ellipse I_f parallel to E, and the resultant stator current by I. Thus, since I and E are no longer at right angles, their product will no longer be accurately zero, but since they are still in quadrature it will be wattless (reducing to zero at synchronism, however, since in this machine low frequency magnetization is made use of).

In the above discussion only the secondary E.M.F. and current distributions are considered, but it is easy to show that primary E.M.F. and current distributions are reciprocal also. In order to bring this out in its clearest form we paraphrase the discussion in *Single-phase Commutator Motors*, pages 51, 52, 53, in two parallel columns on page 92.

Shunt type machines. There is by no means so large a variety of shunt type as of series motors having different modes of operation though substantially the same characteristics. Although the number of constructional modifications which may be suggested is, of course, endless, all practicable types of motor reduce, in the end,

to what is essentially one machine. This is due to the following facts.

Consider the ordinary shunt motor operated on alternating current. If the armature current is approximately in phase with the

The Repulsion Motor Rotor E.M.F.	The Inverted Repulsion Motor Stator E.M.F.
Since the rotor is short-circuited along the axis OP there can be no E.M.F. along that axis, and the rotor E.M.F. ellipse is, therefore, a straight line along OR perpendicular to OP.	Since the stator is short-circuited along the axis OP there can be no E.M.F. along that axis, and the stator E.M.F. ellipse is, therefore, a straight line along OR perpendicular to OP.
Flux Distribution	*Flux Distribution*
Such a rotor E.M.F. distribution can only be produced at speed K by a flux ellipse whose axis lies along OP and OR, that along OR being of length a, say, that along OP being Ka, where K = speed synchronism. In this case the "transformer" E.M.F. and E.M.F. of rotation along the short-circuited axis OP will cancel, leaving a straight line distribution of rotor E.M.F.	Such a stator E.M.F. distribution can only be produced by an exactly similar flux distribution, which may be represented to an appropriate scale by exactly the same straight line ellipse.
Stator E.M.F.	*Rotor E.M.F.*
The stator E.M.F. due to this flux distribution will also be represented to a suitable scale by exactly the same ellipse as the flux.	This purely single-phase flux ellipse induces in the rotor an E.M.F. ellipse whose axis lies along OP and OR, that along OR being of length a say, and that along OP being Ka.
Terminal E.M.F.	*Terminal E.M.F.*
This ellipse must touch a line perpendicular to OQ, and at a distance OE from the origin equal to the maximum terminal E.M.F.	This ellipse must touch a line perpendicular to OQ, and at a distance OE from the origin equal to the maximum terminal E.M.F.

line E.M.F., which it should be, of course, for operation on good power factor, it will be in quadrature with the flux, which lags 90° behind the line E.M.F. In order, therefore, to keep the flux and current in phase, we need to supply the field with an E.M.F. leading 90° on the armature or line E.M.F.

There is but one way to generate such an E.M.F. if we are to retain a purely single-phase motor, and that is by means of a pair of brushes arranged on the armature at right angles to the load brushes or those which carry the main current. The practical
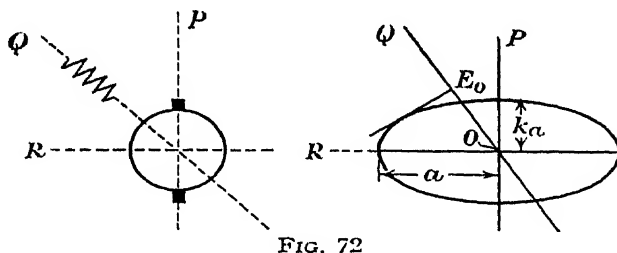
FIG. 72
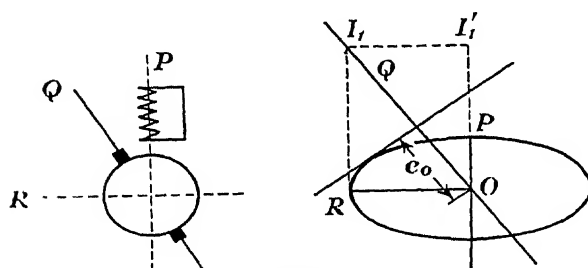
FIG. 73

machine in which this principle is carried out in its simplest form is usually called the Atkinson commutator induction motor. It consists of a distributed single-phase winding on the stator, a pair of short-circuited brushes co-axial with the stator winding, and another pair at right angles thereto, also short-circuited.

The E.M.F. of rotation induced in this latter pair of brushes by the primary flux leads 90° on the line E.M.F. and is, therefore, capable of producing the field we require.

This commutator armature with its two pairs of short-circuited brushes may be replaced by a plain squirrel-cage armature giving us the ordinary single-phase induction motor. Such a machine is of the " rotating field " type, and is, therefore, capable of low-frequency magnetization, otherwise known as compensation, by a well-known method. Such a machine is the single-phase representative of the polyphase or circular field machines discussed above, and, in fact, a polyphase commutator machine of which one phase

has been disconnected will, if of the shunt type, continue to operate on single phase just as an induction motor will. Our principles of classification do not necessitate any distinction being drawn between single-phase and polyphase machines so long as the fields of both remain circular. The principal utility of these single-phase commu-

Inductance
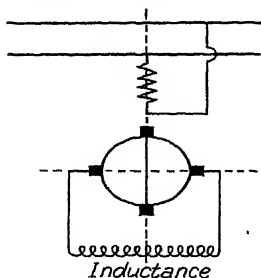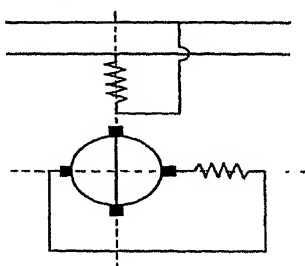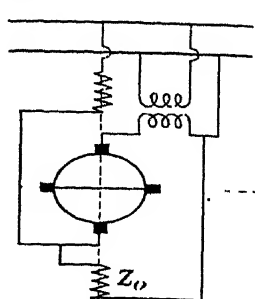
FIG. 74   FIG. 75   FIG. 76

tator shunt machines is as adjustable speed machines, and two methods exist for varying the speed from synchronous—

1. By introducing power into the appropriate rotor circuit (see Fig. 76) by means of a transformer, in exactly the same way as in the case of three-phase motors. Our principles as mentioned above do not necessitate a distinction being drawn between the single and the three-phase case.

2. A method peculiar to the single-phase motor, consisting in weakening the field perpendicular to the stator axis by means of reactance or an auxiliary coil placed in series with the field brushes. In this case, the rotor power is no longer zero, but wattless, being that consumed by the reactance or by the auxiliary coil. This method of weakening the field is shown in Figs. 74 and 75.

But besides these classifications, every type may be built for variable pole numbers. One method of changing the number of poles is by changing the connections of the conductors, the effect being to alter the number of bands of current while the phase difference between adjacent bands remains the same ; so that, for instance, instead of a small number of wide bands of current, the machine presents a large number of correspondingly narrower bands. A second method,
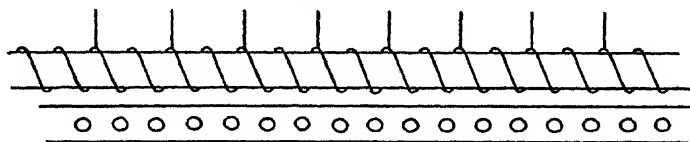
FIG 77
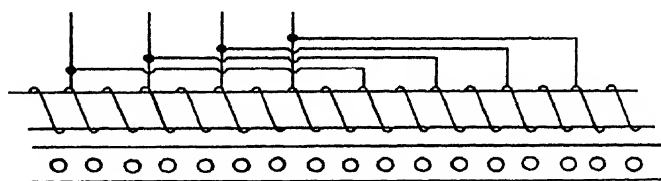
FIG. 78

on the contrary, provides for varying the phase difference between adjacent bands of current, the number and width of the bands remaining unaltered. These two methods are more fully explained in Chapter XXI, page 185.

These methods, particularly the second, can be applied to many other types of dynamo-electric machines, as well as to the induction motor. Fig. 77 shows, diagrammatically, an induction motor, with a ring winding having eight tappings by which an 8-phase supply can be connected to it, so as to produce either 2 poles or 4 poles. Fig. 78 represents an ordinary induction motor having a fixed number of poles, i.e. 4 poles. Its winding has the same tappings, but they are cross-connected, so that the motor has but four terminals for connection to a 2-phase supply. The change that has been made, therefore, in changing from a constant pole number to a variable pole number type, is to omit the cross-connections and to vary the pole number of the motor by varying the phase difference between the eight terminals. The cross-connections being no

longer necessary, the winding need not fulfil the conditions that they impose, and, the best number of phases in the supply is a prime number which in none of its connections to the motor will admit of cross-connections being inserted.

If the motor of Fig. 77 is provided with a wound rotor tapped off to a number of slip-rings in a suitable manner, and these slip-rings be fed with direct current, the motor operates as a synchronous

FIG. 79

FIG. 80

motor. Instead of supplying to the slip-rings E.M.F.'s differing in phase, constant E.M.F.'s of magnitudes proportional to a number of equidistant ordinates of a sine curve are supplied.

Or alternatively, the supply to the rotor may be obtained from a variable pole generator of the type described below.

It is well known that the ordinary induction motor can be converted into a commutator motor by the addition of a commutator to its rotor winding with brushes upon it joined to the mains directly or through transformers, and that this addition makes phase compensation and self-excitation possible. In precisely the same way the induction motor, just described, can have a commutator

and brushes added to it and operate as a variable pole commutator machine. Fig. 79, for instance, shows a shunt commutator machine so constructed, the brushes being connected to the mains through transformers, while Fig. 80 shows a substantially identical machine provided with cross-connections which convert it into a fixed pole-number shunt commutator machine. If collector rings are attached to tappings on the armature winding of Fig. 79, chosen according to the principles explained elsewhere, a frequency convertor is obtained, which, as will appear below, may fo. a particular setting of brushes operate as a rotary convertor. A switching operation
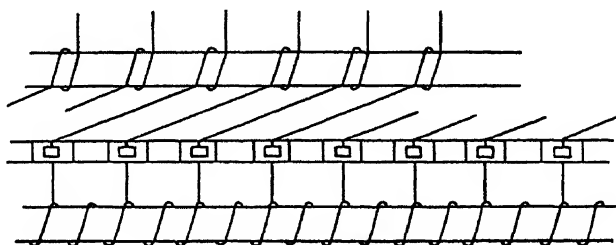
FIG 81
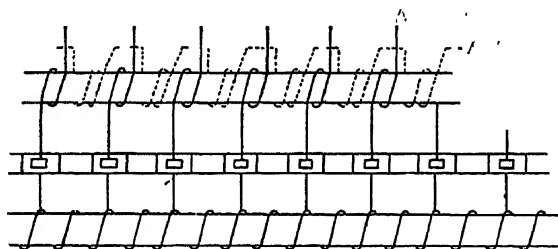
FIG. 82

must, of course, be performed on those collector rings whenever the number of poles is changed. Fig. 81 illustrates in the same diagrammatic manner a variable pole series commutator machine with star-connected stator winding, the rotor winding being connected into the star point. Fig. 82 shows a shunt conduction commutator motor, of which the neutralizing winding is shown in full lines and the field winding dotted. There must be no connection between adjacent sections of the neutralizing winding, but, on the contrary, it is star-connected, the rotor taking the place of the star point, and this must be borne in mind when designing the neutralizing winding, as not every type of winding is capable of being arranged to give the required distribution of ampere-conductors when arranged in star. There is a conductive type of machine corres-ponding to each inductive type, and this holds good of variable pole machines. Fig. 83 is a diagram of the ordinary form of phase

advancer, and Fig. 84 shows the connections of a phase advancer adapted for use on different numbers of poles, for instance, for use in connection with any of the induction motors hitherto described.

Any of these motors will, if its rotor is mechanically driven, generate current, and so constitute a variable pole generator. The motor of Fig. 82, for instance, if mechanically driven, forms a variable pole generator. That the machine will excite itself when mechanically driven has been fully shown.

In Figs. 85 and 86 is shown a ring-wound shunt conduction motor, substantially identical with Fig. 82. Only three sections of
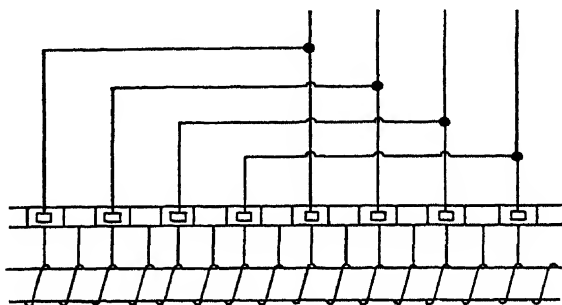
FIG. 83
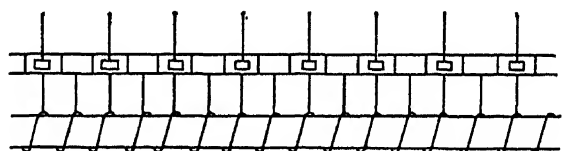
FIG. 84

the neutralizing winding are shown (dotted) for the sake of clearness. It should be noted that in a neutralized machine the relative position of brushes and neutralizing winding must be kept fixed, and, therefore, instead of moving the brushes we must move the points of connection of the field winding. In many cases, however, a neutralizing winding is unnecessary.

In the arrangement shown in these figures, 13-phase alternating current will be supplied from the brushes, and in the limiting case to be dealt with fully below direct current can be taken from the brushes, giving, in effect, a 13-wire system. Or supply might be taken from any less number of the brushes.

Of course, the differences of potential between the brushes of such a direct-current machine are not equal, but the potentials of a number of brushes spaced over a pole-pair or 360 electrical degrees are proportional to equally spaced ordinates of a complete sine curve. In other words, the potential of each brush is directly proportional to the sine of the angle in electrical degrees by which

the brush is displaced from a zero-potential point on the commutator.

As will appear below, the pole number will be variable by mechanically moving the brushes over the commutator, with the exception that, in a machine with a neutralizing winding, the
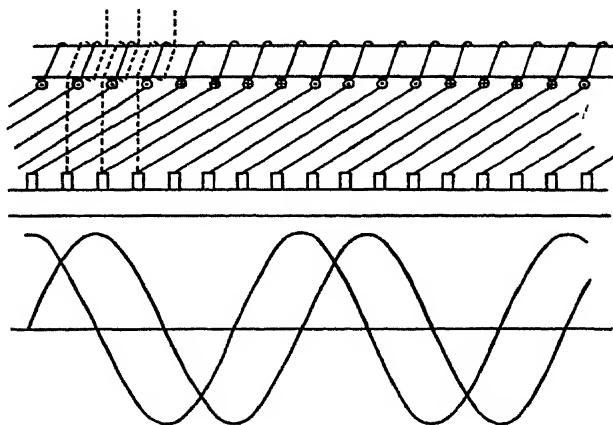
FIG. 85
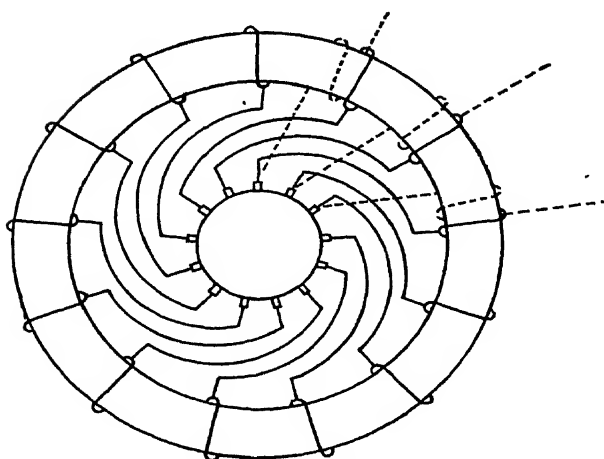
FIG. 86

brushes having to be kept fixed, some form of switching device must be employed to alter correspondingly the tapping points on the field winding instead.

As an example, if each brush and the field tapping to which it is connected are displaced by half the circumference, the distribution is 2 pole, and the machine will generate direct current If this displacement is slightly varied the supply becomes alternating of

frequency depending on the exact angle of displacement. The corresponding displacement for a 4-pole distribution is a quarter the circumference, and so forth.

The arrangement shown in Figs. 85 and 86 may be driven, and will give a 13-phase supply from its brushes to supply, say, a 13-phase induction motor. The unique feature is then obtained that the speed of the motor is variable as desired by merely shifting the brushes or the field tapping points of the generator.

If in such a ring winding, in fact, the ampere-conductors per slot are set up as ordinates of a curve, and likewise the densities per tooth as ordinates of another curve, the two curves will be in quadrature, and the ampere-conductors per slot at each point will be proportional to the difference in tooth densities on either side of the slot. These curves are shown at the bottom of Fig. 85.

Considering a ring-wound machine of the same type as shown in Fig. 86, in which each armature section is connected to a ring-wound field section, and taking the case when the machine operates on a definite pole number. Consider the E.M.F.'s in any section of the machine included between two neighbouring brushes. This section includes a section of the armature winding and of the field winding. From elementary principles it follows that the sum of the E.M.F.'s round such a section must be zero. The E.M.F.'s around this section are three in number. First of all there is the voltage drop, due to the current (Ir). This drop is in phase with the field current. Secondly, there is the voltage induced by the flux cutting the stator winding (ILp where $p = 2\pi \times$ frequency). This voltage is in quadrature with the field current. Thirdly, there is the voltage induced by the flux cutting the rotor winding. This is proportional to the field current, and may be represented by IMs, where s is the speed of the machine and M is the coefficient of mutual induction. The phase of this voltage induced in the rotor may be adjusted as desired by moving the position of the points of connection or tappings of the field winding relatively to the armature brushes as mentioned above. For this is the characteristic feature of this general type of machine. Suppose that the voltage in the rotor be in advance of the field current by an angle α, then the three voltages in question are represented in Fig. 87, where ON is the current drop equal to Ir. OP is the voltage induced in the stator windings equal to ILp. OR is equal to the E.M.F. induced in rotor windings equal to IMs, and the angle NOR is α. As stated above, the sum of these three E.M.F.'s must be zero.

By resolving horizontally and vertically, this condition gives us the two following equations—

$$Ir = IMs \cos \alpha$$

$$ILp = IMs \sin \alpha$$

As we have above assumed the number of poles fixed, the fre-quency and, therefore, p will take up such a value as to satisfy the second equation, and the saturation will take up such a value as to satisfy the first equation, because both M and L are similar functions of the saturation, being very nearly inversely proportional to it, and it, therefore, cancels out from the second equation.

It will be easily seen, for example, that if α has the value 0° or 180°, the frequency will be equal to zero, and in the latter case the machine will, therefore, excite with direct current.

It may be interesting to consider other possibilities. If α is equal to 90°, cos α will be zero, and the rotor voltage could never balance the current drop. Hence the machine would not excite. In a similar manner, if α is negative or greater than 180°, the rotor voltage could never balance the E.M.F. induced in the stator windings. and again the machine could not excite, except by a reversal of the direction of rotation of the flux, in which case, the angle is again less than 180°. Hence the condition for self-excitation is that α lies between 90° and 180°, but obviously the value of α depends on the number of poles, α being halved when the number of poles is halved and vice versa. Therefore, if the machine cannot excite on one pole number, it will excite on another which will satisfy the above conditions. In conclusion it follows that, in the machine shown in Fig. 86, the pole number corresponding to a particular position of the points of connection of the field winding will be such as to permit of the angle at which the voltage induced in the rotor in any section is in advance of the phase of the current in the field section to which it is connected to lie between 90° and 180°. Also, in the limiting case in which it is exactly 180°, the machine will excite with direct current. There is a finite number of these limiting cases, one corresponding to every integral number of pairs of poles.

Except in this limiting case, alternating currents will be produced, and the current in any field section will lag by a certain angle behind the E.M.F. in the section to which it is connected. For a high frequency it may lag nearly 90°. Assuming a lag of 90°, which, of course, is a limiting case which can never actually happen in practice, then there is no current drop. It follows that the E.M.F.'s induced in the stator and rotor must balance one another. Hence the displacement of rotor section relatively to the stator section to which it is connected will now be 0 or 180°, or an integral number of pole pitches. For any position of the brushes the frequency and wavelength will so adjust themselves that the number of pole pairs may be integral, and the angle of lag between armature E.M.F. and the field current less than 90°.

Thus, if the windings of the machine are well distributed a gradual motion of the tappings produces a gradual change of frequency until, when the distance between field and armature sections

approaches 180°, there will be a change in pole number and a similar cycle recurs. In some cases several pole numbers may co-exist, forming harmonics of the lowest or 2-pole arrangement.

SUMMARY

We may now summarize the results of our discussion, and make some final suggestions as to the best methods of classification—

1. The classification to which we have been led by the above discussion first of all subdivides our machines into

(*a*) Circular field or constant intensity field machines, including all direct-current and balanced polyphase machines, all the standard types in fact (constant or variable pole-number).

(*b*) Elliptic or variable intensity field machines, including single-phase and unbalanced polyphase apparatus (constant or variable pole-number).

(*c*) Multiple polarity apparatus, such as the internal cascade machine and the split-pole convertor, whose operation depends on the presence of harmonics in the main field. No detailed discussion of this class is attempted (constant or variable pole-number).

(*d*) We may regard homopolar machines as a fourth class.

2. It is shown from general mechanical considerations that electric power is developed in both the primary and secondary element of the general induction machine which is taken as typical, and that before an operable machine can be produced, some method of disposing of this secondary power must be decided on. Various methods by which it may be reduced to zero, as is done in all standard machines, are discussed. Other methods of disposing of it are by its utilization on a separate apparatus (cascade sets), by commutator frequency transformation and returning to the line (some commutator machines), or by rendering it purely reactive. This can be done in elliptic field machines only.

3. A second important difference depends on the use to be made of the commutator which may be absent, on the primary, or on the secondary. Except in direct-current machines, the commutator is only useful for obtaining adjustable speed.

4. A third difference, not so important as the above two, depends on the method of magnetization. Two different methods of magnetization are distinguished, viz. " high frequency " magnetization leading to the appearance of considerable reactive power on the line, as in the ordinary induction motor, and " low frequency," as in the synchronous and compensated types.

5. Lastly, many types may be connected either series or shunt.

6. A table is constructed showing all possible combinations of these five sets of alternatives, and it is shown that they cover all known types of " constant intensity field " machines, and that

when the place of any apparatus in the table is assigned it can only be constructed in one way.

7. Coming to the elliptic field machines, we find that the secondary power can be reduced to zero or a purely wattless form in many ways without either secondary current or E.M.F. being zero.

8. Our second principle of classification, according to whether the commutator is on the primary or secondary, leads us to distinguish elliptic field series type machines into two classes : pulsating field machines— the neutralized series motor and the inverted repulsion motor, and " elliptically rotating field " machines— the ordinary and compensated repulsion motor, etc. It is shown that a certain reciprocity or duality exists between these types whereby stator E.M.F. and current distribution on the one type is exactly the same as rotor E.M.F. and current on the other. The two types are distinguished by different constructional features, salient poles in the one type and uniform air-gap in the other, and by differences in commutation.

9. Shunt type machines when purely single phase are invariably of the rotating field type, as there is only one way of producing the requisite shunt field.

It is no part of the object of this present work to give a detailed description of well-known apparatus, and, accordingly, nothing further will be said about the standard types.

The large class of single-phase commutator motors referred to above has already been discussed by the writer in a volume under that title, and nothing further need, therefore, be said about these. On the other hand, comparatively little has been published either on internal cascade machines or on variable pole machines, and Parts III and IV will, consequently, be devoted to these. It seems desirable as well to devote some further attention to the methods and apparatus used for low-frequency magnetization (or compensation as it is usually called), and also to the methods of employing polyphase commutator machines to utilize the secondary power delivered by induction machines at speeds different from synchronism. Part V is devoted to this.

Finally, a table (given on page 104) may be constructed by reference to which a characterization of any dynamo-electric machine may be carried out.

The importance of conservatism in nomenclature has already been pointed out, and, in fact, it is undesirable to give any names (especially complicated ones) to possible types of machine until they assume commercial importance.

It may be desirable, perhaps, in some cases to be able to characterize a type in the most compact manner possible, and for this purpose the very convenient Dewey decimal system (already adopted in such learned and authoritative works as Whitehead and

Type of Field	Constant or Variable Pole Number	Disposition of Secondary Power	Commutator	Magnetization	Series or Shunt
Constant[1] Intensity } (1)	Constant Pole number } (1)	Zero (·01)	None (·001)	High frequency (·0001)	Series (·00001)
Cyclic Intensity } (2)	Variable Pole number } (2)	Wattless (·02)	On primary (·002)	Low frequency (·0002)	Shunt (·00002)
Multiple Pole number } (3)		In cascade (·03)	On secondary (·003)		
Homopolar (4)		Returned to line by transformer }			

Russell's *Principia Mathematica*) seems suitable. The entries in the above table are, therefore, each accompanied by its appropriate number on the Dewey system. Thus, on this system,

1·11221 would be the direct-current series motor, and

2·11211 the inverted repulsion motor. Some possible combinations of numbering will be found self-contradictory.

[1] This means constant under particular conditions of operation It does not exclude variations in field strength when conditions of operation change —for instance, when the load changes in a series motor

PART III

CHAPTER XIII

COMBINATIONS OF INDUCTION MACHINES

The " cascade " or tandem connection of electrical machines. We now come to a class of machines usually known as the cascade-connected type. It represents a different solution of the problem of how to dispose of the secondary power of the general dynamo-electric machine, or, to adopt our mechanical analogy, of the power given out by the third shaft of our differential gear.

The best-known type of cascade-connected machine is a cascade-connected pair of induction motors, and we shall accordingly discuss these first.

In a cascade-connected pair of induction motors the primary of one motor is connected to the line, and the secondary of the same motor is connected to the primary of another. The secondary of this second machine may be short-circuited directly or through resistance, or it may be connected to a third machine, and so on.

By this means the electrical power flowing from the secondary of the first machine is converted into mechanical power by the second, and the problem arises as to what disposition is to be made of the mechanical power thus rendered available. This problem is usually solved by coupling the first and second motor together, so that they run at the same speed. They may also be coupled by gearing having a fixed velocity ratio. If the two machines are uncoupled both will be capable of running efficiently under variable speed conditions, but there will be a fixed relation between the speeds of the two machines. By coupling the two machines together we limit the set to a speed or speeds satisfying this fixed relation. We must now investigate this point further in the case where the two machines are both electrically and mechanically coupled.

We have for the primary machine

$$S = W_1 \left(f_1 - f_2\right)$$

and, for the secondary

$$S' = W_2 \left(f_1' - f_2'\right).$$

Since the machines are mechanically coupled, $S' = S$.

Since they are electrically coupled $f_1' = \pm f_2$

Here the $\pm$ sign indicates that the field of the second motor may revolve either the same way as, or opposite to, the first.

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Neglecting the slip of the second motor, we have

$$f_2' = 0.$$

Making these substitutions we get

$$W_1 (f_1 - f_2) = \pm W_2 f_2 = S$$

or eliminating f_2 and putting $W_1 f_1 = S_0$

$$S = \frac{W_2}{W_1 \pm W_2} S_0$$

S_0, of course, is the synchronous speed of the first motor.

If $W_1 = W_2$, $S = \frac{1}{2}S_0$ or infinity, according as we take the plus or minus sign in the expression above.

The number of poles p is proportional to $\frac{1}{W}$.

Expressing S in terms of p we get

$$S = \frac{p_1}{p_1 \pm p_2}$$

Hence we obtain our final rule.

A pair of cascade-connected motors runs at a speed corresponding to the sum or difference of their numbers of poles.

When two machines are connected to run at a speed corresponding to the difference in their numbers of poles they are said to be in differential cascade.

Taking the case where $W_1 = W_2$, it is easy to see apart from all calculations that when the machine runs at half speed the secondary frequency is half the line frequency. If we apply half the line frequency to the second motor it will run at half the speed corresponding to the full line frequency. As this is the speed of the first motor they may be coupled, and will then be in equilibrium.

Let us construct a mechanical analogue to this set by means of a pair of differential gears as shown in Chapter IX. In this chapter we saw how our induction motor might be compared to a differential gear, in which the intermediate shaft was braked by a brake giving a torque proportional to the speed of rotation of the intermediate shaft.

Our present arrangement, then, must be compared to a pair of such gears in " series," or cascade, in which shaft B of the first gear is connected to the corresponding shaft B_1 of the second to correspond to the mechanical coupling, and shaft C is connected to shaft A' to correspond to the electric coupling.

We saw above that in each of these gears we had

$$B = C - A$$
$$B' = C' - A'$$

In virtue of the two couplings and the braking of shaft C', we have also

$$C' = 0$$
$$B' = B$$
$$A' = C$$

Substituting

$$B' = -C = B$$

or
$$-C = C - A, \qquad C = \tfrac{1}{2}A$$

this gives

$$B = C - A = \tfrac{1}{2}A - A = -\tfrac{1}{2}A$$

Thus we conclude that in a pair of gears, such as are shown in Fig. 87, the shafts B and B' go at half the speed of A.

This may easily be seen without calculation as follows—

Since C' is fixed, A' and B' must have speeds that are equal and opposite, therefore B and C must be equal and opposite. Since also B is equal to the difference between C and A, these conditions can only be satisfied by having B and C each equal to $\tfrac{1}{2}A$.

Let us consider what happens when the first machine—that connected to the line—runs in the neighbourhood of synchronous speed, say, with a very small slip.

We must remember at this point that the second motor has a certain amount of primary resistance. When the first motor runs with a very small slip, the secondary motor is subjected to a very low voltage and a very low frequency. This voltage will be almost entirely consumed by the primary resistance, the voltage induced in the second secondary being extremely low.

In this case, therefore, the second motor rotates idly at the higher speed, and the secondary of the first motor is in effect short-circuited through the primary of the second.

Thus we arrive at the conclusion—

A pair of cascade motors can run at the normal speed of the primary motor as well as at the cascade speed.

Another form of cascade machine electrically identical, but mechanically different, from that last described, is the following—

Instead of connecting the rotor of the first to the stator of the second machine, we may connect the rotor of the first to the rotor of the second, in this way eliminating the necessity for collector rings. This is shown in Fig. 89.

The question now arises, In which direction should the flux of the second motor rotate? It is clear that if the primary is stationary and short-circuited, the flux must revolve in the direction opposite to that of the shaft, since it must be almost stationary with respect to the (fixed) stator. Hence, while in our previous type (primary rotor connected to secondary stator) the secondary flux goes in the

same direction as the shaft, in our present type the flux goes in the opposite direction to the shaft.

We may, however, set the secondary stator in rotation as well as the rotor, in which case the flux may go in the same direction as the shaft, and we get an example of a type of cascade machine having three elements all going at different speeds.

An instance of such a cascade machine would be the following,
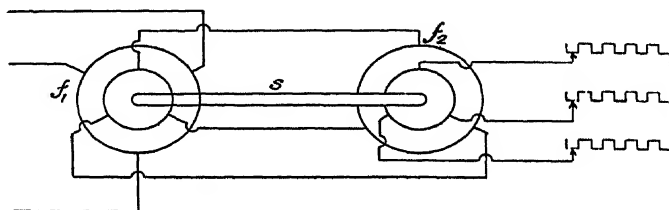
Fig. 87
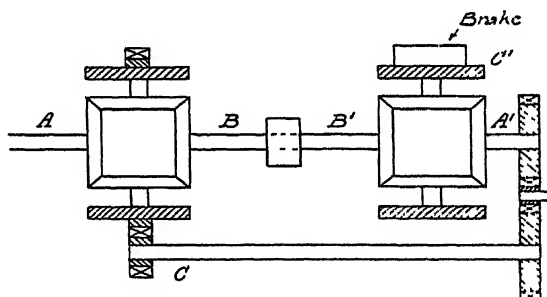
Fig. 88
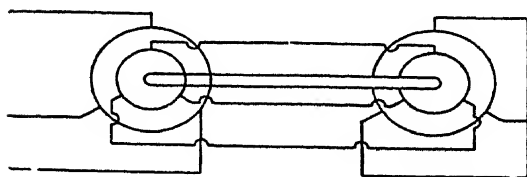
Fig. 89

in which an inverted direct-current convertor is used to feed an induction motor—

The induction motor primary is mounted on the shaft of the convertor, and its flux runs the same way as the convertor armature.

The squirrel-cage armature of the induction motor, therefore, runs at a speed which is the sum of the speeds of the convertor, and the induction motor flux relative to the primary. Such an arrangement supplies a means of producing a high speed in the squirrel-cage rotor, while the direct-current motor continues to run at a low one. Inverted, it forms the turbo convertor described below.

The above brief description is merely inserted here to give some idea of the type of cascade machine having three relatively rotating parts. Another convenient example will be the following—

Consider a synchronous generator revolving field type of which the field and armature are shown at A_1B_1 (Fig. 150), connected to an induction motor of which the primary and secondary are shown at CD. Let both the field and armature of the alternator be capable of revolution, the armature carrying with it the primary of the induction motor. The connection between armature and primary, of course, is such that the motor flux goes the opposite way to the primary itself. The speed will then adjust itself, so that the motor flux is approximately stationary with respect to the (fixed) stator and its squirrel-cage winding.

Suppose both the windings B and C have the same number of poles. The frequency in B is the same as that in C. The relative speed of C and D and, likewise, the relative speed of A and B are each the same multiple of this frequency, and must, therefore, be equal.

Thus we conclude—

If D is stationary, B and C will go at half the speed of A, or $B = \frac{1}{2}(A + D)$.

This relation, however, is exactly that of the speeds of the three shafts of a differential gear

Hence the above arrangement may be regarded as *an electrical differential gear*.

We may note that in the more ordinary type of cascade set, the same relation is obeyed by the frequencies in the three members.

We have

$$f_2 = \tfrac{1}{2}(f_1 + f_3)$$

where f_2 is the secondary or intermediate frequency, f_1 the line frequency, and f_3 that of the secondary of the second motor.

Thus the two types are reciprocal as regards speed and frequency.

Induction motors cascaded with synchronous machines. It is not difficult to see that the chief difference between the induction and synchronous machine lies in the method of magnetization, as has been discussed at length above.

We may, therefore, substitute a synchronous machine for the second induction machine of our cascade set.

The only difference we need make in our arrangements is to wind what was before the secondary of our second machine as a continuous current magnet.

In the present arrangement the characteristics will be somewhat different from those discussed in the last chapter. In the first place there will be no starting torque, and the speed will be rigidly limited to the synchronous speed deduced above.

In addition to this, such a set possesses, as a rule, two sources of magnetizing power—the line and the field magnets of the synchronous machine, and, hence, hunting is not impossible, although it has not been found to give rise to serious trouble in practical machines.

Such a set, therefore, finds its principal applications as a generator or convertor rather than as a motor.

We shall discuss here two of its applications—

1. As frequency convertor.
2. As " motor convertor."

Since in a synchronous set the speed and secondary frequency is quite independent of the load, it follows that such a set will give a fixed ratio of conversion, independent of the load.

The chief points, therefore, for our consideration are—

1. What will be the voltage regulation of such an apparatus ?
2. What must be the relative proportions of the two machines ?

Dealing with the second question first. We must note that our machine is now a convertor, and neither takes in nor gives out mechanical power, while the induction element necessarily gives a positive or negative torque at all speeds differing from synchronism. It follows, therefore, that the synchronous element gives a nearly equal and opposite torque, since the resultant of the two must be merely enough to overcome the mechanical friction.

Since the two machines are mechanically coupled together their speeds are also equal, and, hence, their horse-power outputs are equal. It will be clear that the arguments we have used are equally true, whether the number of poles on the two machines is the same or not.

If we suppose that the induction element is operating below synchronism it is a motor, and drives the synchronous element as a generator. The following will be the relations between the powers of the two elements—

Induction Machine

Primary power input	=	Mechanical power output + secondary electrical power output

Synchronous Machine

Mechanical power input + secondary electrical power conducted in from induction machine	=	Electrical power generated + electrical power flowing through to terminals from induction machine

But our set may also run above the synchronous speed of the induction machine, if there are different numbers of poles in the two elements and it is connected in differential cascade.

In this case the induction machine will be a generator and will receive mechanical power from the synchronous machine, furnishing to it in return an amount of electrical power equal to the total primary power + the mechanical power it receives from the secondary.

Let us consider, by way of example, a set arranged for the purpose of frequency conversion. In this set the electrical power is supplied to the primary of the induction machine, and the electrical power flowing out of the secondary is led into the synchronous machine, which acts as a " booster," and from which it passes to the line.

We have now to consider the relation of primary and secondary frequency for various numbers of poles.

Starting, as in the last chapter, with the formula

$$S = W'_1 (f_1 - f_2) = \pm W_2 f_2$$

we get

$$f_2 = f_1 \frac{W_1}{W_1 \pm W_2} \qquad \text{on eliminating } S$$

or putting

$$W_1 = \frac{1}{p_1} \quad \text{and} \quad W'_2 = \frac{1}{p_2}$$

$$f_2 = f_1 \frac{p_2}{p_1 \pm p_2}$$

Substituting again for f_2 in the original formula, we get

$$S = f_1 \frac{W_1 W_2}{W_1 \pm W_2} \text{ or } S (p_1 \pm p_2) = f_1 \text{ and } S p_2 = f_2$$

These are the fundamental formulae regulating the speed, and the numbers of poles required for a given change of frequency.

Let us illustrate the above formulae by the rather awkward case of transformation from 60 to 25 cycles.

In accordance with the formula

$$f_2 = f_1 \frac{P_2}{P_1 + P_2}$$

and if

$$f_2 = 25 \text{ and } f_1 = 60$$

since

$$\frac{25}{60} = \frac{10}{24}$$

we may most conveniently take 10 poles for our synchronous machine, and 14 for the induction. We then have 10 + 14 = 24.

Now suppose the set is differentially connected, and the primary still supplied with 60 cycles.

line 8 : " For satisfactory working and symmetrical distribution of the forces, the two numbers of poles for the rotor winding must be such that when these numbers are divided by their greatest common factor, the one dividend must be odd and the other even, whilst the common factor itself must be greater than 2."

On page 408 of his paper in Vol. 52 of the *Journal of the Institution of Electrical Engineers,* he states that, to satisfy the condition of magnetic balance, the greatest common factor between the two numbers of poles must be a number greater than 2. Any two numbers of poles must have a common factor of 2, since they are necessarily even. By the Hunt rule they are also to have a common factor other than 2, and, hence, it is clear arithmetically that their highest common factor must be a multiple of 2. More concisely stated this simply means that the two numbers of pairs of poles shall have a common factor. The effect of this is that the circumference is divided into two or more identical zones or sections, so that any values of magnetic density, current, etc., which occur at any one point, occur also at a diametrically opposite point or at 3, 4, 5, etc., equally spaced points, according as the common factor between the numbers of pairs of poles is 2, 3, 4, 5, or any other number.

The effect of this is, of course, to ensure magnetic balance, since identically the same magnetic pull as occurs at any one point will occur at two or more equally spaced points. We shall see below that this condition, while correct and sufficient, is by no means necessary. In the first place it is clear that, if the necessary condition of magnetic balance is that the same magnetic density must occur at two or more equally spaced points round the circumference (the condition to which Hunt's rule leads us), then this condition must be satisfied not only by cascade machines, but by every other type of machine if it is to be magnetically balanced. But the briefest study will show that it is not satisfied, for instance, by a 2-pole induction motor or any other 2-pole machine, since there is no point in such a machine which has at every instant the same magnetic density as any given point. Hence it is clear that Hunt's rule cannot be perfectly general.

The explanation of this apparent difficulty is as follows- -

The magnetic attraction at any point on the circumference is proportional not to the magnetic density but to its square. Hence, not merely points having equal density, but those having equal and opposite densities, will give rise to the same magnetic attraction. In the case of the 2-pole machine diameterically opposite points have equal and opposite magnetic densities, and, hence, the squares of these two densities are equal and the magnetic pull at diametrically opposite points is the same. The most general condition is, therefore, that at every instant any value of the square of the magnetic density which appears at any po'nt shall also appear at

one or more other points equally distributed round the circumference with respect to the first.

For convenience in application it is necessary to translate this generalized rule into a relation between numbers of poles, and we find that we may distinguish two distinct ranges of cascade motors which will be magnetically balanced—

(*a*) The Hunt type, in which the numbers of pairs of poles have a common factor.

(*b*) A new type, in which the numbers of pairs of poles are prime to one another and both odd.

In fact, in all machines having odd numbers of pairs of poles, diametrically opposite points have equal and opposite magnetic densities. Hence, if two fields both having an odd number of pairs of poles are superposed, and the values of the densities due to each at any given point are X and Y, say, so that the resultant density at that point is $X + Y$, then at a diametrically opposite point the densities due to each field will be $-X$ and $-Y$, and the resultant density $-(X + Y)$. The squares of the densities at the two diametrically opposite points will thus be equal. Thus we find it is possible to generalize Hunt's rule for magnetic balance, and thereby to disclose an entirely new range of cascade machines, many of which have very valuable properties.

Viewing the question of magnetic balance from a somewhat different standpoint it appears as follows—

Let the wave of flux density of one pole number be $\beta_1 \sin (m_1 x - pt)$

where $m_1 = \dfrac{\pi \times \text{number of poles}}{\text{circumference}}$

$p = 2\pi \times \text{rotor frequency}.$

That due to the other $\beta_2 \sin (m_2 x - pt)$,
and let x be measured from an origin fixed relative to the rotor.
Resultant flux density $\beta = \beta_1 \sin (m_1 x - pt) + \beta_2 \sin (m_2 x - pt)$.
m_2 may be taken negative to account for the case where the two fluxes travel opposite ways with respect to the rotor.

Magnetic pull is proportional to β^2

$$\beta^2 = \beta_1{}^2 \sin^2 (m_1 x - pt) + \beta_2{}^2 \sin^2 (m_2 x - pt)$$
$$+ 2\beta_1 \beta_2 \sin (m_1 x - pt) \sin (m_2 x - pt)$$

This may be resolved into a series of sine waves.

$$\beta^2 = \beta_1{}^2 \tfrac{1}{2} \left\{ 1 - \cos 2(m_1 x - pt) \right\}$$
$$+ \beta_2{}^2 \tfrac{1}{2} \left\{ 1 - \cos 2(m_2 x - pt) \right\}$$
$$+ \beta_1 \beta_2 \left[\cos (m_1 - m_2) x + \cos \left\{ (m_1 + m_2) x - 2 pt \right\} \right].$$

This expression consists (besides a constant term uniform round the periphery) of (*a*) a wave leaving the number of poles of the one

flux, (*b*) a wave having the number of poles of the second flux, (*c*) a wave having the sum of the number of poles of the two and doubled frequency, (*d*) a wave having the difference of the number of poles of the two fluxes and stationary with respect to the rotor.

This series of waves (with the constant term) represents the magnetic pull at all points round the circumference.

Where the sine wave is positive the pull is inward, say, towards the circumference. Where it is negative it is outward.

Plot a sine wave on a circular base representing the circumference.

Two cases arise—

1. If there is a " two-pole " wave, the pull on, say, the upper half of the circumference is towards the circumference that is down-
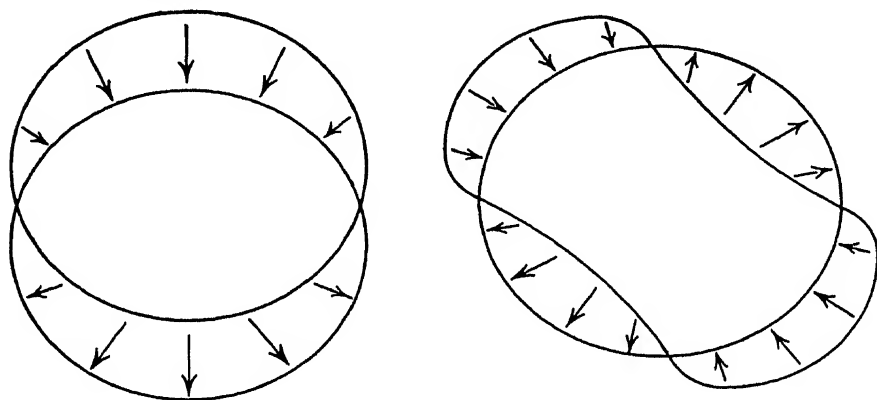
Fig. 90

ward in the figure (*a*) and that on the lower half of the circumference is away from the circumference, that is still downward in the figure. That is, both halves of the wave produce a pull having the same direction in space, and hence the magnetic pull is unbalanced.

2. If the wave has more than two poles, Fig. (*b*), then the zones in which the pull is towards or away from the circumference are symmetrically distributed so that there is no unbalanced magnetic pull.

Hence, we get the final rule for magnetic balance, embracing both the earlier ones and indicating fresh possibilities. *In a machine having fluxes of two different pole-numbers, the pole numbers must differ by more than two to secure magnetic balance.*

The cascade machine has hitherto been considered to be essentially a low speed type. This arises from the facts that the number of poles in the cascade motor must be the sum of the number of poles of two distinct machines, and that both numbers of pairs of poles are sufficiently high to permit of there being a common factor between them. The new range of machines does not necessitate

there being any common factor between the two numbers of pairs of poles. Hence, it permits of the production of machines of a higher speed than has hitherto been possible. Described below in detail is a combination having 2 and 6 poles which gives a cascade speed corresponding to 8 poles, i.e. 50 per cent higher than the highest cascade speed obtainable from machines built under Hunt's rule.

We now have to consider whether any limitation in the pole combinations is required in order to enable the windings to be mutually non-inductive. Where a winding of the type described by Hunt is used as a stator winding, it is necessary that the numbers of pairs of poles when divided by their greatest common factor shall be odd in one case and even in the other But it is possible to design types of stator winding, other than that described by Mr. Hunt, to which this limitation does not apply, and examples of these will be found below. Obviously, since in the new types of mechanically balanced machines both numbers of pairs of poles must be odd, the condition imposed by Mr. Hunt's stator winding would not be met, and it is necessary to have recourse to other methods, which will be found described below.

In the general case, where two distinct stator windings are used, it may readily be shown that almost any two stator windings containing no closed circuits will be mutually non-inductive, there being only a few exceptions, practically confined to cases where the number of poles in one case is a direct multiple of that in the other. Thus, by using two distinct windings on the primary member, neither of which contains any closed circuits, we can produce an internal cascade motor capable of operating on practically any combination of poles, with the possible exception of poles which are exact multiples of one another. Therefore, in studying the rotor windings as we shall do below, we need not be handicapped by the thought that pole combinations which give rise to a possible rotor winding cannot be used because an appropriate stator winding cannot be found for them.

Rotor Windings. It will be generally agreed that the rotor winding is both that part of the internal cascade machine which is most difficult to understand, and also the part which differentiates it from every other class of apparatus. An attempt has, therefore, been made to consider it from an entirely novel and very fundamental standpoint, in a manner quite independent of any other method. This has the advantage of making clear some very interesting features of these windings. It is shown below that a winding adapted to any given cascade speed can operate with any balanced combination of fields which gives rise to the given cascade speed. For instance, a rotor adapted to run with a 20-pole cascade speed can operate with either a combination of fields giving 8 and 12 poles,

or a combination of fields giving 16 and 4 poles. Now, considered as a variable speed motor, perhaps the most striking feature of the internal cascade machine is that when the slip-rings are short-circuited the motor rises to the speed corresponding to the number of poles for which the primary is wound. Thus, a machine having a cascade speed corresponding to 12 poles and a primary wound for 8 poles will, when the slip-rings are short-circuited, rise from the 12-pole speed to the 8-pole speed. If the primary is arranged for 4 poles, however, it will rise from the 12-pole speed to the 4-pole speed. The same characteristic is retained in the case just mentioned. For instance, if the stator winding is arranged to give either 4, 8, 12 or 16 poles, the motor will rise from its 20-pole cascade speed to 16-pole speed, to 12-pole speed, to 8-pole speed, or to 4-pole speed, according to which of these numbers of poles the stator winding is arranged for. Hence, instead of the three-speed cascade motor as hitherto developed it is found possible by this means to obtain a five-speed cascade motor. Similarly, a cascade motor arranged to give a cascade speed corresponding to 28 poles can be arranged with the same rotor winding to operate on 4, 8, 12, 16, 20 and 24 poles, rising to any one of these speeds, on short-circuiting the slip-rings, according to which of them the stator winding is arranged for.

CHAPTER XV

IT has been mentioned that the essential part of the cascade motor is its rotor winding, and it is accordingly essential that this should be clearly understood. One of the clearest ways of regarding the subject seems to be the following. Many people are familiar with the acoustic phenomenon known as " beats," and most electrical engineers are acquainted both with the method of synchronizing an alternator by means of the alteration in the flickering of a lamp as we approach synchronism, and with the explanation thereof.
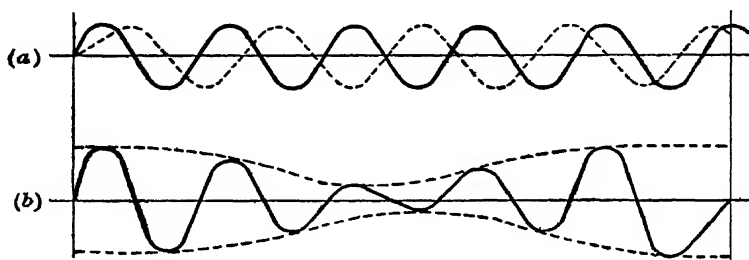
FIG 91

The compound winding of the cascade rotor is an application of this phenomenon of " beats " in another field. " Beats " in music, or the flickering of the synchronizing lamp, are due to the following cause. Let there be two E.M.F. waves of different frequency which are plotted in Fig. 91, in which the ordinates represent E.M.F., and the abscissae time. Starting with both waves approximately in phase (passing through zero together), after a certain number of periods, due to the difference of periodic time, the two waves become opposite in phase and the amplitude of the resultant wave (Fig. 91 (b)), which was first of all the sum of the amplitudes of the two component waves, now becomes their difference. After a further number of periods the waves are in phase again, and the resultant is again the sum of their amplitudes. The period of this cycle, or " beat," is the difference between the periods of the two component waves.

As an application of this phenomenon will be described below, it is desirable to dwell upon it further.

Another method of studying the subject which is often adopted is that of the clock diagram, whereby each wave is represented by a revolving vector whose projection on a vertical axis, say,

represents its value at any moment in one complete revolution. Since the periods of the two waves are unequal, the rates of revolution of the two vectors are unequal.

Fig. 92a shows the two vectors, the resultant being the vector sum of the two (projected on the vertical as usual). Clearly the
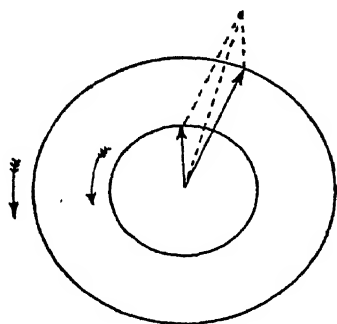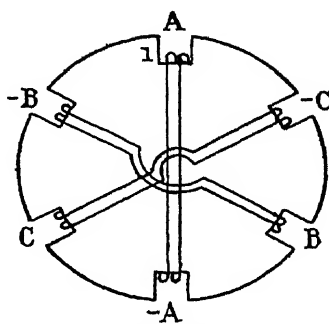
FIG. 92a   FIG. 92b

resultant is a maximum when the two vectors have the same direction, and a minimum when they have opposite directions. The relative speed of the two vectors is clearly also the difference of their actual speeds, and, hence, the frequency of the beat will be the difference between the frequencies of rotation of the two vectors. Thus, when the two vectors have accomplished a relative movement of 180°, which is clearly a half-period of relative motion, the resultant curve has changed from maximum to minimum.
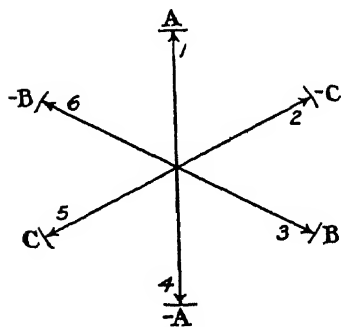
FIG. 93

Now let us see how the above remarks apply to the cascade motor. Consider first a plain drum armature or rotor with 6 slots (Fig. 92b) arranged for three-phase currents represented as vectors in Fig. 93. Let us set out the ampere-conductors per slot in the form of a vector diagram. In slot 1 the ampere-conductors are in phase A, and may be represented by a vertical vector of suitable length. Similarly in slots 3 and 5 the ampere-conductors are in phases B and C respectively, and may be represented by vectors of the same length, since there is the same number of conductors in each slot, making 120° and 240° respectively with the first. In slot 4 we have the return conductors of slot 1. Hence the ampere-conductors of slot 4 are equal and opposite to those of slot 1, and, similarly, those of slots 6 and 2 are equal and opposite to those of 3 and 5.

Examining the diagram obtained by this process, we see that the vector for each slot is of equal magnitude and set out from a common origin, and that on making the circuit of the rotor from slot to slot the vectors corresponding to slot 1 may be regarded as rotating and moving through equal angles for equal distances travelled through, as measured round the circumference. With only 6 slots the vector proceeds by steps, but clearly in an ideal machine we should have an infinite number of slots each carrying currents differing in phase by a very small angle proportional to the distance from slot to slot. In this case, as we travel uniformly round the rotor, the ampere-conductor vector in our diagram would also rotate uniformly.

So far we have only considered a 2-pole rotor. Now suppose we have 12 slots and a 4-pole rotor (Fig. 94). The vector diagram
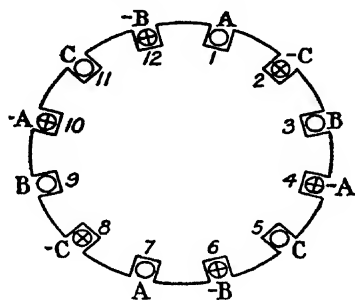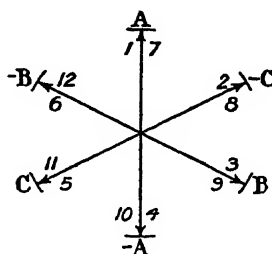
FIG 94   FIG. 95

remains exactly the same as before, but the revolving vector makes a complete revolution in travelling from slot 1 to slot 7, or two complete revolutions travelling round the whole circumference. Each vector, therefore, represents the ampere-conductors in two slots, such as 1 and 7, 2 and 8, etc., instead of in one (see Fig. 95). In general it is clear that for n pairs of poles the vector makes n complete revolutions while we go once round the rotor. Comparing therefore the same rotor wound for, say, 2 and 4 poles, we see that with the 4-pole winding for a given distance, say one-twelfth, travelled round the circumference, there is double the angular displacement of the rotating vector compared with that on 2 poles. In other words, on 4 poles the rotating vector revolves at twice its 2-pole speed (see Fig. 96).

This fact enables us to distinguish a rotating vector drawn in this way from an apparently similar one representing the rotating flux or ampere-turns due to such a rotor on a time basis. Such a rotating vector revolves only half as fast in the 4-pole case as in the 2-pole, the speed being inversely proportional to the number of poles, while the speed of the vector drawn in the above-mentioned manner is directly proportional thereto. In all matters of this

kind the utmost clearness as to fundamentals is essential, and by proceeding to deduce results without making sure of fundamentals serious errors may be committed. Therefore no apology is needed for carefully distinguishing the present diagram from a time diagram.

Such a diagram, when a continuous curve, is the laws of the vector as a function of x or 0 measured circumferentially.

A convenient practical method of drawing a diagram of the above type is as follows—

Draw a circle of radius equal to the length of the rotating vector ; divide the circumference into any suitable number of divisions, and
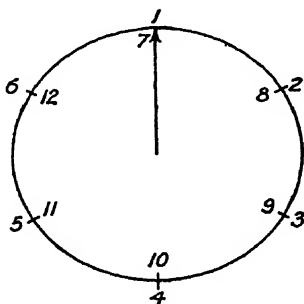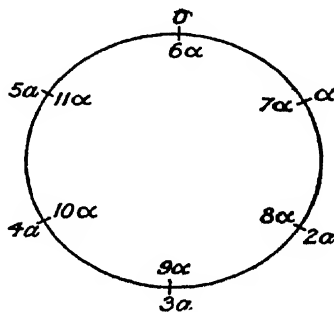
FIG. 96   FIG. 97

number these divisions 1 to n. Determine the angular displacement corresponding to any one division. This will be

$$\frac{(360° \times \text{number of pairs of poles})}{\text{Number of divisions}} = a$$

Starting from any point mark off angular distances a, $2a$ . . . na (which will be a multiple of 360°) along the circumference of the circle representing the revolving vector. Mark the points so obtained 1, 2 . . . n, corresponding to the number of divisions of the circumference. For instance, with $n = 12$ and 2 pairs of poles, $a = 60°$, and we get the result shown in Fig. 97, in which it is clearly unnecessary to repeat the angle in marking each division.

Having explained how we may draw a vector diagram in the form of a rotating vector to represent the ampere-conductors of a polyphase winding on an ideal machine having a very large number of slots and phases, we may proceed to apply this diagram to the cascade motor.

Returning to the 6-slot rotor of Fig. 92, let us now suppose that, in addition to the 2-pole winding we considered before, the rotor is fitted with a 4-pole winding as shown in the outer circle of Fig. 98, the six bars composing it being imagined joined in star at one end of the armature and to the terminals .at the other. The vector

diagrams corresponding to these two windings are shown in Fig. 99. If we consider any particular slot, say slot 2 for instance, the resultant ampere-conductors in it are the vector sum of those due to the 2-pole winding and to the 4-pole winding. That is to say, we have to add the vectors from Figs. 99 (*a*) and (*b*) which correspond to the same point on the circumference, i.e. the vectors marked 2 in the above diagrams, whose resultant is shown in Fig. 99 (*c*).

The general rule, therefore, for finding the resultant vector of ampere-conductors for any number of windings on the same member is to take the resultant of all the vectors of the original windings corresponding to the same point on the armature.

FIG. 98

Let us apply this to the case where the ampere-conductors in both windings are represented by uniformly revolving vectors, which for the sake of clearness we may suppose to revolve, one at twice the rate of the other, in opposite directions. By choosing a particular ratio of magnitude, i e. the 4-pole vector equal to 1·732 times the 2-pole vector, we get a case corresponding to one of the cases discussed by Mr Hunt, which will accordingly permit us to compare our method with his.

Fig. 100 shows a diagram proportioned and arranged in this manner, the inner or 2-pole vector rotating clockwise with half the

Two-pole winding   Four-pole winding
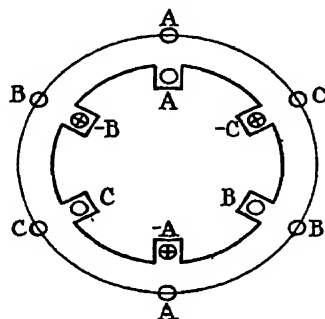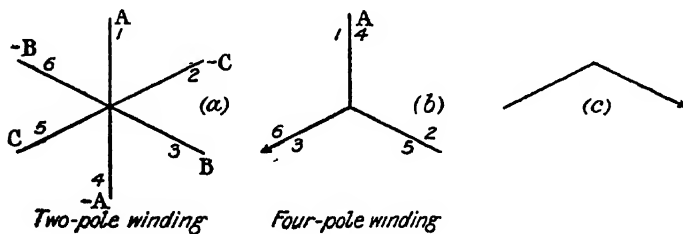
FIG 99

speed of the outer. The numbers on the inner circle show that the circumference of the rotor has been divided into 24 equal parts independently of the number of slots (if any). The same numbers appear on the outer circle angularly displaced by an amount twice as great. A particular phase appears twice at points diametrically opposite on the rotor, so that it is clear that the vector makes two complete revolutions in the same time that the inner vector makes one. The circumference is divided into a definite number of parts to enable us to identify the vectors on the two circles which

correspond to the same point on the rotor. The number of divisions is chosen merely to give a sufficient number of points to enable us to draw a smooth resultant curve, and not with any reference to the number of slots.

It will be seen that we have now arrived at a diagram identical in principle with Fig. 92a, by which we illustrated the " beats " of a synchronizing lamp, except that in the former the different positions of the rotating vector correspond to different instants of time instead of to different points on the circumference of a rotor. Fig. 92a is a " time " diagram and Fig. 100 a " space " diagram, otherwise they are identical in principle.

By means of the two rotating vectors of Fig. 100 let us plot out the resultant curve of ampere-conductors round the rotor, due
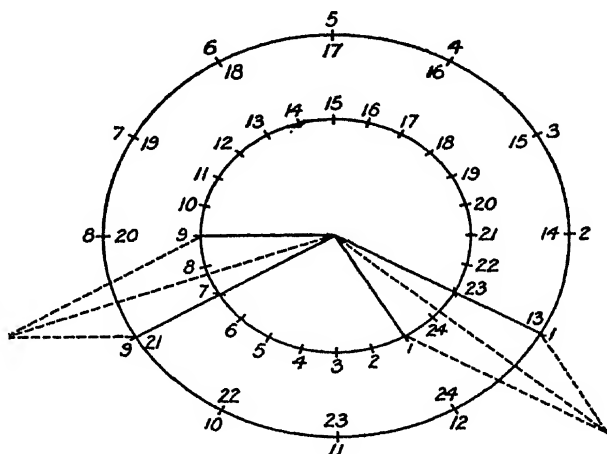
Fig. 100

to 2-pole and 4-pole windings rotating in opposite directions. This is done by first forming the vector sum of the two component vectors corresponding to point 1, marking its extremity as the resultant at point 1 ; next forming the resultant of the two vectors corresponding to point 2, marking its extremity as the resultant at point 2 ; and so on round the circumference. In this way we plot out point by point a curve having three symmetrical lobes, which gives us the resultant ampere-conductors both in magnitude and in phase at any point round the circumference. This curve is shown in Fig. 101 and, as it is of the utmost interest and importance, we shall devote some space to studying it and its relation to other methods of regarding the subject.

Consider, for instance, Hunt's investigation given in his paper above referred to. In this paper he considers 4-pole and 2-pole windings rotating in opposite directions, as we have done (for

the present investigation has been modelled to give results as comparable as possible with his). He chooses a definite number of slots (12) and winds into these slots two distinct windings, so connected and of such pitch that the ampere-conductors per slot of the 4-pole winding are 1·732 times those of the 2-pole winding. These two windings are connected in series in such a manner that

Ratio:- 4-pole 1·73
2-pole 1·0
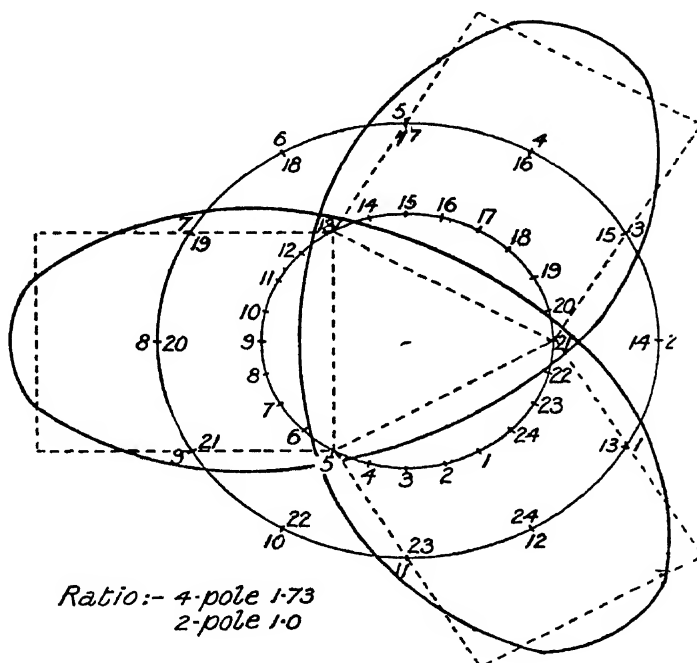
Fig. 101

the R.M S. currents in all conductors are equal in magnitude, and a symbol is attached to each conductor indicating its phase.

A diagram of these two windings in series is given in Fig 102. By a suitable arrangement of the relative position of the two windings, Hunt then succeeds in showing that in certain slots there are bars carrying equal and opposite currents which he then cancels. This process, it is clear, is one of finding the resultant ampere-conductors of two distinct component windings at a particular slot. That is to say, Hunt's process and the one illustrated above are the same in principle.

Nothing could be more conclusive or more graphic than Hunt's process where it is applicable, but the existence of so many different variables, for instance, (1) number of slots, (2) conductors per slot on the first winding, (3) conductors per slot on the second winding, (4) pitch of first winding, (5) pitch of second winding, (6) relative

displacement between the two windings, (7) number of phases, (8) connection of windings (star, mesh, etc.), makes it a process very limited and difficult to apply. On the other hand, the vector method of drawing the resultant eliminates nearly all of these variables, and enables us to study cases where the use of Hunt's
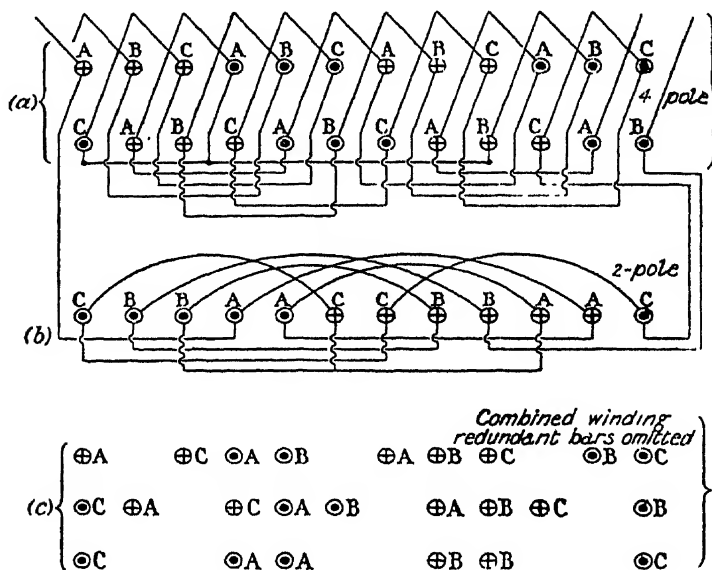
Fig. 102
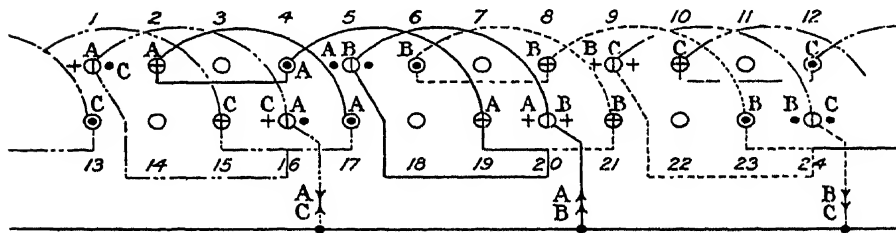
Fig. 102*a*

method is impossible, or is at least extremely difficult of application.

We shall now, for comparison with Fig. 101, draw a diagram of ampere-conductors represented as vectors for the case of an armature having 12 slots and carrying the winding developed by Hunt. Hunt's method of indicating the phases of his currents differs so widely from the vector method that it is necessary first of all to prepare a little dictionary of symbols, whereby we may translate his symbolism into vector diagrams, as given in the table at the top of the next page.

Symbol	Magnitude	Phase
A ⊕	1·0	0°
B ⊕	1·0	60°
C ⊕	1·0	120°
A ○	1·0	180°
B ○	1·0	240°
C ○	1·0	300°

That is to say, where Hunt marks a bar as in the left-hand column, the phase of the bar is that shown in the right-hand column.
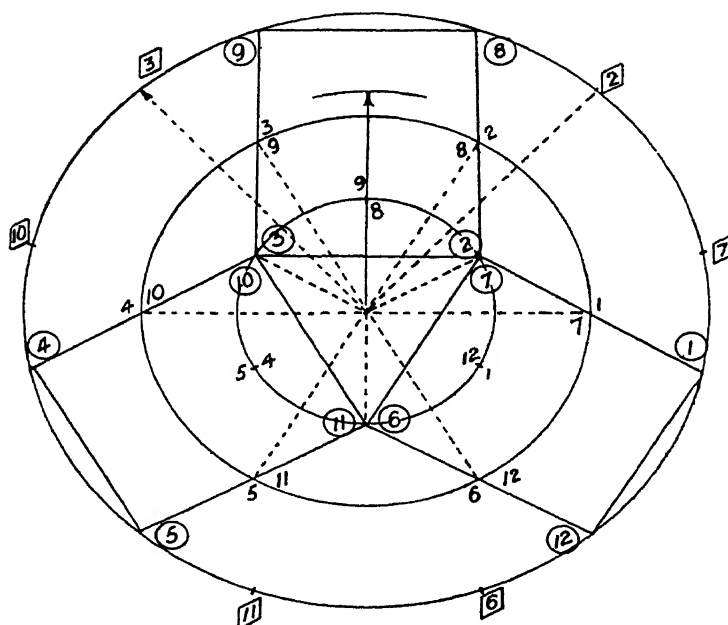
Fig. 103

With the aid of this table we may draw the vector diagram corresponding to each of Hunt's component windings. First of all we note, as Hunt points out, that the phase difference between top and bottom conductors in the 4-pole winding is always 60°. Hence the resultant ampere-conductors per bar will be 1·73 times those in any one conductor. We also note that the phase difference between conductors in adjacent slots is always 60°. In the 2-pole winding adjacent slots have the same currents in them in pairs as 2 and 3, 4 and 5, etc.

From these data we may draw Fig. 103, containing two concentric circles of radii in the ratio of 1·73 : 1, their circumferences being numbered in the manner shown. By producing the resultant of

vectors to points on the two circles similarly numbered, and join-
ing the extremities of these vectors, we get Fig. 103 which corre-
sponds to Fig. 101, except that the vectors change from one value
to the next in steps instead of gradually. In Fig. 101 this polygon

(a)   (b)   (c)
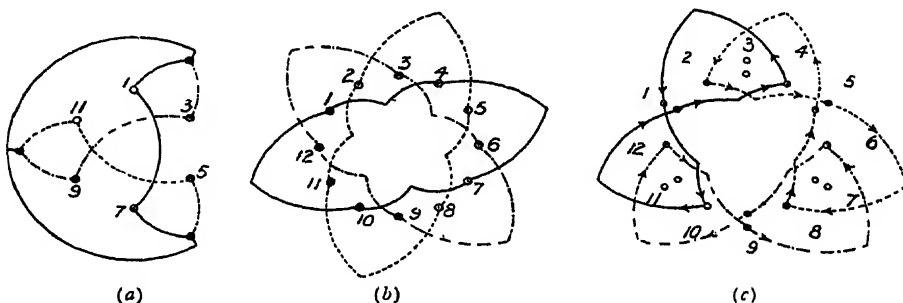
FIG. 104

is shown dotted and superposed on the 3-lobed figure derived from
the previous calculation. It will be seen that this polygon repre-
sents an approximation as close as can be obtained with only
12 slots. In a practical case there would be more slots, say not
less than 48, which would give
a still closer approximation.
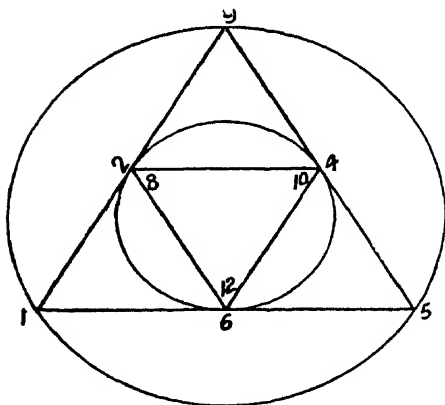
FIG. 105

Hunt has shown so con-
clusively the identity of his
two component windings with
his resultant winding that it is
unnecessary for us to compare
again this latter with our ideal
3-lobed figure. Already, how-
ever, we begin to see what is
the essential characteristic of
the cascade rotor, viz. that
certain parts of the armature
have a greater number of
ampere-conductors per slot than
others.

In addition to the winding
we have just analysed, which is the one chiefly employed in practice,
Hunt has described, chiefly in his patents, several other windings.
It will be instructive to draw corresponding ampere-conductor
diagrams for these windings also.

Referring, therefore, to Hunt's British Patent No. 15 711/1906
and his Figs. 1, 2, and 3 (reproduced in Fig. 104 here), let us
endeavour to translate the resultant diagram (Fig. 104 (c)) into
a vector diagram in the same manner as before.
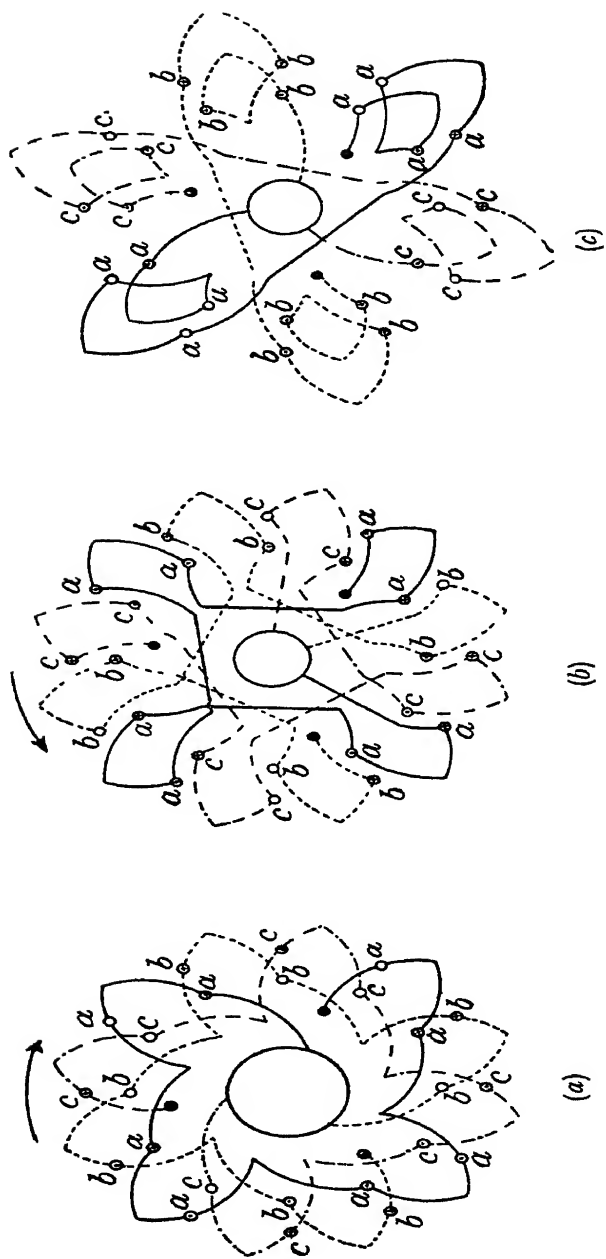
FIG 106

The same table as previously used will serve to translate Hunt's symbols into vectors.

In the polygon (triangle) obtained in Fig. 105 we clearly have an approximation to a 3-lobed figure such as that of Fig. 101.
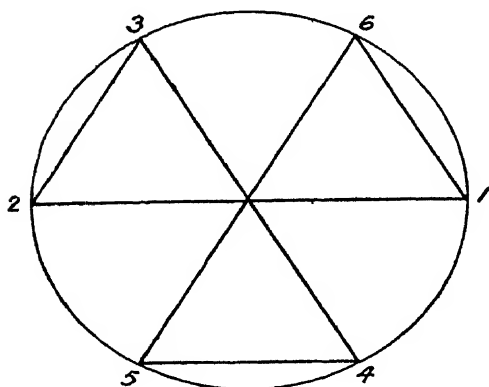
FIG. 107

Another curious winding is shown in Hunt's Figs. 4, 5, and 6 (reproduced here in Fig. 106, (*a*) and (*b*) representing the components, and (*c*) the resultant winding.

Another 3-lobed figure of a rather imperfect type is shown in Fig. 107, derived from Fig. 106 (*c*).

It should now be obvious from the examples we have discussed that *any* winding which yields a vector diagram approximating sufficiently closely to our original lobed figure can be used as a cascade secondary. For instance, (*c*) in Fig. 106 may be much improved by the addition

2 pole – 4 pole diagram for 36 slots
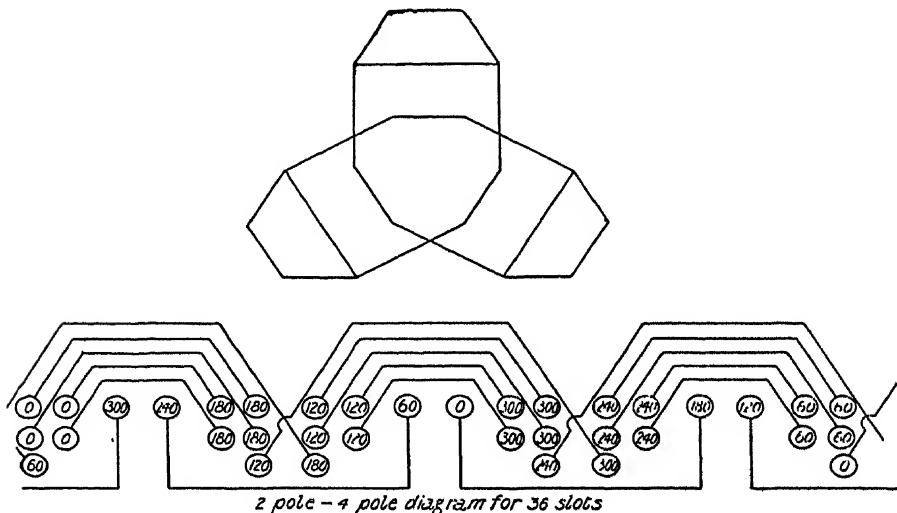
FIG. 108

of some further conductors, thereby making it practically equal to Fig. 103. Diagrams of this improved winding and its vector diagram are shown in Fig. 108.

Fig. 108 is a vector diagram and diagram of connections for a

winding having 36 slots connected as shown, which has merely been built up empirically so as to give a vector polygon as close as possible to the ideal curve, which it approaches very closely. No attention has been paid to determining whether such a winding can be arrived at by superposing two other windings, and the principal object of explaining it is to make it clear that this is not in any sense a necessary condition, and that the sole condition to be met is that it shall conform as closely as possible to the ideal curve. From the diagram of connections it will be seen how the fundamental characteristic of the cascade rotor winding, i.e. that of having a periodic variation corresponding to the " beats " or ampere-conductors per slot, is shown, since the number of conductors per slot varies from 1 to 3.

We have thus shown how actual windings may be constructed in various ways to give a distribution of ampere-conductors per
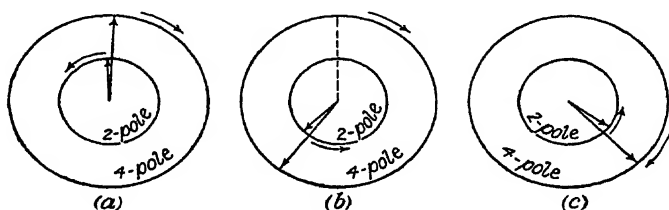
FIG 109

slot approximating to that of the 3-lobed curve derived from two uniformly rotating vectors, one of which revolves through double the angle traversed by the other for equal displacements along the periphery of the rotor.

Two questions at once suggest themselves. It has been pointed out above that cascade motors may be built for many other ratios of numbers of poles than 2 . 1. What curves will arise in these cases, and how can we fit windings to them, and what determines the number of maxima or lobes ? Let us now consider these questions, taking the second one first.

It is clear in the first place that the maxima can only occur when the two revolving vectors coincide in direction. Consider again the case where one vector is revolving twice as fast as the other and in the opposite direction, and assume that at a certain point in the periphery the two vectors coincide in direction and the resultant curve, therefore, has a maximum. This is shown in Fig. 109 (*a*). Now consider Fig. 109 (*b*), where the slower, say the 2-pole. vector has moved through 120°. Since the faster vector, say the 4-pole, goes twice as fast it will have moved through 240° in the opposite direction, and will, therefore, have arrived at the same point. Hence we have another maximum. Similarly, when the slower

vector has moved through 240° (Fig. 109 (*c*)), the faster vector will have moved through 480° (360° + 120°) in the opposite direction, and they will again coincide in direction, giving a third maximum. Finally, when the slower one has moved through 360° the faster one will have traversed 720°, and we return to the position from which we started. This explains why, in the case of a 2 : 1 speed ratio, we have a 3-lobed curve.

Let us now endeavour to express this in the form of a general rule, taking first the case of vectors revolving in opposite directions. When one of the vectors has traversed an angle θ assume that the other has traversed an angle $\theta \times b/a$, where b/a is the ratio of the numbers of pairs of poles of the two fields reduced to their lowest terms by dividing by any common factor. Then, if when $\theta = 0$ the two vectors coincide, they will again coincide when

$$\theta = 360° - (\theta \times b/a)$$

i.e. when

$$\theta = \frac{a \times 360°}{a + b} \text{ or } \frac{360°}{1 + (b/a)}$$

This will be called the characteristic angle.

Thus the angle between successive maxima is

$$\frac{a \times 360°}{(a + b)}$$

The number of maxima will be the smallest number by which this angle can be multiplied to make it a multiple of 360°.

This number will be $(a + b)$, since a and b have no common factor.

Thus our final rule is—

The numbers of maxima in the curve of resultant ampere-conductors is the sum of the numbers of pairs of poles in the two oppositely revolving component fields, after any common factor has been divided out.

A few numerical examples may make this rule clearer.

In the case we have just considered (Hunt's case) $b/a = 2/1$,

$$\therefore \theta = \frac{360°}{(2 + 1)} = 120°, \text{ and there will be } (2 + 1) = 3 \text{ maxima.}$$

Take the case of 4 and 6 poles or multiples thereof, which will also be balanced as has been shown above. Here $b/a = 3/2$,

$$\therefore \theta = \frac{2 \times 360°}{(2 + 3)} = 144°, \text{ and the number of maxima is } (2 + 3) = 5.$$

Here we notice for the first time a fact that it will be necessary for us to investigate fully, viz. that curves having the same number of maxima can correspond to different pole combinations. For instance, take the case of 2 and 8 poles or multiples thereof, another balanced combination. Here

$$b/a = 4/1, \therefore \theta = \frac{360°}{(4 + 1)} = 72°$$

and the number of maxima is again 5. Before investigating this, however, we must deal briefly with the case where both fields revolve in the same direction.

If both vectors rotate in the same direction but at different rates, starting from a position of coincidence, they will again coincide when the faster has turned through the same angle as the slower plus some multiple of 360°, that is, when the difference between the two angles is a multiple of 360°.

Expressing the above statement as a formula we get

$$\theta - \frac{\theta b}{a} = 360°, \quad \therefore \ \theta = \frac{a \times 360°}{(a - b)}$$

Take the case of 2 and 6 poles, also shown to be balanced. Here $b/a = 3/1$, $\therefore \ \theta = 360°/2 = 180°$, and the number of maxima for the same reason as before is $(a - b) = 2$. The negative sign refers only to direction of rotation, and may be neglected. Thus, for vectors rotating the same way our final rule is—

The number of maxima in the curve of resultant ampere-conductors is the difference between the numbers of pairs of poles in the two similarly revolving component fields, after any common factor has been divided out.

As regards the magnitude of these maxima, it is clear that this will be equal to the arithmetical sum of the magnitudes of the two component vectors, and the minimum value to their arithmetical difference. Hence, if they are equal the minimum value will be zero.

Several instances occur in the course of the present investigation where it is necessary to work out the difference of two equal vectors, E.M.F.'s or currents for instance, having a given phase difference. For two equal vectors, the well-known formula for the third side of a triangle having two sides a, b, and an included angle of θ ; viz. $a^2 + b^2 - 2\,ab \cos \theta = c^2$ reduces to $c = 2a \sin \frac{1}{2}\theta$. For instance, the E.M.F.'s in the two bars forming one turn of any coil differ in phase by an angle equal to $(180° \times$ coil pitch$)/($pole pitch$) = a$, say, and the resultant E.M.F. per turn is $E_1 = 2E$ (per bar) $\times \sin$ $(90° \times$ coil pitch$)/($pole pitch$)$, the latter quantity being frequently called the chord factor.

Again, if two turns are connected in series reversed, as in the case of turns round adjacent N and S poles of a machine, then the E.M.F.'s in the two turns will differ in phase by $\gamma = 180° \times$ (pitch of corresponding bars of the two turns)/(pole pitch), and the resultant E.M.F. of the two turns will be $E_2 = 2E_1 \sin \frac{1}{2}\gamma = 4E$ (per bar) $\sin \frac{1}{2}a \sin \frac{1}{2}\gamma$. The same formula enables us to calculate the star currents in the windings, each of which forms the resultant of two mesh currents, and is used in several places in the present paper.

RATIO OF THE FLUXES

In several of the windings, notably the Hunt star-mesh winding, each phase consists of two coils joined in series and in opposition. This is clearly shown in Fig. 110, in which the three circuits in the Hunt winding are shown by different kinds of line. It will be worth while, therefore, for us to study how to determine the ratio of the fluxes in such a circuit. We have just seen that the E.M.F., whatever the number of poles is,

$$E_2 = 4E \text{ (per bar) } \sin \tfrac{1}{2}\alpha \sin \tfrac{1}{2}\gamma \text{ (per circuit),}$$

where $\alpha = (180° \times \text{coil pitch})/(\text{pole pitch})$, and $\gamma = (180° \times \text{pitch}$ of corresponding bars)/(pole pitch).

Corresponding bars, for instance, might be those marked 30° and 240° in the top layer of Fig. 21. [?]
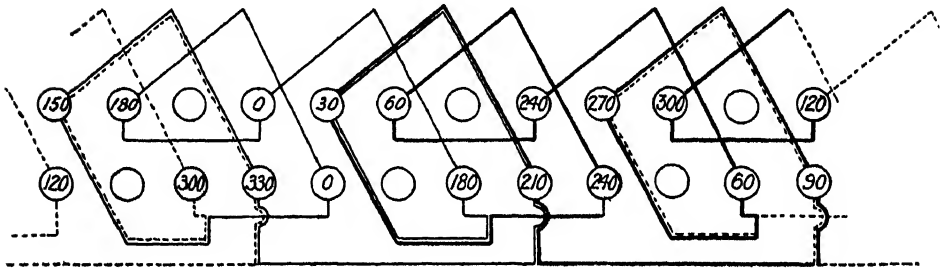
FIG. 110

It is clear that α and γ will be different for the two distinct numbers of poles of the cascade machine, and that the sum of the E.M.F.'s due to the two fluxes must be zero. Hence, if B_1, B_2 are the maximum densities due to the two fluxes, we have

Max. E.M.F. per bar $= V_1 B_1$ for the first flux.

Max. E.M.F. per bar $= V_2 B_2$ for the second flux.

V_1 and V_2 being the speeds of the fluxes relative to the rotor. The equation determining B_1/B_2 will be

$$V_1 B_1 \sin \tfrac{1}{2}\alpha_1 \sin \tfrac{1}{2}\gamma_1 = V_2 B_2 \sin \tfrac{1}{2}\alpha_2 \sin \tfrac{1}{2}\gamma_2$$

This ignores the breadth coefficient of the coils, which we shall have to discuss directly.

In some windings, for instance that shown in Fig. 102, we have Coil pitch = pitch of corresponding bars in the two coils. In this case $\alpha = \gamma$, and the formula becomes

$$V_1 B_1 \sin^2 \tfrac{1}{2}\alpha_1 = V_2 B_2 \sin^2 \tfrac{1}{2}\alpha_2$$

Taking this formula first, let us apply it to certain instances—

Case	No of Slots	Pitch of Coil	Poles	α_1	Sin ½α	V	(Sin ½α)²	δ	Sin $\left(180 - \dfrac{\delta}{2}\right)$
1	12	1—4	2	90°	0·707	2	0·5	30	0 965
	12	1—4	4	180°	1 0	1	1	60	0·866
2	20	1—4	4	108°	0·81	3	0·655	36	0·95
	20	1 - 4	6	162°	0 99	2	0 98	54	0 88
3	20	1 - 4	2	54°	0 455	4	0 207	18	0 99
	20	1—4	8	216°	0 96	1	0 92	72	0·81
4	16	1—4	2	67 6°	0·557	3	0 301	22½	0 98
	16	1 - 4	6	203°	0·98	1	0 96	67½	0 83

Case 1. $B_1 \times 2 \times 0\cdot5 = B_2 \times 1 \times 1$; $B_1 = B_2$.

This is Hunt's case, for one of his windings.

Case 2. $B_1 \times 3 \times 0\cdot655 = B_2 \times 2 \times 0\cdot98$; $B_1 = 0\cdot99 B_2$.

Case 3. $B_1 \times 4 \times \cdot207 = B_2 \times 1 \times 0\cdot92$; $B_1 = 1\cdot11 B_2$.

Cases (2) and (3) are the cases discussed below, in which identically the same winding is operated on two different combinations of poles, enabling us to get 4, 8, 12, and 16 poles with a 20-pole cascade speed. The calculation just made shows that the ratio B_1/B_2 is about the same on both pole combinations.

Case 4. $B_1 \times 0\cdot301 \times 3 = B_2 \times 0\cdot96 \times 1$; $B_1 = 1\cdot06 B_2$.

This is an example of the new type of magnetically balanced machine described below.

These results will be somewhat modified by the influence of " breadth factor " when the coils in series extend over several slots instead of one only. We assumed above E (per bar) $= VB$. In the Hunt winding, for instance (Fig. 102), it will be noted that each section, between the star point and the point where the winding reverses, i.e. the point where the coil joined to a collector ring is tapped in, consists of two turns in neighbouring slots spaced apart therefore by one-twelfth of the circumference, or 30° of phase on 2 poles and 60° on 4 poles. The E.M.F.'s in it will, therefore, differ in phase by these amounts. Calling these angles of angular spacing δ_1 and δ_2, our final formula becomes

$$V_1 B_1 \sin \tfrac{1}{2}\alpha_1 \sin \tfrac{1}{2}(180 - \delta_1) = V_2 B_2 \sin \tfrac{1}{2}(\alpha_2) \sin \tfrac{1}{2}(\alpha_2) \sin \tfrac{1}{2}(180 - \delta_2)$$

In the table above, the values of these quantities are calculated. Applying these to the various cases considered, we get—

Case 1. $B_1 = 0\cdot895 B_2$. Hunt states on page 413 of his paper : " The flux per pole of the 2-pole field is equal to 1·73 times the 4-pole flux." This gives us $B_1 = 0\cdot866 B_2$, i.e. the value to which we approximate as the number of slots gets larger. Hence our results are in practical agreement with those of Hunt.

Case 2. From the previous calculation corrected, $B_1 = 0\cdot915 B_2$.

Case 3. From the previous calculation corrected, $B_1 = 0.915B_2$.

The result of this correction is to show that the ratio of the densities is identical in the cases where the same winding is operated on two distinct combinations of poles.

Case 4. This gives $B_1 = 0.9B_2$.

It is perhaps desirable to point out that the ratio of the diameters of the two circles on which the ideal curve of ampere-conductors is based is merely a measure of the ratio of ampere-conductors per slot in two component windings, and *not* of the fluxes or magnetic densities. These calculations, in fact, will be sufficient to show clearly the principle on which the determination of the relative densities must be based. Every winding will consist of a number of exactly similar closed circuits. Calculate the E.M.F. in one such circuit due to the flux of one number of poles. Then calculate it as due to the flux of the other, and equate the two. This will give an equation determining the relative densities or fluxes per pole.

CHAPTER XVI

CONSTRUCTION OF SECONDARY WINDINGS

ONE of the most important features of the cascade motor is the possibility of changing from cascade speed to any of the basic speeds, and several methods of carrying this out may be explained. The two salient characteristics of the cascade rotor are—

1. Both the fields on which it operates rotate with respect to it at speeds much greater than that of slip.

2. The ampere-conductors per slot are different at different points of the circumference.

Since in practice it is not convenient to make the slots of different sizes, they must be of a size to accommodate the maximum ampere-conductors required, and this involves some of them being only partially filled. It is well known that, if a cascade set of two machines be brought up to the synchronous speed of the primary machine, it will continue to operate and carry load at that speed, the second machine, which now receives only slip frequency, merely acting as an impedance, with some self-induction, in the secondary of the first. In the case of the internal cascade motor, in which the currents corresponding to the second motor circulate in the windings of the first, and this second motor has no actual independent existence this possibility receives its full development because at slip frequency the secondary E.M.F. is chiefly consumed by resistance. There is, therefore, little occasion for any secondary flux to be generated, and, consequently, the secondary machine does not act appreciably as an impedance in the secondary of the primary machine. Of course, since all the slots are not filled, such a winding will not give the same torque as one of normal type. The problem, therefore, is to cause the motor to change from cascade speed to one of the basic speeds, and it will not do this unless some further means are adopted. What we require, in order to do this, is some means of stopping the rotation of the fluxes with respect to the rotor. One effective means of doing this is to apply another winding quite independent of the cascade winding which, when it is short-circuited through slip-rings, will stop the rotations of the fluxes with respect to the secondary member. Winding (4) (Fig. 127) illustrates this method. Another method is to introduce further connections into the existing cascade winding, thereby producing further closed circuits which will no longer be equal in number to the sum of the numbers of pairs of poles of both fields. Winding (5) (Fig. 128) is an example of this.

In certain cases, as, for instance, in Hunt's star-mesh winding,

good results may be obtained by what is in effect a combination of both these methods, while in others it is necessary to use them separately. One convenient way of combining the two methods which is sometimes applicable, is to close each phase of the cascade winding through a phase of the auxiliary winding, instead of short-circuiting it. We have already described the Hunt star-mesh (Fig. 110). To apply to this winding the two distinct methods described above, in this manner, we should connect the upper bars in slots 3, 7. and 11 to the junction points of sections 2 and 4, 6 and 8, 10 and 12, and short-circuit the lower bars of these three sections, thereby bringing about the closing of the required circuits with only three slip-rings. Let us consider the nature of the winding
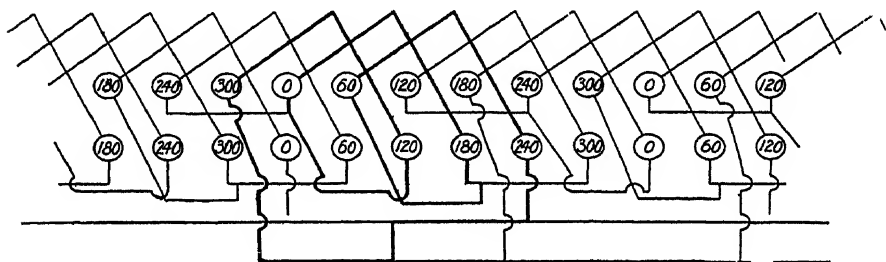
FIG. 111

so produced (Fig. 111), and similarly for any increased number of phases.

The diagram showing the circuits in skeleton form (Fig. 111) indicates that there are 6 circuits, each of the star members of the winding being common to two of these circuits. With 4 poles the mesh-connected or even-numbered coils will differ in phase by 120", and with 2 poles by 60°. The current in the star-connected coils, being the resultant of that in the two-mesh-connected coils between which it lies, will in the case of 4 poles be equal to that in the mesh-connected coils, and in the case of 2 poles will be 1·732 times as great. Fig. 111 shows a 12-slot winding fully drawn out similar to Hunt's Fig. 7, one complete circuit of the six which exist when the slip-rings are short-circuited being shown in heavy lines. In Fig. 110, for the sake of comparison, the same winding is shown as operating on cascade, the three circuits of which it is essentially composed being shown by three different types of line. It will be seen that every star coil is common to two circuits.

We have hitherto dealt chiefly with windings having three phases, largely in order to enable us to compare our results with those of Hunt. We shall now describe a number of windings adapted for, say, 5 phases, both for the sake of variety and because windings having a higher number of phases lead to a very important generalization

already alluded to above, whereby we are enabled to obtain a greater number of speeds in the cascade motor. In connection with these 5-phase windings, the most appropriate method of obtaining the basic numbers of poles will be described. Diagrams will also be given of the vector polygon for each winding, showing how nearly this polygon approximates to the ideal curve.

We have seen that any winding having the appropriate number of phases is compatible with the existence of two fluxes of the numbers of poles given by the rule above stated, and that one of the chief characteristics in such a winding must be the variation of the ampere-conductors per slot from point to point round the circumference, the number of maxima and minima being equal to the number of phases. We have already seen that the number of phases required is equal to the sum of the numbers of pairs of poles in the two fields. A single-phase wind-ing for $(m + n)$ pairs of poles is a winding having the requisite character-istics, i.e. consisting of

FIG 112

$(m + n)$ sections capable of being the seat of a balanced system of $(m + n)$-phase currents, and also having $(m + n)$ maxima and minima. Every phase will be repeated twice to comply with Hunt's rule for magnetic balance Such a single-phase winding is shown in Fig. 106 (c).
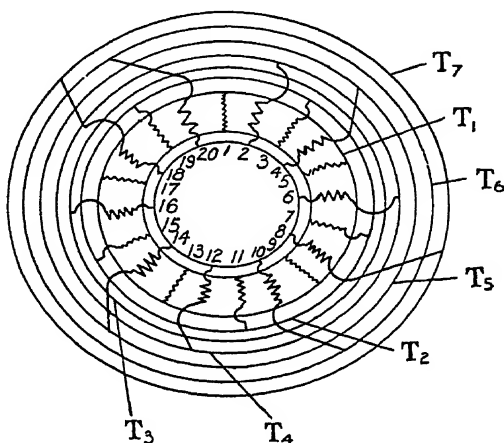
Clearly the phase difference between adjacent sections on $(m + n)$ pairs of poles will be $\dfrac{360}{2(m + n)} \times (m + n) = 180°$ (since when repeated twice there will be $2(m + n)$ sections), which proves the winding to be single-phase. The phases of the cascade winding may be connected in many ways, either in star or mesh, or short-circuited independently on themselves. A convenient standard arrangement which may be used for many windings is connected in star, and if alternate terminals are joined to each single-phase terminal (as T_1 and T_2, Fig. 112) they form a parallel type single-phase winding with $(m + n)$ local circuits, in which the single-phase currents will flow from each terminal through the star to the next. This is shown by the odd-numbered coils of Fig. 112, as connected to rings 1 and 2. Such a single-phase winding forms a convenient starting

point in developing various types of cascade winding. We shall illustrate these for the case of 4 and 6 pairs of poles throughout, but it will be clear that the same principles are applicable to any other combination of poles modified for cascade work.

It will now be desirable to describe a number of windings adapted for cascade working, and also how they may be modified to enable them to operate on any of the basic number of poles.

Winding (1). Of the various types of winding described in the present paper the following is one of the most interesting and
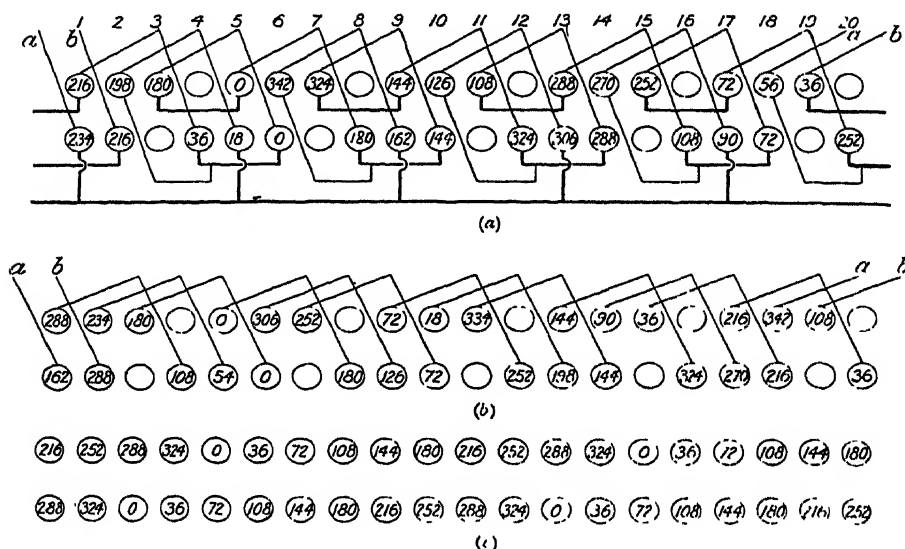
Fig. 113

important, and we shall, therefore, devote considerable space to it. It is shown in Fig. 113 (*a*) and described in detail below. The distribution of ampere-conductors produced by it is superposed on the ideal curve of Fig. 114 (*a* and *b*), showing that the approximation is quite as close as in any of the machines as described and built by Hunt. In this connection, as a preliminary step, a somewhat clearer and simpler form of connection diagram is desirable. Hunt usually denotes a single section (which may in practice consist of one or several coils in series) in a 2-layer drum winding by two small circles, one in the top and one in the bottom layers, joined by a curved line. It serves considerably to clarify the diagram, in the author's opinion, if we merely join these circles by a straight line as short as convenient, and distinguish the lower layer by using a blackened instead of a plain circle (see Fig. 115). The symbols formed in this way may be numbered successively round the circumference in order to show their relative position in the
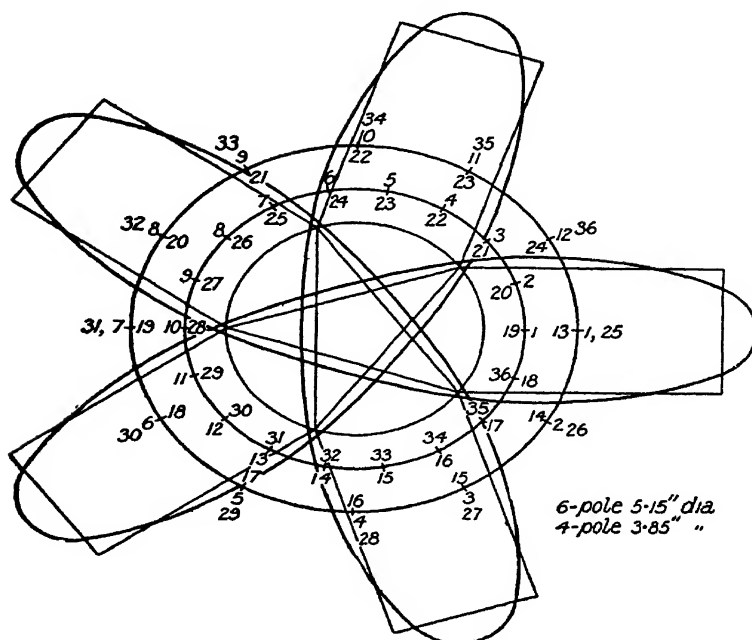
FIG 114a
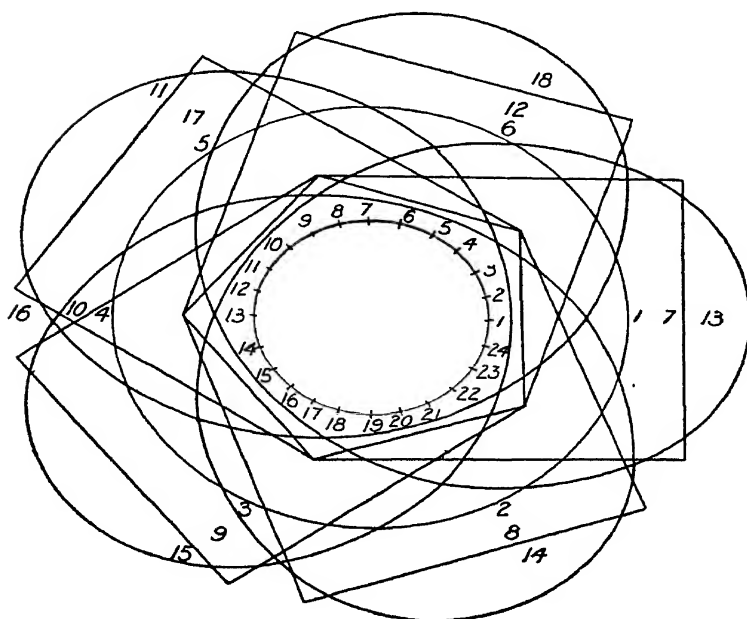
FIG. 114b

winding, and interconnected to form a key diagram (see Fig. 116), of which (*a*) is identical with Fig. 110, and (*b*) with Fig. 113 (*a*).

Consider any point where three sections are joined together, say, 12, 1, and 2 (Fig. 116). Suppose in the first place that the same currents flow in the same directions in 12 and 2. Then, from the connection of the circuits, it is clear that their arithmetical sum will flow in section 1. If, however, these currents are displaced in phase by an angle *θ*, then their resultant or vector sum will flow in section 1. The resultant (not difference) of two unit currents making
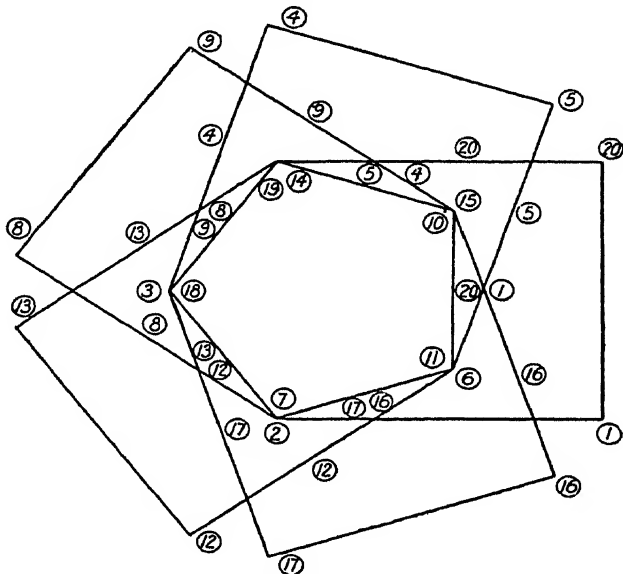
Fig. 114*c*

an angle *θ* is $2 \sin \frac{1}{2}(180 - θ)$. Calling the currents in 12 and 2 the mesh currents, and that in 1 the star current, we have

$$\text{star current} = 2 \sin \tfrac{1}{2}(180 - θ) \times \text{mesh current.}$$

Applying this to the case in which sections 12 and 2 are 120° out of phase, we get the familiar result

$$\text{star current} = 2 \sin 60° \times \text{mesh current.}$$

The winding shown in Figs. 116 (*b*) or 24 (*a*) is made up of units consisting of two coils in series reversed, 10 units of this character being connected in mesh instead of in star. Numbering the turns in the different slots from 1 to 20, we note that all the even-numbered coils are connected in mesh, alternate ones being mutually reversed, thus the turn lying in slots 1 and 4 is connected in series with that lying in slots 3 and 6 by a connection between the lower conductors of slots 4 and 6, which has the effect of causing mutual

reversal of the two turns. The turn in slots 3 and 6 is again connected in series with that in slots 5 and 8 by a connection between the upper conductors of slots 3 and 5, and so on. In this way all the even-numbered coils are connected. Of the odd-numbered coils, only those whose number, deducting 1, is divisible by four are connected to the two even-numbered coils between each of which such coils lie. The upper bar of such odd-numbered coils is connected to the junction of the two lower bars of the two coils between which it lies.
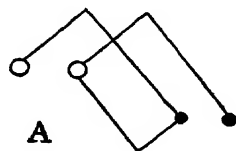
FIG 115

The winding of Fig. 113 (*a*) as arranged to fill 20 slots, contains five closed circuits as shown in Fig. 116 (*b*). For instance, the five circuits contain sections 1, 2, 4, 5; 5, 6, 8, 9; 9, 10, 12, 13; 13, 14, 16, 17; 17, 18, 20, 1. These five circuits are evidently distributed evenly round, the circumference, and each carries one phase of a balanced 5-phase system of currents. A 5-phase system is capable of two orders among its phases, viz. 0°, 72°, 144°, 216°, 288°, and 0°, 144°, 288°, 72°, 216°, and these two orders give rise to two distinct cases, both of which we must study. Five of the sections
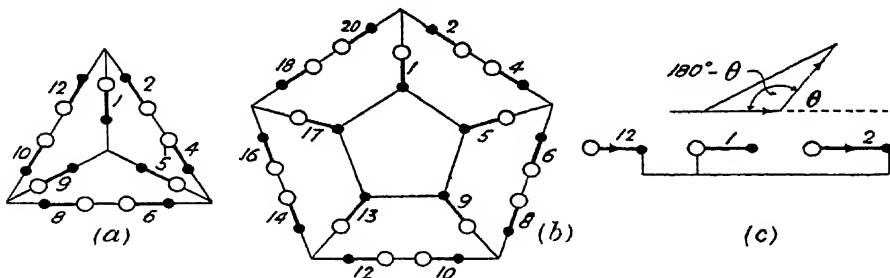
FIG 116

—the star sections—are each common to two of the closed circuits, and if we consider that the positive direction of circulation of all the currents round the closed circuits is clockwise, the current of one section will be flowing inward when that of the next (say, 72° displaced) is flowing outward. The difference between a vector at 0° and another equal to it at 72° will be $2 \sin 36° = 1·17$. If the vectors had been at an angle of 144° we should have had $2 \sin 72° = 1·9$; or if they had been at 120°, $2 \sin 60° = 1·732$. This shows that if the phase difference between adjacent circuits is 72° (the 2- and 8-pole cases) the star current will be 1·17 times the mesh current, while if it be 144° (the 4- and 6-pole cases) it will be 1·9 times the mesh current. In Hunt's case the ratio is 1·732.

Drawing out the 20-slot winding for each of the two cases as in Fig. 113 (*b*), choose five equally spaced bars, say, the top bars in slots 5, 9, 13, 17, 1, and mark them with angles, say, 0°, 72°, 144°, 216°, 288°. The connections of the winding now enable us to mark the angles corresponding to all the other bars. For instance, the top bar in slot 3 and the bottom bar in slot 8 must be opposite in phase to the top bar in slot 5, and similarly for other bars bearing the same relation to the remaining bars already marked. The bottom bar in slot 6 must also be opposite in phase to the top bar in slot 3, and similarly for corresponding bars. We have now marked all but the star bars. From the way these are connected, it is clear that the bottom bar in slot 9 is intermediate in phase between the bottom bars in slots 8 and 10, since it carries the sum of two equal currents flowing in these two bars. We accordingly mark it $126° = \frac{1}{2}(72° + 180°)$, and similarly for corresponding bars. The top bar in slot 2 must be opposite in phase to this, and so on similarly.

In Fig. 113 (*a*) an angle of 144° has been substituted for 72°, the marking being otherwise carried out in the same way.

Having prepared the marked diagrams, we next draw two circles of relative radii corresponding to the currents in the mesh and star bars respectively, calculated as above from the phases of the sections, and divide both into as many parts as may be necessary to give us all the angles we require (20 in this case). Take the " mesh " circle (the inner one as a rule) ; at the point corresponding to the angle with which a bar is marked insert the slot number of that bar. For instance, if a bar in slot 1 is marked 0° we mark the number 1 opposite the point on the circle we have chosen to represent 0°. Having marked all the slot numbers carrying mesh bars in this way, we do the same on the other circle for the star bars. We then draw the vector sum of all vectors drawn from the centre to either of the circles and marked with the same slot number, and mark its extremity with that slot number, which is the resultant value of the ampere-conductors per slot. In this way we construct one point corresponding to each slot. Joining these points in order, we get Figs. 114*a* and 114*b*, the former representing the order 0°, 144°, 288°, and the latter the order 0°, 72°, 144°.

We have described in detail how the ideal curve is drawn ; therefore it is only necessary now to point out how the radii of the oppositely revolving vectors of this curve may be obtained from the polygonal diagram corresponding to a finite number of slots. To do this draw circles tangential to the polygon both internally and externally. The maximum of the nearest ideal curve will be slightly greater than the radius of the outer circle, and the minimum slightly less than that of the inner— how much can only be estimated. We have already seen that the maximum value of the

ideal curve was equal to the arithmetical sum of the oppositely revolving vectors and its minimum to their arithmetical difference, so they can be calculated from these data. It may be worth mentioning also that the side of the polygon between two angles marked with two neighbouring slot numbers represents in magnitude and phase the magnetomotive force acting in the tooth between the two given slots. As will be seen, the approximation on 2 and 8 pairs of poles (Fig. 114*b*) is quite as close as on 4 and 6 pairs of poles (Fig. 114*a*), which proves from another standpoint that the same winding can operate with both pair of fields.

In Fig. 113 (*c*) the phases of the currents in all conductors with a winding short-circuited in a suitable manner such as that described above, operating on 8 poles, are shown. These phases are arrived at by attributing to conductors spaced apart by one-tenth of the circumference phases differing by 144°, when the connections of the winding enable us to determine the phases of all the other conductors. Averaging the phase of the two conductors in each slot, we find that they differ uniformly by 108°, and, hence, the winding is quite satisfactory as an 8-pole winding.

Winding (2). In order to complete the description of the secondary windings it will now be desirable to describe a winding adapted for the case of 2 and 6 poles, which depends on the new rule worked out above as regards magnetic balance.

Working out the number of phases corresponding to this case from the rule given above, we find that the winding must be adapted for four phases. It has been pointed out that any of the windings described above may be adapted for any other number of phases, and, hence, the 5-phase windings previously described may be reduced to 4-phase windings by omitting one-fifth of their conductors. For instance, windings described for five phases in 40 slots may also be arranged for four phases in 32 slots, the pitch of the coils, etc., remaining the same. Just as in the winding for 40 slots, each of the five phases was repeated twice and the winding was, therefore, adapted for 4 and 6 pairs of poles, so in a 32-slot winding each of the four phases would be repeated twice, and the winding would be adapted for 2 and 6 pairs of poles. But in the cases we are now studying this is not necessary, as a winding for 1 and 3 pairs of poles is magnetically balanced, unlike a winding for 2 and 3 pairs of poles. Hence, one half of the 32-slot winding can be omitted, and an adequate 4-phase winding could be wound for 16 slots only.

On working out the diagram of ampere-conductors for this winding it will be found quite as close to the ideal curve as that of any of the other windings. It is shown superposed on the ideal curve in Fig. 117.

Winding 3. Take a winding having 60 slots containing numbers
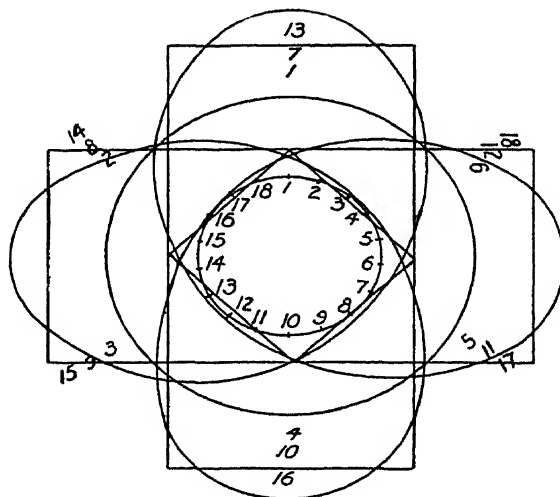
FIG. 117
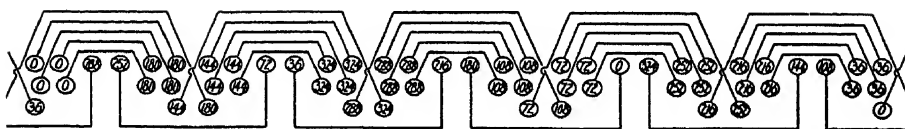
FIG. 118
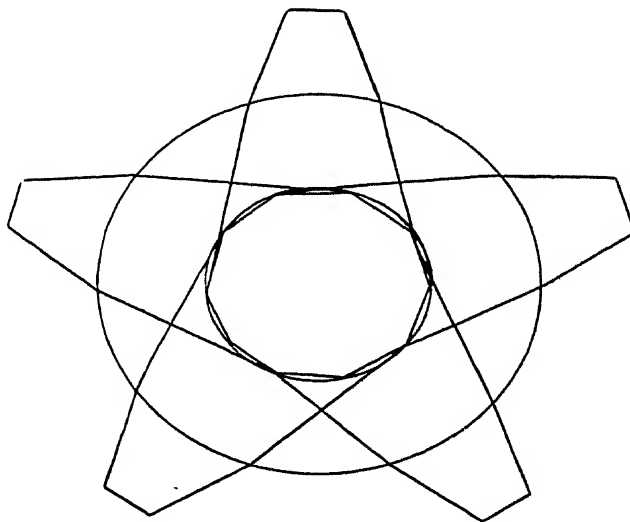
FIG. 119

of conductors varying from 1 to 3, having the phase shown in the annexed table, and all carrying equal currents. Only one-half of this winding need be considered (slots 1 to 30) since the remainder merely repeats it for the sake of magnetic balance. Connecting in series all conductors having opposite phases, such as 0° and 180°, it will be found that this winding is in the main a single-phase concentric winding arranged for five pairs of poles (see Fig. 118) (the sum of two and three pairs of poles) all the poles of which may be short-circuited on themselves (although diametrically opposite coils may be placed in series) so that they may carry currents in the different phases required by the given combination of pole numbers. If the resultant ampere-conductors of this winding be set out as a vector diagram (Fig. 119) it will be seen that a 5-lobed curve is generated, which is very close indeed to the ideal curve already drawn (Fig. 114a). All these phases may be independently short-circuited or preferably connected in star, alternate ones being connected to each single-phase terminal, as described above in connection with Fig. 112, when they form a 10-pole single-phase

PHASE OF CONDUCTORS

Slot		Top	Middle	Bottom
1	31	0	0	36
2	32	0	0	—
3	33	288	—	—
4	34	252	—	—
5	35	180	180	—
6	36	180	180	144
7	37	144	144	180
8	38	144	144	180
9	39	72	—	—
10	40	336	—	—
11	41	324	324	—
12	42	324	324	288
13	43	288	288	324
14	44	288	288	—
15	45	216	—	—
16	46	180	—	—
17	47	108	108	—
18	48	108	108	72
19	49	72	72	108
20	50	72	72	—
21	51	0	—	—
22	52	324	—	—
23	53	252	252	—
24	54	252	252	216
25	55	216	216	252
26	56	216	216	—
27	57	144	—	—
28	58	108	—	—
29	59	36	36	—
30	60	36	0	—

winding of the type mentioned above. The two single-phase terminals will be short-circuited when the winding is being used as a cascade, and, as mentioned above, the diagram only shows one-half of it occupying 30 slots, since the other half is exactly similar. It will be desirable, before terminating the description of this winding, to explain how it can be adapted to act on either of the basic numbers of poles. It is clear from the table that there is room in the slots for another almost exactly similar winding which, since the original winding is practically a single-phase concentric winding for 10 poles, makes altogether a 2-phase 10-pole concentric winding. Such a winding has 20 sections, each of which is short-circuited on itself. Now the phase difference between 20 equally spaced sections on 12 poles, say, will be $(360 \times 6)/20 = 108°$, and
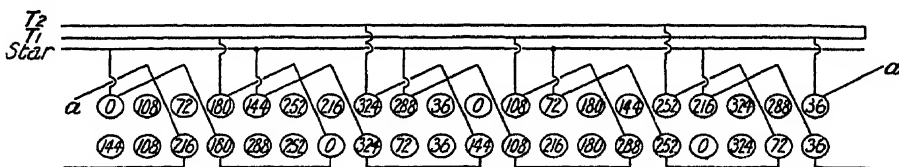
FIG. 120

currents having this phase difference will circulate in adjacent sections if these are independently short-circuited.

In many types of standard polyphase rotor the phase difference between adjacent sections is 120°, and, hence, a phase difference of 108° is clearly not too great. The same argument applies even more strongly to the case of 8 poles. Clearly the 10 parallels of the second phase winding cannot be permanently joined up, as they would prevent the operation of the machine on cascade speed. Diametrically opposite coils of the second star-connected winding may, however, be permanently connected in parallel, for instance, terminals T_3 to T_7 (Fig. 112), where the even-numbered coils represent the extra winding, and five slip-rings are used to make the other parallel connections when required. Fig. 119 shows the vector polygon corresponding to this winding. It will be seen that it is remarkably close to the ideal curve shown for the case of five phases in Fig. 114a. This winding is clearly a generalization to five phases of the winding shown in Fig. 108.

Winding 4. Another convenient winding for the same purpose is a two-coil-per-slot drum winding having 40 slots (see Fig. 120). A winding having a pitch one to four is wound in these slots, two full and two blank coming alternately, if we consider the upper conductors only. The winding is divided into 10 units, each unit being made up as follows. The turn consisting of the upper conductor of slot 1, and the lower of slot 4 is connected in series reversed with the turn consisting of the upper conductor of slot 4, and the lower

of slot 7 by a connection between the lower conductors of slots 4 and 7. Ten circuits similar to this are formed, having their initial conductors in slots 1, 5, 9, 13, 17, 21, 25, 29, 33, and 37. These 10 circuits are connected in star, for instance, alternate ones being joined to the two single-phase terminals, T_1 and T_2 in the diagram, these being short-circuited when the winding is used on cascade. To enable this winding to operate on its basic numbers of poles of 8 and 12, the first method described above may be used, viz. that of a second winding in the empty slots, in order to prevent rotation of the two fields with respect to the rotor. This can clearly be done, the winding being joined in star similar to the cascade winding, diametrically opposite coils being connected permanently together and five slip-rings being used to short-circuit the five circuits thereby obtained, as in Fig. 112.

Fig. 120 shows this winding, the circles denoting the individual conductors, the straight lines the connections between them, and the figures inscribed in the circles the phases of the currents in each conductor. The conductors joined up by the connections shown represent the cascade winding, and those not joined by connections the extra winding for stopping the rotation of the fields with respect to the rotor. Clearly, since the cascade winding is unaltered, whether we are running at the cascade speed or any other speed, the relative phases of the currents in it will not vary.

The phases of the currents in the different conductors may be obtained as follows—

The 10 units of which the winding is composed carry 5-phase currents, as has been pointed out, diametrically opposite units being in phase. Hence, if we mark the conductors in the slots 1, 5, etc., mentioned above with phase 0°, 144°, 288°, 216°, and 72°, in order, the connections of the circuit will enable us to obtain the phases of all the other conductors. For instance, if the top conductor in slot 1 has phase 0°, the lower conductor in slot 4 must have phase 180°, the lower conductor in slot 7 must have phase 0°, while the upper conductor in slot 4 must also have phase 180°. As regards the extra winding, the phases of this will be intermediate between those of the cascade winding. Its initial conductors will lie in slots 3, 7, 11, etc., and these should, therefore, be marked with phase 72°, 252°, and so on, when the same process as before enables us to get the phases of the other conductors. Having done this we may mark below each slot, which, of course, contains two conductors, the average phase of the two, and it will be found that the average phase of the conductors per slot will vary uniformly round the circumference, the phase difference between adjacent slots being $(360 \times 4)/40 = 36°$, and thus the winding is a uniform 8-pole winding. This process may also be repeated on 12 poles, giving an exactly similar result.

The best way to prevent the rotation of the fields with respect to this winding is to insert entirely distinct windings in the empty slots, this second winding being connected up and short-circuited precisely like the first, when the machine is running on its basic pole speeds.

Winding 5. This winding is a variation of winding (4), having double the number of conductors, and is formed by connecting the following pairs in series to form a unit in a manner similar to that previously described. The turn consisting of the upper conductor of slot 1 and the lower of slot 4 is connected directly in series with that consisting of the upper of slot 2 and the lower of slot 5. The
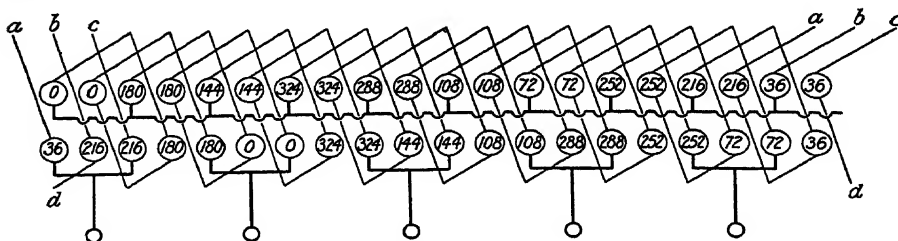
FIG. 121

FIG. 122

lower of slot 4 is connected to the lower of slot 7, and the turn consisting of the lower of slot 7 and the upper of slot 4 is connected directly in series with that consisting of the lower of slot 6 and the upper of slot 3. There will thus be eight conductors in series instead of four, and all the slots are now filled by a winding consisting of pairs of coils alternately reversed as described above. In this case the number of conductors does not vary from slot to slot, the variation of ampere-conductors being due to the varying phase difference between the upper and lower conductors in any slot, the currents in the conductors in some slots being nearly in phase and in others widely out of phase.

Assuming that conductors differing in position by one-tenth of the circumference are 144° out of phase, as before, the connections of the winding enable us to fill in the phases of all the other conductors, and we can readily draw the vector diagram of resultant ampere-conductors, which will be found to be very close to the ideal curve. It may be connected in exactly the same way as winding (4), but is shown in Fig. 121 with both terminals of every set of 8 conductors joined to a common point.

Since with this winding all the slots are full, obviously a second

winding cannot be used to get the basic speeds, and the second method described above must be employed, namely, that of producing short circuits between parts of the cascade winding. Each unit of this winding consists of two turns in series connected in opposition to another pair of turns in series, the whole four turns
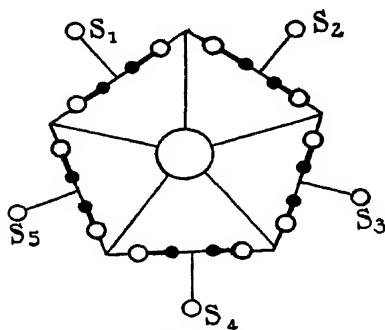
FIG. 123
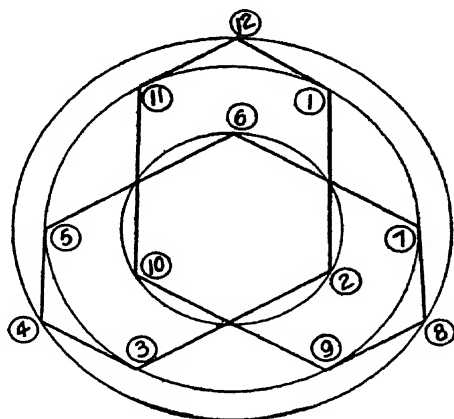
FIG. 124

then being short-circuited. Introducing a short circuit between the first pair of turns and the second pair of turns, i.e. joining S_1, S_2, S_3, S_4, S_5 to the common star-point, or even to one another, we have what is required, and when this has been done every pair of turns on the winding is short-circuited independently. By connecting
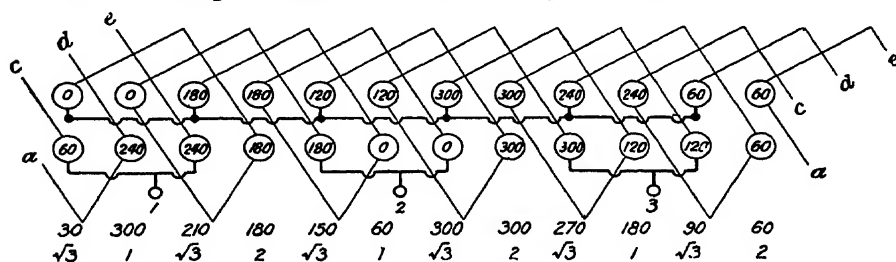
FIG. 125

opposite sections together permanently, this also involves 5 slip-rings. There will now be, as in winding (4) 20 sections each independently short-circuited, and, for the same reason as already mentioned, this is sufficient to make a satisfactory 8- or 12-pole winding. The diagrammatic scheme of the winding is shown in Fig. 123.

Fig. 122 shows the phases of the conductors in all the slots when the winding is operating on 8 poles, whilst Fig. 124 shows the vector diagram corresponding to this winding for the case of three phases. The winding for the 3-phase case is shown in Fig. 125.

CHAPTER XVII

STATOR WINDINGS

It is necessary now for the sake of completeness to describe stator windings corresponding to, and capable of use with, the rotor windings already described. In the case of a machine having a single pair of fields, that is, having only two distinct numbers of poles, it is always possible to use two distinct windings.

It may be shown without difficulty that any number of sections equally spaced round the circumference of a machine will have induced in them a balanced system of polyphase E.M.F.'s on any number of pairs of poles, which is not a multiple of the number of sections. Hence, if all these sections are connected in series, the sum of the E.M.F.'s will be zero. Thus, to obtain two mutually non-inductive windings for different numbers of poles, all that is necessary in general is to arrange them, say, in star, devoid of parallel circuits, and then, with a few exceptions, practically confined to cases where one number of poles is a direct multiple of the other, they will be quite mutually non-inductive.

The advantage of a single winding is, of course, very great, and it is desirable, therefore, to describe how such a single winding may be used in several of the cases described above. In particular, where it is desired to use two or more pairs of basic numbers of poles requiring primary windings having three or more speeds, it would often be very inconvenient to use distinct windings for all these numbers of poles.

Let us confine ourselves firstly to the Hunt type of magnetically balanced machine, in which the primary and secondary numbers of pairs of poles, when divided by their greatest common factor, give in one case an even, and in the other an odd, number as quotient. Consider four sections arranged equidistantly round the circumference, which we may refer to as a, b, c, and d (see Fig. 126), and consider the phase differences between them on different numbers of poles. These are shown in the table below.

No. of Poles	Section			
	a	b	c	d
4	0	180	0	180
8	0	0	0	0
12	0	180	0	180
16	0	0	0	0

We have here chosen the numbers of poles corresponding to the rotor winding having 5 phases, which has been fully described above, as by describing this winding we complete the description of the cascade machine having these characteristics. Connect the four sections *a, b, c d*, as follows—

A in series with *b* and *d* in series with *c*. The pairs *ab* and *dc* are now connected in parallel to form a closed circuit, in which the beginning of section *a* is connected to the beginning of section *d*, and the end of section *b* to the end of section *c*. Four terminals are now brought out from the junction points of each pair of sections. Such a closed circuit we may call a Hunt unit, and it has the following properties, which may be verified by reference to the table on previous page. With 8 and 16 poles, sections *a* and *b* are in phase and likewise sections *c* and *d*, hence the E.M.F.'s in *a* and *b* are added arithmetically, and their sums are exactly equal to the sum of those in *c* and *d*. With 4 and 12 poles, *a* and *b* will be opposite in phase and

Fig. 126

likewise *c* and *d*, and, consequently, the E.M F.'s round the closed circuit will be zero. Let us number the terminals as follows—
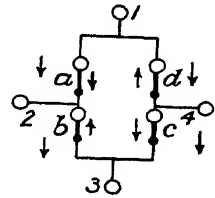
Terminal 1 between sections *a* and *d*.
    ,,   2   ,,   ,,   *a* and *b*.
    ,,   3   ,,   ,,   *b* and *c*.
    ,,   4   ,,   ,,   *c* and *d*.

If now we connect terminals 1 and 3 in series with the line, and a resistance across terminals *c* and *d*, the primary currents for 8 and 16 poles will flow between 1 and 3, and for the secondary currents between 2 and 4. If we wish to make 4 and 12 the numbers of primary poles, we must cause the primary currents to flow between terminals 2 and 4. Tracing the circuits between 2 and 4 we note that *b* and *c* form one branch and *a* and *d* the other, these being connected in series and opposition. It will be seen from the table that sections *b* and *c* are opposite in phase on 4 and 12 poles, and, hence, their E.M.F.'s will be added if these two sections are connected in opposition. For the same reason as described previously, secondary currents capable of producing an 8- and 16-pole field will not circulate round the closed circuit, but can only flow between terminals 1 and 3.

Hence, if we can build up a winding adapted for, say, 8, 12, and 16 poles (neglecting the case of 14 poles which will be seldom required) out of these units we shall have a winding which will answer the required purpose. To do this we require a winding containing at least 24 sections, giving rise to six Hunt units (see Fig. 127), In such a winding, between sections *a* and *b* of any given

unit, that is, any two neighbouring sections, there will be sections belonging to five other units, making six in all. We may call these units one to six, numbering them in order from section *a* of

(a)

FIG. 127

12-pole
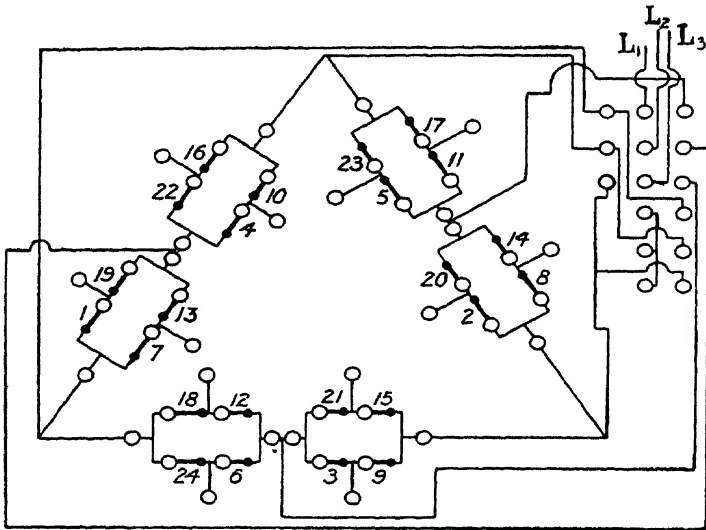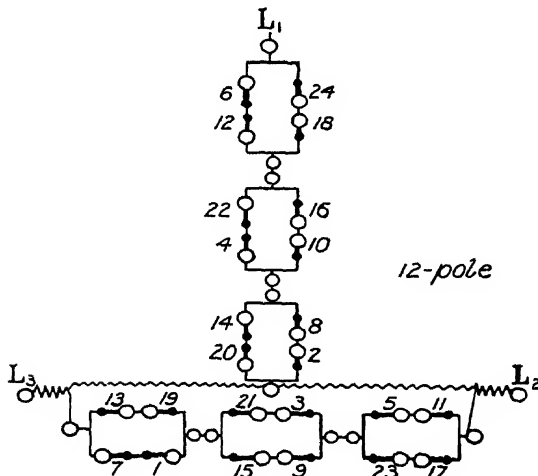
FIG. 128

unit 1 to section *b* of the same unit. Unit 1 is thus the unit containing section 1.

To connect these six units for 16 poles we may connect units 1 and 4, 2 and 5, 3 and 6 in series, and these three sets in mesh. To

obtain 8 poles connect units 1 and 4, 2 and 5, 3 and 6 in parallel and connect these pairs in star. To obtain 12 poles connect units 1, 3, and 5 in series, and 2, 4, and 6 in series. Connect units 2, 4, and 6 between one line and the central point of an auto-transformer connected across the other two lines. Connect units 1, 3, and 5 across tappings on the same auto-transformer adapted to give a voltage 0·866 times the line voltage. Care must, of course, be taken, as has been done in Fig. 128, to see that the North and South poles of each phase on all numbers of poles are in correct sequence.

We now come to the new type of magnetically balanced machine in which both the numbers of pairs of poles are odd, and we shall describe the case of 2 and 6 poles, as this is one of the most important practically. In the case of 2 and 6 poles, opposite sections will be opposite in phase, on both the primary and secondary numbers of poles (we here assume that the primary number of poles is 6), and, therefore, the Hunt winding is inapplicable, but it is possible to make the sum of the secondary E.M.F.'s equal to zero round the local circuits by methods other than those used by Hunt.

FIG. 129

To form one branch of the circuit, let us take three instead of two equally spaced sections in series. On 6 poles all these three sections will be in phase, and the three E.M.F.'s will consequently be added arithmetically (see Fig. 129). On 2 poles they will have phases 0°, 120°, and 240°, and the sum of their E.M.F.'s will, therefore, be zero, as shown in Fig. 130.

FIG. 130

Circuits, therefore, made up of any number of branches, each consisting of three equally spaced sections, will have zero E.M.F. round them on 2 poles, and can be used as branches of a primary circuit on 6 poles.

In order to obtain a winding which can act satisfactorily as a secondary on 2 poles, it is necessary to have three such circuits in parallel, and this may at first sight give rise to difficulty.

A satisfactory winding may, however, be constructed as follows, 54 sections, each of which may conveniently occupy one slot, being required. In the table (page 157) are shown the phases of these 54 sections both on 2 poles and on 6 poles, each phase of the 6-pole winding being constructed from 18 of these sections, 6 sections in each branch of a three parallel winding. Each of these 6 sections forms a complete 6-pole winding in itself, being connected in the order shown in Fig. 131, in which is also shown the primary terminals A and B, and the secondary terminals C_1, C_2, C_3, and D_1, D_2, D_3. Between the junction point A, and C_1, C_2, C_3, each branch consists of two diametrically opposite sections in series connected in
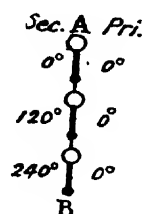
opposition so that their voltages add on both the 2-pole and 6-pole circuits. The three branches are exactly equally spaced round the circumference, so that C_1, C_2, C_3 are precisely equi-potential on 6 poles. On 2 poles, however, the three branches form a balanced 3-phase circuit, which fulfils the condition that the sum of the currents measured towards a junction point such as A shall be zero. The sections between C_1, C_2, C_3, and D_1, D_2, D_3, and between D_1, D_2, D_3, and B are arranged among themselves in a precisely similar way.
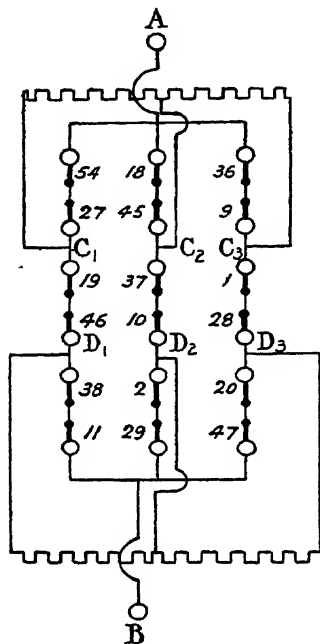
Fig. 131

Now consider the sections in any one branch; sections 54 and 27 reversed give phase 0° on 2 poles; sections 19 and 46 reversed give nearly phase 120°, in fact, $126\frac{2}{3}°$; sections 38 and 11 reversed give nearly 240°, in fact, $253\frac{2}{3}°$. By the use of overlapping sections, several sections being wound into one slot, these slight variations in phase angle can be avoided. But, as will be seen directly, this complication would not be justified. If we take the sum of three equal vectors having the exact phase angles mentioned above (see Fig. 132), it will not be precisely zero, but will nevertheless be only about 6 per cent of the E.M.F. across any branch, say, from A to B.

Consider how a cascade motor having this primary winding will operate when C_1, C_2, C_3, D_1, D_2, D_3 are short-circuited. Since they are exactly equi-potential points with respect to the 6-pole E.M.F.'s, no effect will be produced on the 6-pole primary currents. Each of the three local secondary circuits is exactly balanced within itself, and no currents from the one need be closed through any of the others. Hence, both as a primary and as a secondary winding, it is quite as effective as any of the windings described by Hunt. During the starting period only, when C_1, C_2, C_3, D_1, D_2, D_3 are open, there will be a residual 3-phase E.M.F. of about 6 per cent the magnitude of that in the circuit between A and B, thus causing an induced current to flow round the closed circuit between A and B, having a resistance three times that of the local circuits between A and C_1, C_2, C_3. This residual current, therefore, will be fully effective in producing starting torque. When we commence to close the rheostats further, a much larger current will begin to flow through the circuit so established, and a powerful starting torque

is produced until, when they are completely short-circuited, the motor operates as above. The effect of this residual current is not serious, and may be further diminished by connecting only corresponding branches together, forming thereby three star points instead of one, and connecting an additional rheostat in circuit with them.

Were it not for the desirability of giving resistance starting in many cases, a much simpler type of cascade winding for a 2- and 6-pole motor might be employed, making use of only 18 sections, i.e. 6 per phase, each containing three parallels, and each branch of
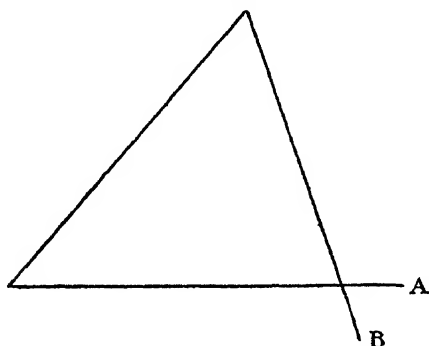
FIG. 132
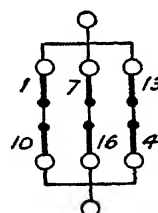
FIG. 133

the parallels consisting of two diametrically opposite sections connected in series and opposition. With 2 poles the three parallels in any one phase form a balanced 3-phase circuit short-circuited on itself, while with 6 poles it is one phase of a normal 3-phase winding. Its precise arrangement is shown in Fig. 133. For many two-speed motors, which do not require heavy starting torque, this winding may be preferable to the former one.

Alternate sections in series are relatively reversed, as shown in Fig. 129, so that, although sections 54 and 27, for instance, are shown in the table as having phases 0° and 180°, the E.M F.'s in these sections are arithmetically added owing to the manner of connection.

Section	Phase		Section	Phase		Section	Phase	
	Two Poles	Six Poles		Two Poles	Six Poles		Two Poles	Six Poles
Terminal A			A			A		
54	0°	0°	18	120°	0°	36	240°	0°
27	180°	180°	45	300°	180°	9	60°	180°
Terminal C_1			C_1			C_3		
19	$126\frac{2}{3}°$	20°	37	$246\frac{2}{3}°$	20°	1	$6\frac{2}{3}°$	20°
46	$306\frac{2}{3}°$	200°	10	$66\frac{2}{3}°$	200°	28	$186\frac{2}{3}°$	200°
Terminal D_1			D_2			D_3		
38	$253\frac{1}{3}°$	40°	2	$13\frac{1}{3}°$	40°	20	$133\frac{1}{3}°$	40°
11	$73\frac{1}{3}°$	220°	29	$193\frac{1}{3}°$	220°	47	$313\frac{1}{3}°$	220°
Terminal B			B			B		

CHAPTER XVIII

BRIEF DESCRIPTION OF VARIOUS CASCADE MACHINES

MANY different types of internal cascade machines have already been described. In such machines the secondary winding is not connected to the line, and when the machine is intended to run at the cascade speed only, the secondary winding may be permanently short-circuited. The primary winding carrying the line currents, which is also arranged to carry circulating currents of low frequency, is commonly connected in two or more parallels. This primary winding produces what may be called the primary flux, having a number of poles which may be called the primary pole number.

Such a machine in the case where the pole numbers are in the ratio of 2 : 1, and where there are two parallels in the stator winding, may be adapted to synchronous working for the purpose of improving its power factor in the following manner—

A suitable portion of the windings of one phase, containing half the conductors of that phase, may be connected to corresponding portions of the other phases in a star-point ; the remainder of the windings in that phase may be connected to the remaining conductors in the other phases in a second star-point. A constant potential difference may be maintained between the two star-points thus formed, by means of a separate exciter, direct-coupled to the main machine or driven in any other suitable manner. A direct current, therefore, flows in at one of the star-points, circulates round the windings, and returns through the other star-point, producing what may be called the secondary field having half (or sometimes double) the primary number of poles. When a cascade-wound secondary member is placed inside a primary excited in this manner, it will run synchronously at its cascade speed and have all the properties of a synchronous motor, while, in addition, it may be used as a synchronous generator.

Such a machine is started as an induction motor by well-known means, and the synchronizing switch is closed when the machine reaches synchronous speed as nearly as may be judged, whereupon it will become synchronized and operate as already described.

Fig. 134 shows a synchronous cascade type of motor such as is already known, in which the necessity for the use of a separate exciter is avoided.

A second winding B, similar to that of a direct-current armature and attached to a commutator, is fitted to the rotor in addition to the permanently short-circuited cascade secondary winding A, already referred to.

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In Fig. 134, the two distinct windings on the cascade secondary members are shown by the reference letters *A* and *B*, while the two star-connected portions of the primary winding, each containing half the conductors, are referred to as *C* and *D*. The commutator is indicated at *E*, and the brushes at *FF*. It will be seen that the commutator is connected between the two star-points of the windings *C* and *D*.

The operation of Fig. 134 will be described, for the sake of brevity, as if the primary flux were arranged for 4 poles and the secondary for 2 poles.

In order to prevent any induction from the flux due to the primary currents, which we may call for brevity the 4-pole flux,
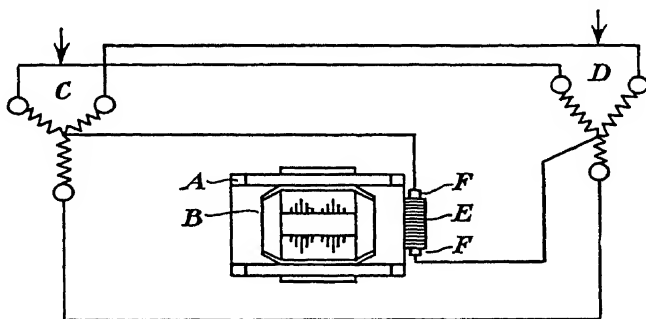
FIG. 134

the pitch of the coils of this second winding may be conveniently taken at twice the polar pitch corresponding to 4 poles. This renders it impossible for the 4-pole flux to produce any E.M.F. in the coils of the auxiliary winding attached to the commutator.

The brushes resting on this commutator may be connected, as just mentioned, between the two star-points already referred to, and will serve, when the machine has reached synchronous speed, to maintain the constant potential difference between them which is required to enable the machine to continue to operate synchronously. The machine is started as previously mentioned, as an induction motor, the circulating currents flowing in the primary winding at standstill being of full frequency, their frequency being steadily reduced until it becomes zero at synchronism.

The cascade secondary winding produces, as is well known, a second field in addition to that impressed on it by the currents flowing into the primary from the line, which we have called the primary, or 4-pole, field. This second field we shall call the secondary, or 2-pole, field, and the circulating currents above referred to are produced by the 2-pole field cutting the windings on the primary member, their frequency being that of the slip between the secondary flux and the conductors on the primary member.

This field will also cut the auxiliary exciting winding, and will produce across the brushes E.M.F.'s of exactly the same frequency as the circulating currents which it produces in the primary winding, that is, of full line frequency at starting decreasing to zero at synchronism. Hence, if these brushes are permanently connected between the star-points of the primary winding, the current flowing through them will be of the same frequency as the circulating currents on which the operation of starting depends, and, hence, will simply aid in this operation and will not produce any harmful effect. In this way the arrangement described differs entirely from the use of a separate exciter, since an exciter produces continuous current at all speeds, whereas an auxiliary winding such as above described produces currents of slip frequency at all speeds which only becomes continuous current at synchronism. Resistances may be interposed between the commutator and the primary winding for starting purposes.
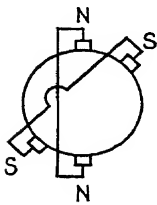
Fig. 135

Another essential difference between such a machine, and a machine having a separate exciter, arises from the fact that the flux to which the voltage occurring on the commutator is due, which has been called the 2-pole flux, is determined in magnitude and phase by the main secondary cascade winding *A*, and, hence, cannot be adjusted at will in order to vary the voltage across the commutator, in the same way in which the voltage across the commutator of an exciter can be adjusted by varying its field strength.

With a normal arrangement of brushes on the commutator *E*, therefore, the voltage across it is fixed, and, therefore, the current flowing into the star-points of the primary winding can be adjusted only by means of resistance. Since a fine adjustment of the synchronous motor excitation is almost always required in practice, a certain amount of such resistance will usually have to be left in circuit during the whole period of operation of the motor, and this in certain cases may be inconvenient. In order to render it unnecessary, the positive and negative brushes resting on the commutator *E* may be made capable of mutual adjustment, as shown in Fig. 133, for the case of a 4-pole secondary flux.

In Fig. 135 let *NN* be the positive brushes resting on the commutator *E*, while *SS* are the negative brushes. By moving the brushes *SS* relative to *NN*, the voltage between *N* and *S* may be adjusted in a manner which does not require the use of resistance. It should be noticed that the auxiliary winding *B* will, as a rule, be arranged to give a very low voltage.

It is clear that the commutator *E* must be proportioned in such a way as to commutate currents of full line frequency at the moment of starting, and, hence, the voltage per segment will be low, and a

commutator so proportioned will permit of the adjustment of brush position after synchronism has been reached.

Although in Fig. 134 the commutator is shown directly connected across the star-point of the primary winding, it may be desirable to open the circuit during the starting period in certain cases, for instance, for the purpose of improving the starting torque.

But the synchronous motor has certain grave disadvantages of which the necessity for synchronizing is only one. Another is the fact that the field ampere-turns are invariable from no-load to full-load. If, therefore, a high overload capacity is aimed at, a very heavy field excitation is required roughly in proportion to the overload required, and this field excitation remains constant at all loads, even though it may never be required. This results in additional heating besides additional cost of construction.

In the ordinary induction motor, on the other hand, the secondary ampere-turns (which correspond to the field ampere-turns in a synchronous motor) are approximately proportional to the load, and, therefore, it is possible to have the very heavy secondary ampere-turns which are necessary at overloads without having the same excessive ampere-turns on normal and light loads. This gives the induction machine a great advantage. It is, therefore, desirable to maintain this characteristic in a self-exciting or unity power factor machine by arranging it as an induction rather than as a synchronous type.

In order to do this it is necessary to arrange for a polyphase exciting winding instead of the direct-current (single-phase) type already known. The machine, in fact, becomes identical with the already-known cascade induction motor, save that means are provided to insert a voltage in the local circuits of the primary winding for the purpose of raising the power factor.

It has already been pointed out that for other ratios of pole numbers than 2 : 1, other numbers of parallels may be required in the stator winding. For instance, windings have been described for pole numbers having ratios of 3 : 1 which have three parallels, and which can, therefore, be arranged to give three star-points instead of two. In order to deal with such combinations, we only require to place a three-phase arrangement of brushes upon the commutator of the auxiliary winding instead of a direct-current (single-phase) arrangement, and, in general, the brushes on this commutator may be arranged to give a number of phases equal to the number of parallels.

The operation of a machine with a polyphase arrangement of brushes is similar to that of a machine fitted with certain types of phase advancer, that is, it will be capable of operating on unity power factor, and may even be caused to reach synchronous speed by suitably moving the brushes.

Another method of utilizing an auxiliary winding with commutator to obtain high power factors, is to make the commutator itself the star-point of the primary winding. For instance, a winding such as that just referred to arranged for a primary pole number of six, and a secondary number of two, and having three parallels, has, in all, nine sections joined to the star-point, three for each phase. If, therefore, we place on the commutator a 9-phase arrangement of brushes and connect one of the sections to each, we thereby make the
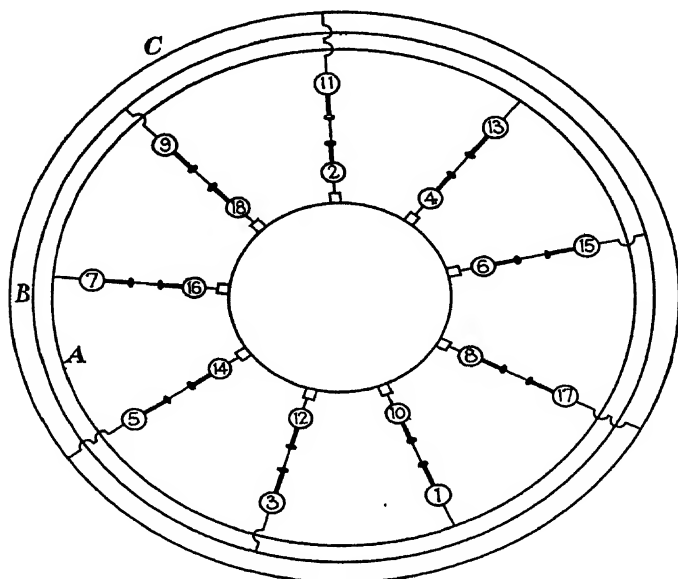
FIG. 136

commutator the star-point, and the results already referred to as regards high power factor may be obtained. In Fig. 136, for instance, the stator is fitted with a winding of 18 sections symbolized in the same way as in Chapter XVI, diametrically opposite sections such as 1 and 10 being connected in series reversed, that is, the end of section 1 being connected to the end of section 10.

Of these pairs, the beginnings of sections 1, 7, and 13 are joined together to form phase A connected to the line, while the beginnings of 3, 9, and 15 are connected together to form phase B, and of 5, 11, and 17 to form phase C. The beginnings of sections 2, 4, 6, 8, 10, 12, 14, 16, and 18 are connected in consecutive order to the 9-phase brushes on the commutator, the direction of rotation round the commutator being chosen with reference to that of the secondary flux. Thus the sections belonging to one primary phase, for instance, 10, 16, and 4, will be connected to brushes 120° apart on the commutator. As already pointed out, the pitch of the

auxiliary winding will be equal to twice the polar pitch of the primary flux, that is, to one-third of the circumference in the case now being considered, since the primary produces a 6-pole flux.

With a winding having this pitch, and, say, two conductors per slot, with brushes spaced and connected in the manner just described, it will be found on tracing the flow of primary currents through the winding that the two conductors in each slot carry equal and opposite currents, and, hence, the primary currents which flow through the commutator when it is used as a star-point are incapable of producing any field.

For instance, in Fig. 137, three coils are shown, each one connected to a commutator segment on which a brush connected to one of the three parallels in one primary phase rests. Hence the
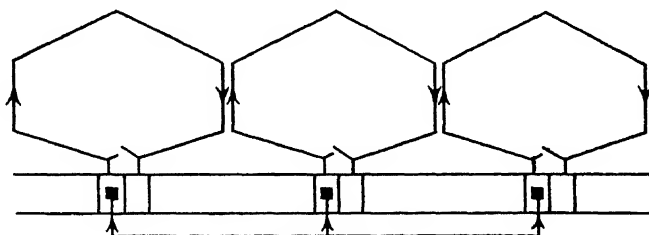
FIG. 137

primary currents carried by these three brushes will be exactly equal.

Tracing the current from each brush, we find that at any given instant it flows up the left-hand side of the coil and down by the right-hand side. Since each coil spans one-third of the circumference, the left-hand side of one of these coils lies in the same slot as the right-hand side of another, and, hence, since the currents carried by the three brushes are equal, the ampere-turns due to these currents in the slots will cancel.

As already pointed out, the 2-pole field produced by the secondary winding of the cascade motor can induce an E.M.F. in the auxiliary winding. By suitably moving the brushgear round the commutator, an E.M.F. may be applied to each local circuit of phase and magnitude suitable to effect phase compensation in the same manner as for an induction motor of the ordinary type.

Fig. 138 shows how a similar construction may be applied to a machine having 4 and 2 poles, which may be regarded as typifying machines whose pole numbers are in the ratio of 2 : 1. Such a machine will have 12 brushes upon the commutator and 12 sections in the primary winding (for a 4/2 pole motor).

Consecutive sections alternately reversed are connected in consecutive order to the brushes on the commutator, as shown in Fig. 4.

That is, the end of section 1 to brush 1, the end of section 3 to brush 3, and so on.

Sections 1, 4, 7, and 10 are connected together to one of the line wires *A*. Sections 3, 6, 9, and 12 are connected to a second of the line wires *B*, while sections 5, 8, 11, and 2 are connected to the third line wire *C*.

Another plan which in many cases might be regarded as preferable is to have only six brushes on the commutator instead of 12,
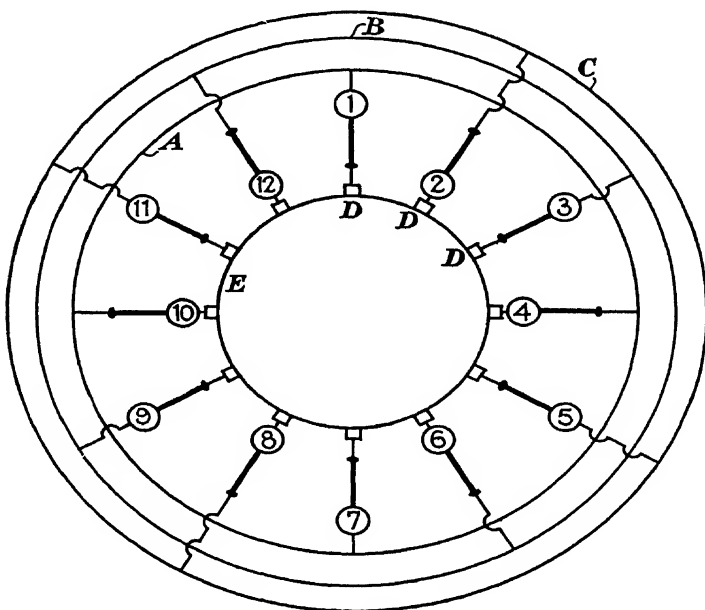
Fig. 138

connected to sections 1, 3, 5, 7, 9, and 11, while sections 2, 4, 6, 8, 10, and 12 are short-circuited in a common star-point.

It should be pointed out that while the number of brush sets is greater when the commutator is made the star-point in the manner already described than when synchronous working is employed, yet the total current to be carried by the commutator for a given type of winding, and, therefore, the brush area, is not increased ; thus, to take a simple example, instead of using 4 brush spindles with 3 brushes on each spindle, we employ 12 brush spindles with 1 brush on each spindle, the result being a commutator of larger diameter and shorter length than that of a separate exciter.

By adjusting the position of the brushes round the commutator, the phase of the E.M.F. inserted in the local circuits of the primary winding may be varied, one position of the brushes giving the best results from the point of view of phase improvement, while a

position electrically in quadrature with it produces a change in the no-load speed.

Resistances may be inserted in this as in previous machines, in series with the brushes for purposes of starting.

In certain cases it may be desirable to employ two windings on the primary member, one connected to the line and a second to the commutator. In such a case, of course, the winding connected to the commutator would have a small number of phases, such as three, requiring three brush sets only. In such a case the two ends
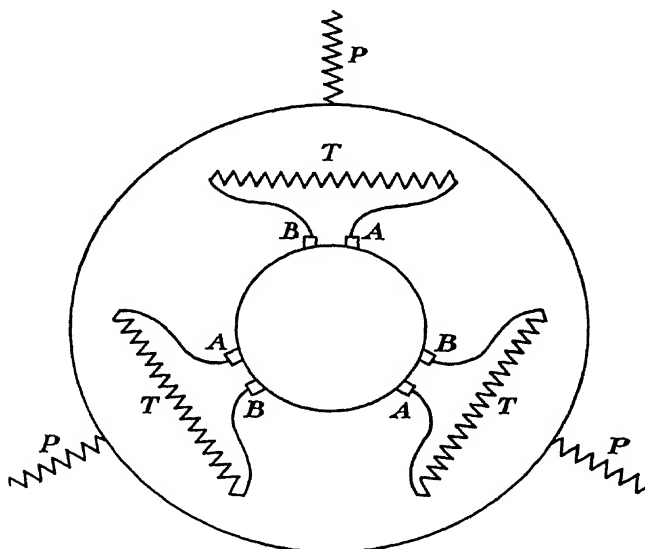
FIG. 139

of each phase will be brought to terminals independently, one end of each phase being connected to the commutator brushes, while the other is connected to a 3-phase starter of standard form for purposes of starting, and short-circuited after the starting operation is completed.

Alternatively, the arrangement of Fig. 137 may be employed for raising the power factor, starting resistances being connected in series with the brushes. In the same way, of course, a separate exciting winding for a synchronous motor may obviously be employed.

Fig. 139 illustrates a type of motor adapted for gradual speed variation, and in which two stator windings are made use of. In Fig. 139 the commutator is fitted with two sets of three brushes, each shown in the figure as *AAA* and *BBB*, these being arranged so that the set *AAA* may be moved clockwise, round the commutator, and the set *BBB* counter-clockwise, the two sets being

arranged in different planes, so that they do not interfere with one another.

PPP indicates the primary winding of the motor, which is a simple star winding containing no parallels and connected direct to the line. *TTT* indicates a second winding on the same member arranged for a different number of poles. For the sake of definiteness, we may assume that the primary winding is arranged for 6 poles, and the auxiliary commutator winding and the remaining winding *TTT* on the primary member for 2 poles.

The winding *TTT* may be wound as a standard 2-pole, 3-phase winding, both terminals of each phase being brought out and con-
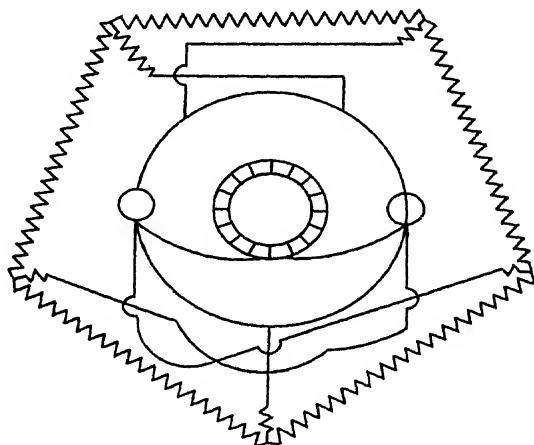
FIG. 140

nected the one end of each phase to the brushes *AAA* respectively, and the other end of each phase to brushes *BBB* respectively.

Clearly, if the brush sets *A* and *B* lie on the same bar, each of the phases *TTT* of the 2-pole winding will be independently short-circuited, while, as the brushes *A* and *B* are separated, a voltage derived from the auxiliary commutator winding may be inserted in the brushes in the circuit of each phase *TTT*. By suitably adjusting the position of the brushes, this voltage may be given any phase or magnitude desired. Since this circuit *TTT* represents the secondary of the second machine of the cascade set, such an E.M.F. may be used to regulate the speed of the machine as well as its power factor, the regulation, of course, being of a perfectly gradual nature. Such a method of regulation is particularly well adapted to give a fine adjustment of speed of a large fan for use in mines, or for other purposes.

In Figs. 140 and 141 is shown a method of economizing copper in certain types of winding attached to a commutator.

In Fig. 141 the commutator is shown attached to a winding entirely separate from, but placed in the same slots as the main secondary winding of the cascade machine. If the said secondary winding of the cascade machine is of the star-mesh type, such a commutator winding may be connected into the star-point of the secondary winding, whereupon it operates rather as the winding of a rotary-convertor than as that of an independent machine.

Such a winding may be constructed precisely as described previously. Taking the case of a machine arranged to give two pole numbers in the ratio of 2 : 3, say, a machine having six primary and four secondary poles. The star-mesh secondary winding of
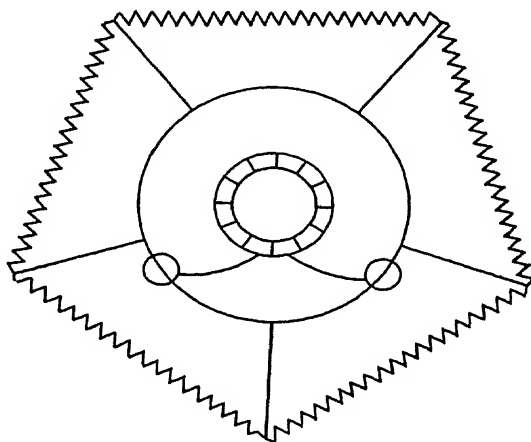
Fig. 141

such a machine will, as shown previously, have five phases. In connecting the commutator winding into the star-point of such a secondary winding, two directions of phase rotation are possible. In the first, the phase rotation of the currents flowing through the tappings of the commutator winding is opposite to its direction of rotation in space, as in Fig. 140 say, and provided the commutator winding is arranged to co-operate with the secondary pole number, for instance, being wound with a pitch of one-third of the circumference, the frequency of the currents flowing through the commutator will be that of the secondary field relative to the stator, that is, the frequency of slip.

If the direction of phase rotation of the currents which flow through the tappings is the same as that of the mechanical rotation, then it follows that, provided the commutator winding is arranged to co-operate with the primary pole number, having a pitch equal to half the circumference say, the currents appearing on the commutator will have the same frequency as that of the primary field

with respect to the stator, that is, the frequency of the line. It has already been pointed out that by giving the commutator winding a pitch equal to that of an even number of primary poles, induction between the commutator winding and the secondary field is prevented. Hence, by adopting one or other of the directions of phase rotation referred to above in combination with the appropriate pitch, it is possible to obtain on the commutator frequencies corresponding to either the primary or secondary flux.

By connecting the commutator winding as a rotary-convertor winding in this way, a certain economy of copper may be obtained, as already pointed out, since the currents in such a winding partially cancel out. The insertion of extra turns, however, in the circuit of the star-mesh secondary winding produces an alteration in the ratio of the primary to the secondary flux densities, which must be taken into account in designing such windings.

The effect of inserting in the circuits of the cascade secondary winding, turns having a pitch different from that of the winding and, perhaps, covering an even number of pole pitches of either the primary of the secondary flux, is to insert further E.M.F.'s in that circuit whose magnitude and phase may readily be calculated on known principles, and from this calculation the exact effect on the ratio of the flux densities may be ascertained.

Such a winding as that shown in Fig. 139, in which the primary frequency appears on the commutator, renders possible the insertion of voltages in the circuits of the cascade secondary winding, for the purpose of regulating the characteristics of the machine. For instance, by connecting a transformer across the brushes of this commutator, the primary of which is in series with the line, voltage proportional to the load may be introduced into such a secondary circuit. This is particularly useful in generators, as it permits the obtaining of a compound characteristic in a simple manner.

In such machines, clearly, the commutator could be used not only for purposes of excitation or speed variation, but to deliver current to an external circuit, the machine thus becoming a convertor. For instance, it could be arranged to deliver direct current from the commutator while absorbing polyphase alternating currents in the primary winding. It would thus be an equivalent of a synchronous motor generator set. Alternatively, it could deliver polyphase alternating current from the commutator of low or even variable frequency, while absorbing power at line frequency on the primary winding, or these functions could be reversed, the machine taking in power at the commutator and delivering it from the primary winding.

CHAPTER XIX

THE TURBO-CONVERTOR

A cascade set containing three relatively moving elements. In addition to cascade sets of the types we have already described, other types exist in which two revolving elements travelling at different speeds, relative to the fixed stator, are used. The only machine of this type to which any great amount of thought has been given is the turbo-convertor, a high-speed, direct-current generator unit, and the present chapter will, therefore, be devoted to describing it.
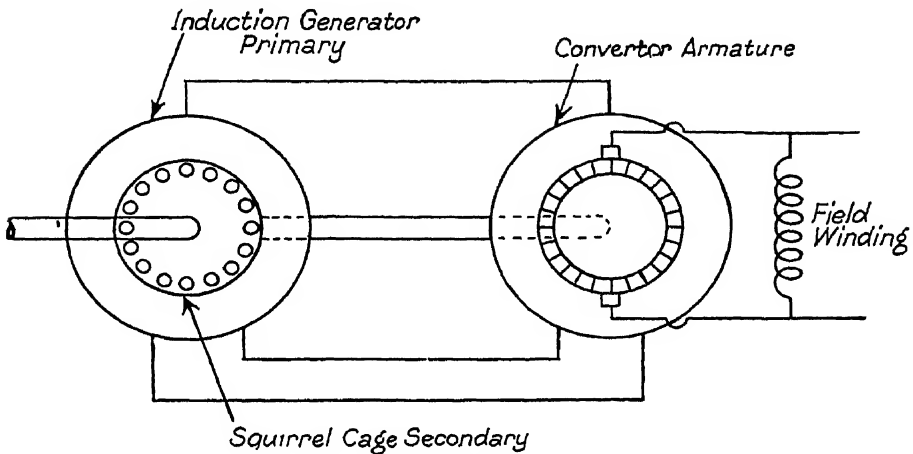
FIG 142

We shall give a fairly full description in order to assist the reader to understand the various other types having three moving elements.

The fundamental difficulty, of course, in the design of a direct-current turbo-generator, is the collection of the current from the rapidly revolving commutator, and for this, in spite of the engineering skill lavished on the subject, there would appear to be no remedy, except reducing the rate of revolution of the commutator. In order to do this, Mr. H. M. Hobart has proposed the use of an alternating-current generator driving a rotary convertor of the ordinary type, but this proposal has not often been adopted in this country, presumably through fear of the cost. Another plan which has had a certain measure of success is the use of double helical machine-cut gears, which are now obtainable in sizes suitable for transmitting large powers.

The present chapter describes another method which may be regarded, to some extent, as a combination of the two mentioned

above, in that an alternating-current generator of the induction type feeding a rotary convertor is employed, the induction generator being used at the same time as a species of electromagnetic gear. By this means it is possible to reduce materially the size of both the rotary convertor and the alternating-current generator, since while in Hobart's proposal it is necessary to have both capable of delivering the full output of the system, on this plan, the output of the system is the sum of the outputs of the component parts, each of which, therefore, need only have half the capacity of the set.

The turbo-convertor, then, consists of an induction generator combined into one machine with a rotary convertor, one member (preferably the primary) being mounted on the convertor shaft and revolving with it, and the other, usually the squirrel-cage rotor, being mounted on the turbine shaft. Fig. 142 shows a diagram of connections of the device.

By mounting the generator primary on the convertor shaft instead of having it stationary, as in Hobart's proposal, we make use of the driving torque required by the induction generator, or, in other words, the resistance which its rotor opposes to being revolved by the turbine, in order to drive the convertor, which is thereby caused to generate direct current in addition to its function as a convertor.

Let us take, by way of example, a 4-pole generator and a 4-pole convertor. Let the convertor be running at 1,500 revs. per min., say, and let the 3-phase induction generator be connected to 3-phase tappings on the convertor armature through the hollow shaft.

At 1,500 revs. per min., 3-phase currents at 50 cycles will flow through the tappings on the convertor armature. These tappings are so connected that the revolving field of the induction generator rotates the same way as the convertor armature. In a 4-pole machine with 50 cycles excitation, the revolving field will also go at 1,500 revs. per min. relative to the primary winding which produces it. Hence the total speed of the revolving field will be 3,000 revs. per min., the sum of its speed relative to its primary and that of the primary itself. The squirrel-cage rotor, and, therefore, of course, the turbine, will go at approximately the same speed as the field. Hence we have obtained an apparatus in which the generator only runs at a fraction of the speed of the turbine.

A little reflection will make it clear that in an apparatus mounted as described, since the driving torque of the induction generator also drives the convertor as a direct-current generator, the torque exerted between rotor and stator of the induction generator must be identically equal to that between armature and field of the convertor. They are, in fact, the same torque exerted at different points.
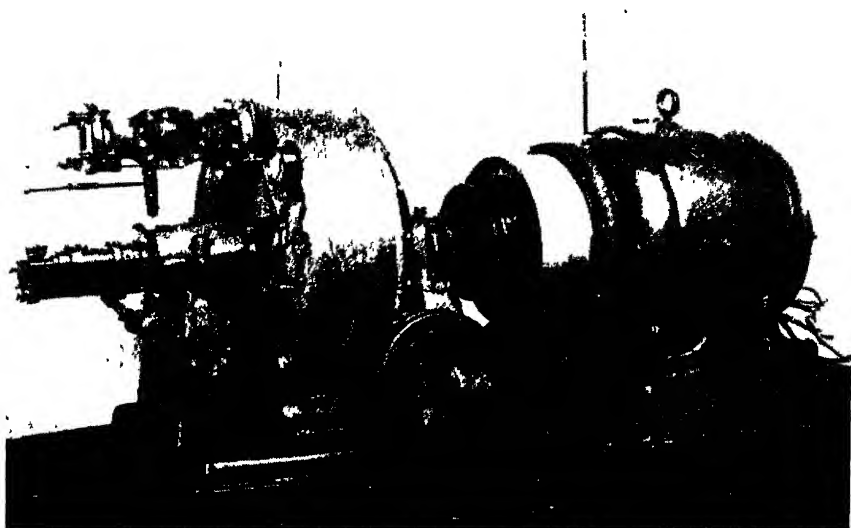
PLATE II
Turbo-convertor ready for operation
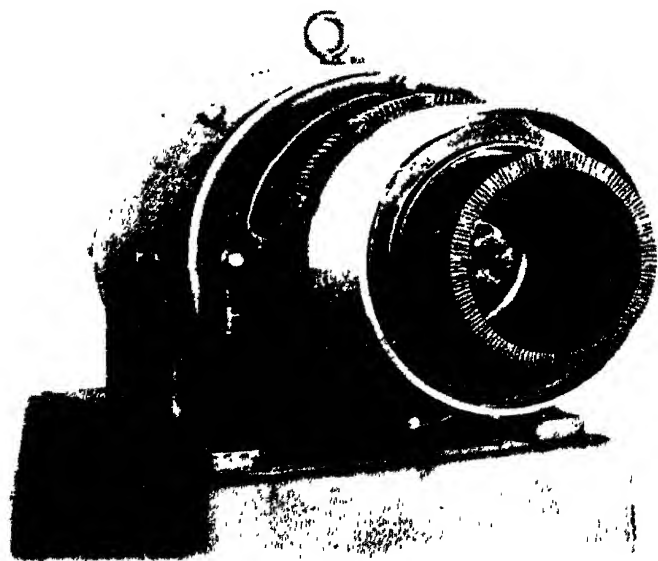
PLATE III
Turbo-convertor dismantled to show induction generator primary

The input of power into any generator, of course, is

Power input = torque × relative speed of inductor and induced parts.

If, for the moment, we neglect the losses, the power input and output will be the same.

We saw above that the torque of the two elements of the turbo-convertor was identically the same. If both have the same number of poles, the relative speed of inductor and induced parts will be the same in each, and, hence, each will generate the same amount of power. In addition to its function as a direct-current generator, the convertor, of course, changes the power of the induction generator, equal in amount to its own, into a direct-current form, and the total power flows out of the commutator of the set.

The reader will not fail to remark the analogy between the present apparatus and the motor convertor.

Before passing on to a more detailed consideration of the characteristics of such a set, it may be well to explain the reason for the use of an induction rather than a synchronous generator, which would appear at first sight to be equally applicable. These reasons are few, but conclusive—

1. If a synchronous generator were employed, it would be necessary to synchronize the two revolving elements every time the set was started.

2. Owing to the presence of two sources of magnetization in the set—one in the convertor and one in the generator field—it would be possible for the set to "hunt" if overloaded, and for it to fall out of step if accidentally short-circuited and be incapable of picking up again.

3. Collector rings would be required on the high-speed element to excite the field.

Characteristics of a turbo-convertor set. The convertor portion of such a set will be distinguished from an ordinary rotary convertor by a number of peculiarities of design—

1. Owing to its function as direct-current generator, it will require more copper on the armature than a standard convertor—exactly the same amount, in fact, as the convertor portion of a motor convertor.

2. As the field of the convertor has to perform the double function of magnetizing both convertor and induction generator, it will necessarily be somewhat heavier than that of the ordinary convertor.

The magnetizing current of the induction generator, in fact, differs 90° in phase from the working current, and circulates in the convertor armature in such a position as to directly demagnetize the field of the convertor. This field, therefore, must be supplied with

an extra number of ampere-turns sufficient to counterbalance the magnetizing current of the induction generator.

In order to calculate the increase in the strength of the convertor field required, we may proceed as follows—

Express the reluctance of the induction generator by means of an effective air-gap in a manner well understood by electrical designers. Multiply this by the ratio of the air-gap magnetic density chosen for the induction generator to that of the convertor.

Excitation must be provided for the resultant reluctance of the convertor calculated as above.

Expressing the above calculation algebraically, it appears as follows—

Let

δc be the convertor effective air-gap.
δg be the generator air-gap.
βc be the convertor gap density.
βg be the generator gap density.

Then

resultant effective air-gap $\delta c + \delta g \, \beta g / \beta c$.

3. The drop in speed of a turbo-convertor set from no-load to full-load would at first sight appear to be considerable, as the " slip " of the induction generator is added to the drop in speed of the turbine.

However, if the ratio of speeds of the turbine and convertor is, say, 2 : 1, the effect of the slip on the speed of the convertor is reduced in the same ratio, so that the resultant drop in speed is much less than might have been anticipated. This is best seen by a short algebraical investigation.

Let

k_0 = the speed of the turbine.
k_1 = the synchronous speed of the induction generator.
s = the slip.
P_1 = the number of poles of the convertor.
P_2 = the number of poles of the induction generator.

Then we have

Convertor speed = $k_0 - s - k_1$.

Frequency generated by convertor = $(k_0 - s - k_1) \times \dfrac{P}{120}$

Synchronous speed of induction generator

$$= \text{frequency of supply} \times \frac{120}{P_2}$$

or,

$$k_1 = (k_0 - s - k_1) \frac{P_1}{120} \times \frac{120}{P_2}$$

Transposing and cancelling,

$$k_1 (P_1 + P_2) = (k_0 - s) P_1$$

or,

$$k_1 = (k_0 - s) \frac{P_1}{P_1 + P_2}$$

Substituting this value for k in the equation for the convertor speed, we get

$$\text{Convertor speed} = (k_0 - s) \frac{P_2}{P_1 + P_2}$$

If we neglect the slip for a moment, this equation shows us that the general rule for finding the " gear ratio " of a set having any numbers of poles P_1 and P_2 is as follows—

" Divide the number of poles on the induction generator by the sum of those on the convertor and the induction generator, and the result will be the ratio of the convertor speed to that of the turbine on no load."

In order to illustrate the influence of the slip on the convertor speed, let us take the rather extreme case of a bipolar induction generator coupled to an 8-pole convertor giving a " gear ratio " of 5 : 1. If the turbine goes at 3,650 revs. per min., say. the no-load speed of the convertor will be 730 revs. per min. Suppose now the slip of the induction generator is 150 revs. per min., or somewhat over 5 per cent. The full-load speed of the convertor by the rule given above will be one-fifth of 3,500, or 700. Thus the convertor has only dropped from 730 to 700 revs. per min., only 30 revs. per min., or about 4 per cent, while the induction generator has slipped 150 revs. per min.

There should be no difficulty in a machine of any size in reducing the slip of the squirrel-cage induction generator to 2 per cent, so that its effect on the convertor speed will be very slight.

It has been thought desirable to discuss the subject of slip in the induction generator quite fully, as otherwise it might appear to have a seriously injurious effect on the regulation of the set. However, the above investigation shows that while the series field of such a set requires to be a little stronger than on an ordinary generator in order to offset this, yet the effect of the slip is of quite small magnitude.

4. Such a set can very readily be employed as a 3-wire set. All that is necessary is that the induction generator be connected in " star " and a tapping led from the neutral point to a collector

ring, the brushes on which are connected to the neutral wire of the system.

5. Starting. It is, of course, obvious that the turbo-convertor cannot be started from the steam-turbine end without auxiliary means. The steam-turbine and squirrel-cage rotor attached will start up alone, while the convertor portion remains stationary without showing any tendency to start. It is necessary to bring the convertor up to a sufficient speed to generate enough voltage to excite the induction generator before the primary and secondary of the latter can get into step.

Methods of starting may be divided into two classes—electrical and mechanical.

Let us take electrical methods first.

1. If a supply of direct current is available, the convertor may be started up as a direct-current motor, when it will bring the turbine
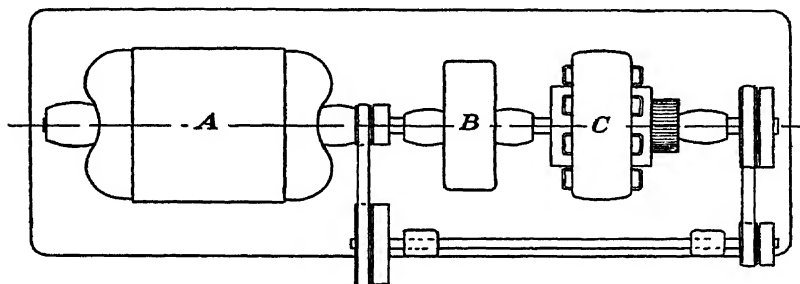
FIG. 143

up with it to its rated speed of twice or more that of the convertor. Steam can then be admitted, and the set will be ready for load. This is by far the best plan where applicable, but, of course, it requires an appreciable supply of electric power to enable it to be used.

2. Another plan requiring only a very small supply of power is the following—

Means are provided for causing a direct current to circulate in one of the phases of the induction generator during the starting period. The generator so excited acts as an electric clutch, which will bring both elements up at the same speed, and may be automatically thrown out of action when the convertor reaches the desired speed.

Coming now to the mechanical methods—

1. One of the best mechanical methods is that illustrated in Fig. 143. By means of an auxiliary idler shaft the turbine is belted on to the convertor by the use of pulleys, which may conveniently be arranged to give the same velocity ratio as the induction generator.

On starting the turbine, the convertor is brought up to its correct speed by the agency of the belts, and as soon as the field switch is closed will be ready for load. The belts may then be run on to loose pulleys, and the idler shaft stopped. This may be done by hand or automatically by means of a solenoid operated by the voltage across the convertor brushes.
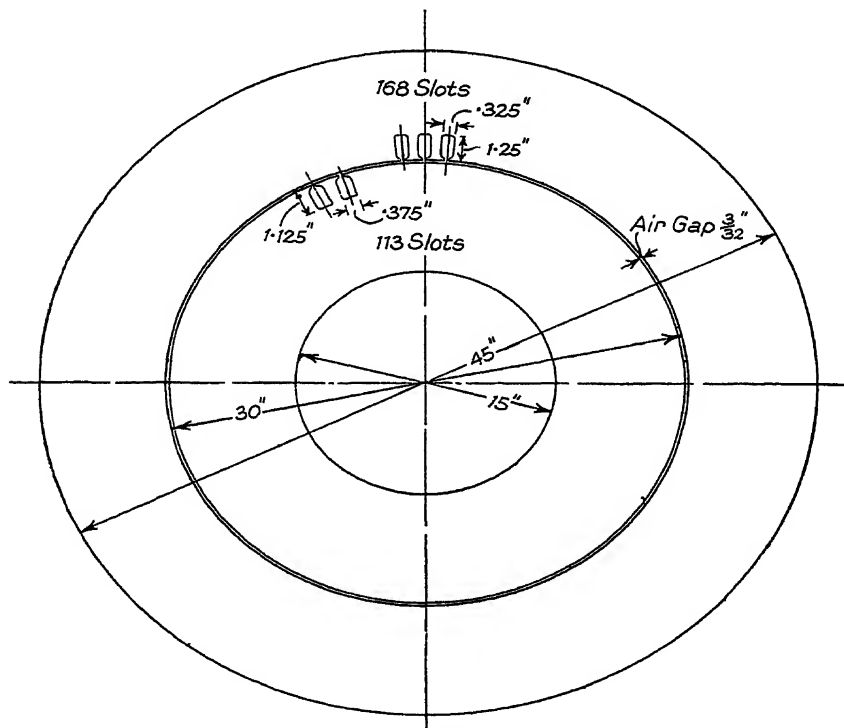
Fig. 144

Gearing could be used instead of belts in the above method, but would probably be less satisfactory.

2. An auxiliary turbine might be used to start the convertor.

3. Primary and secondary of the induction generator might be coupled by a centrifugal clutch releasing when the convertor reached its rated speed.

4. They may be coupled by a clutch operating when the torque between primary and secondary exceeds a certain value. A rudimentary instance of such an appliance is the device occasionally useful for experimental purposes, wherein primary and secondary are tied together by means of a predetermined number of thicknesses of thread.

When the machine reaches an appropriate speed, the torque

between the two becomes sufficient to burst the thread, and the two portions of the generator fall into step. When we come to criticize these methods with a view to picking out one for practical application, we find that all of them, except the first electrical and the first mechanical methods, are subject to the following criticism. So long as everything is in perfect order all of them will operate
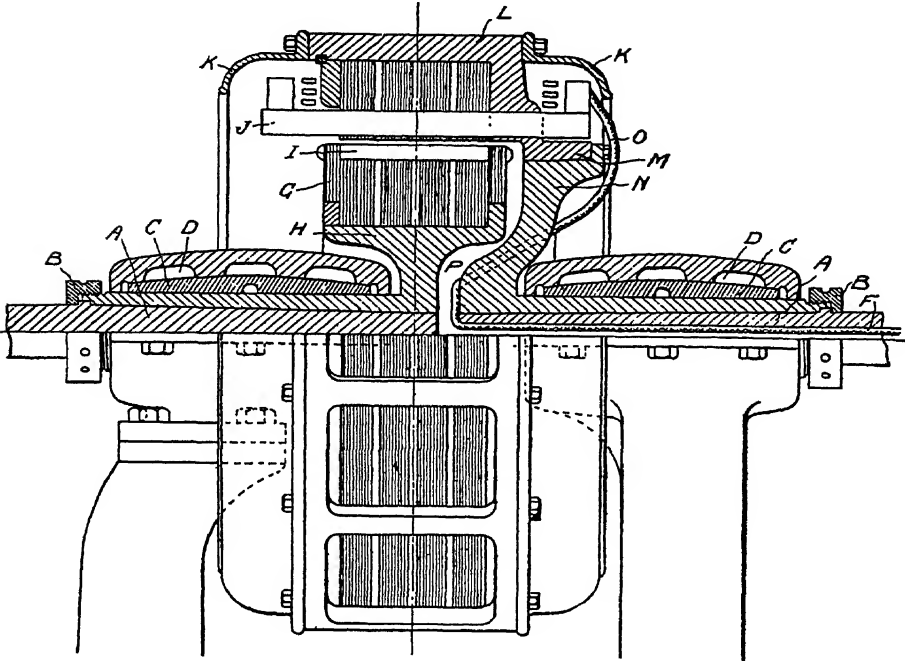
FIG. 145

all right, but if the clutches or switches required failed to operate as expected, the convertor would run the risk of being raised much above its rated speed, the result of which would very likely be disastrous. Means could probably be found to prevent this, but, on the whole, it seems better to adopt a method where this criticism cannot arise.

The first electrical method described is not applicable in the absence of a considerable supply of electrical power, hence we come to the conclusion that the best method of starting is by means of belts and an idler shaft as described.

Mechanical design of the turbo-convertor. Two different forms of mechanical construction are possible for the turbo-convertor. Firstly, the type illustrated in Fig. 145, which may conveniently be called the " fly-wheel " type, or, secondly, that sketched in Fig. 146, which may be called " the spinner " type. In this type

the squirrel-cage rotor forms the inner element, and is driven direct by the turbine. Surrounding this is a second element, the "spinner," capable of free rotation and also supported by means of ball bearings or the like on the same bearing pedestals in which the squirrel-cage rotor runs. This element bears the primary winding of the induction generator on its inside surface, and the armature
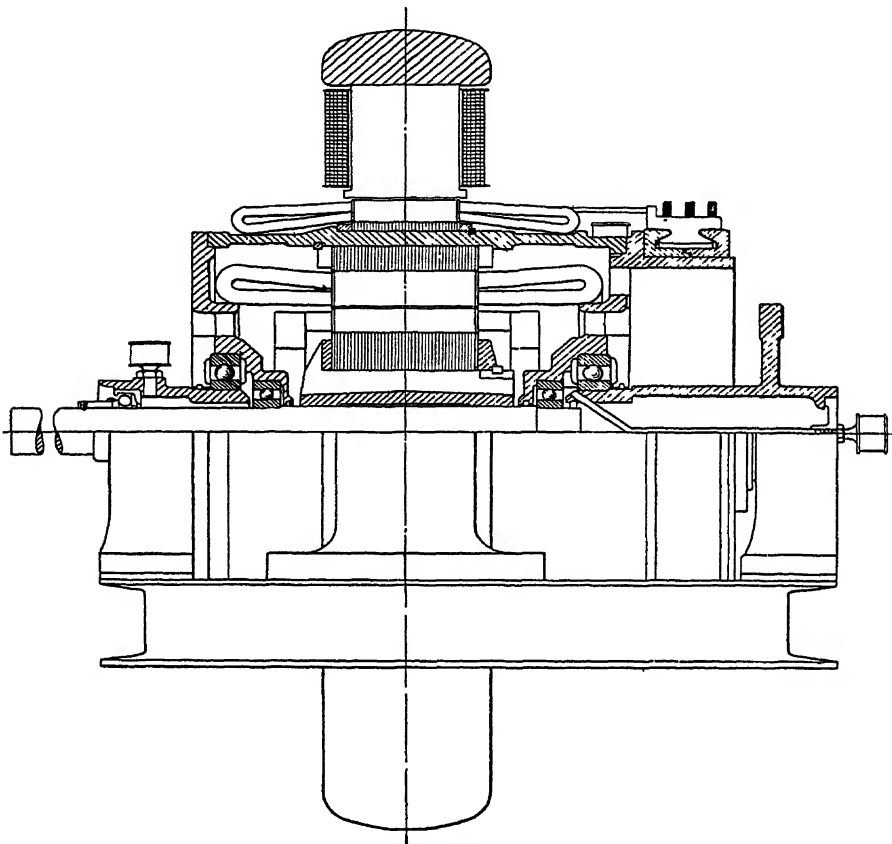
FIG. 146

winding of the convertor on its outside surface, being fitted with a commutator and brushes in the usual way.

Outside of this again is the field ring of the convertor, carrying the pole pieces and their windings, etc.

The cardinal feature of the fly-wheel type, of course, is that both primary and secondary are "overhung," or, in other words, supported by a single bearing.

A description is given below of the design adopted to render this construction unobjectionable. The most essential feature of

any single-bearing construction is to reduce the overhang of the centre of gravity of the overhung mass beyond the bearing nose as much as possible.

It will be found in every case that the design adopted enables us to reduce this overhang to a matter of a few inches beyond the bearing nose even in the most extreme cases, and for sizes of 500 kW. In order to do this, an induction generator design of large diameter and short length must be adopted, of course.

The only point which need cause us any anxiety as regards centrifugal force is the short-circuiting ring of the squirrel-cage. This cannot be made of steel, as it must consist of a high conductivity non-magnetic material. It will be found, on calculation, that for all high-speed work aluminium gives a factor of safety only inferior to that of good steel on account of its extreme lightness. Duralumin is still better in this respect.

As regards conductivity, it is a common-place that an aluminium high tension line is cheaper than a copper one of the same conductivity, so we may feel reassured on this head.

FIG 147

Hence, if we adopt the end-ring construction shown in Fig. 145, in which the end-ring is made of rolled aluminium sheet, we have an ample factor of safety against centrifugal force.

Fig. 145 is a sketch, approximately to scale, showing the mechanical design of the induction generator whose dimensions are given above. It will be seen that the centre line of the secondary only overhangs the bearing nose by little more than an inch, while the slowly revolving primary is overhung by approximately 9 in. We may estimate the weight of this, including everything, as not over 3,500 lb., so that with an overhang of 9 in. only and a speed of 600 revs. per min. we are well within the range of fly-wheel practice.

The cardinal features of the design used to obtain these results are two in number—

1. The primary frame is made of a non-magnetic material in which a number of holes are cast corresponding to the stator slots. Through these holes the stator bars are brought, and so the stator winding is kept outside the frame. An alternative construction not requiring the use of a non-magnetic frame is shown in Fig. 147.

By this means we obtain the following advantages—

(*a*) The overhang of the primary is reduced very much, as we do not have to allow space for the end-connections inside the frame.
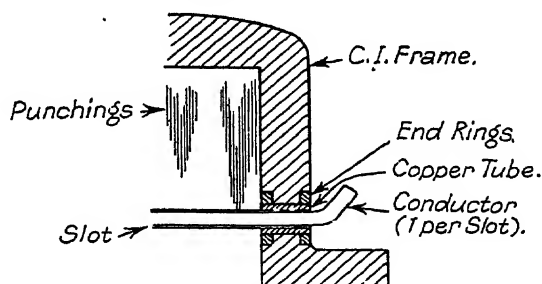
(*b*) The frame abuts solidly on the punchings all round, and there is no hollow space to cause mechanical weakness.

(*c*) There is ample room for the end-connections, and they do not have to be cramped in any way.

(*d*) There is ample ventilation for the end-connections.

2. The other important feature of the design is the arrangement by which the joint between the two hubs on which the primary and secondary of the induction generator are built up is made inside the bearing, instead of on the projecting part of the shafts in the usual way. This enables us to have a quite ample and rigid bearing surface, and yet only have a shaft extension of about 3 in. in the set considered. The distance between the ends of the two bearing brasses is only about 11 in., scarcely more than it would be with an ordinary flange coupling.

The machine, in general, consists of two forced-lubrication, water-cooled bearings of large size, supporting respectively one end of the turbine and the secondary of the induction generator, and one end of the convertor and the primary of the induction generator. Mounted on the turbine shaft by a very long taper fit and keyway is a steel hub on which the secondary of the induction generator is built up. This is built up of punchings, in the usual way. the end-rings of the squirrel-cage winding being of aluminium, in which holes are punched for the bars, these being afterwards riveted over on the outside. The primary is built up within a frame provided with holes on one side, through which the insulated stator bars pass. This frame is mounted by a taper fit of ample area and key-way on another steel hub, similar in general design to that used for the secondary, which is again mounted by a taper fit within the bearing on the convertor shaft. In order to save the space taken up by a nut on the front end, these taper fits are arranged to tighten up from the back as shown.

The leads are brought from the primary winding through the hollow shaft to the convertor.

CHAPTER XX

FURTHER TYPES OF INTERNAL CASCADE SETS CONTAINING THREE RELATIVELY MOVING ELEMENTS

WE saw in Chapter XIX that the turbo-convertor could be built in a concentric form, such as we reproduce in Fig. 148.

In order to build such a machine on the internal cascade principle still retaining the three relatively revolving elements, we have merely to combine windings *B* and *C* reducing the width of the
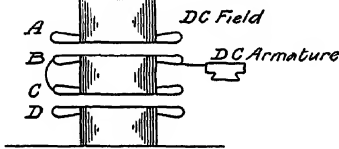
Fig. 148
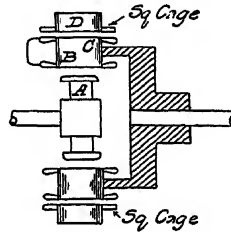
Fig. 150
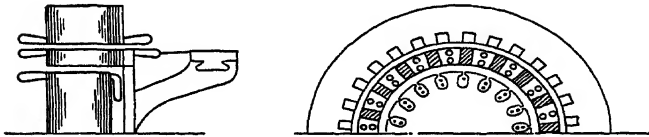
Fig. 149

" spinner " element till it is only just sufficient to carry the windings. The machine then takes the form shown in Fig. 149.

It will be seen that the intermediate, or "spinner," element is now reduced merely to the teeth of the original type. Neither of the two magnetic circuits is closed in the spinner element, but both fluxes simply traverse it and are closed in the two main elements.

Any number of intermediate forms can be constructed, in which the two fluxes are closed partially in the spinner element and partially in the two main elements.

The characteristics of the machine are essentially those of the externally cascaded turbo-convertor, but it admits of many modifications of construction.

Besides the modification shown, we may place the squirrel-cage rotor of the induction generator on the spinner and the primary on the high speed element.

This involves collector rings on both moving elements, which are entirely avoided by the previous arrangement and so far is a disadvantage. But the principal problem of such an arrangement as we are discussing is to obtain a sound mechanical construction for the " spinner." By placing a squirrel-cage rotor on the spinner in which the bars are permanently joined to a short-circuited end-ring we obtain a means of mechanical support. For by suitably designing such a squirrel-cage rotor we can make it support the teeth and the other winding on the spinner. It must not be forgotten that the flux due to the direct-current field also passes through the spinner, and means must be devised to make the squirrel cage non-inductive with respect to this flux. We shall return to this point later. Instead of the squirrel-cage rotor we may substitute a direct-current windings as we saw when discussing motor generators, but this would not have the same advantages.
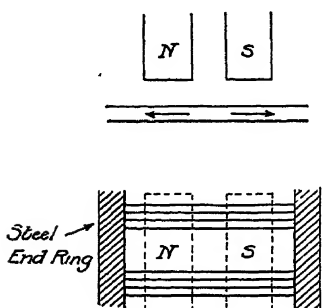
Fig. 151

Another arrangement, perhaps superior to the last, is the following, shown in Fig. 149.

In this arrangement both machines are synchronous, and the direct-current fields of both are on the stationary member. Neither of the fluxes, therefore, rotates with respect to the stator, and, therefore, it need not be laminated but may be of ordinary solid construction. The spinner bears a winding adapted to one polarity of the direct-current field and the high speed rotor a winding adapted to the other. Both moving elements require collector rings.

Let us consider this arrangement a little more in detail.

Take a case where the windings on the stator have 2 poles and 6 poles respectively, the flux wave being a 2-pole wave with a more or less pronounced triple harmonic.

If the high speed winding is connected as a 2-pole winding, the coils being made to cover one-third of the circumference or a complete wavelength of the 6-pole flux, they will be non-inductive to it and generate a frequency corresponding to the 2-pole component only.

If the low speed winding on the spinner is a 6-pole winding, as, for instance, a 2-circuit, 6-pole type, it will generate a frequency at the slip-rings corresponding to the 6-pole component only. If

the two sets of slip-rings are connected together the two frequencies must be the same, or the spinner will run at one-third the speed of the rotor running, say, at 500 if the rotor goes at 1,500. Such a spinner experiences no mechanical torque as the winding on it is purely a rotary convertor winding.

Mechanical construction of the spinner. We now come to the

Wave forms of Flux

Variable Pole fully excited as N-Pole

Variable Pole weakened as N-Pole

Variable Pole reversed & fully excited
as S-Pole
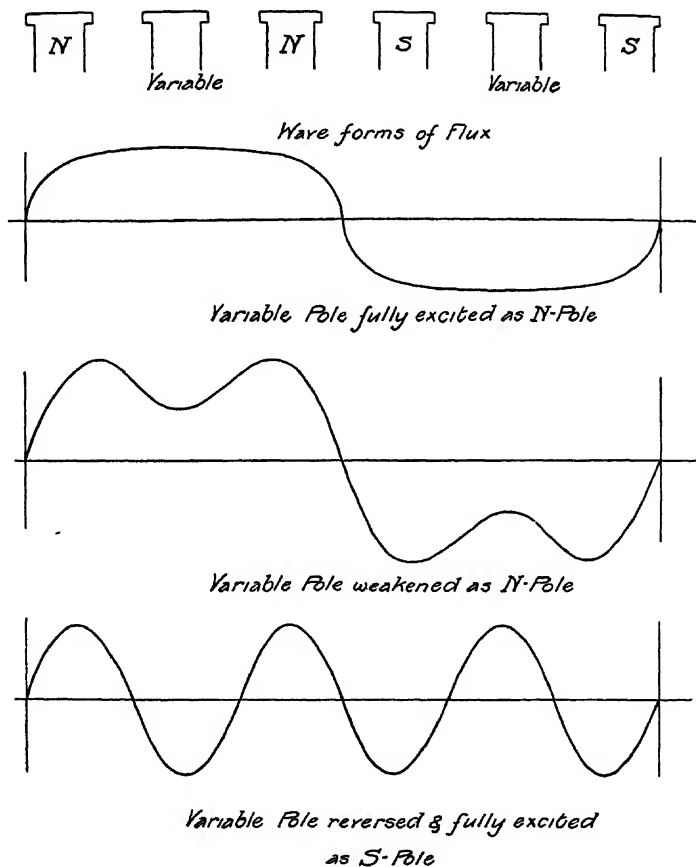
FIG. 152

problem of the mechanical construction of the spinner. The chief difficulty about this is the supposed necessity of laminating the magnetic parts. The following construction, however, may be used to avoid this.

If we have N. and S. poles on any machine longitudinally or axially adjacent to one another (Fig. 151), and so arranged that any conductor of a squirrel-cage winding cuts both of these together, no electromotive force will be induced in any of them and, therefore,

no current will flow in the squirrel cage as a whole, notwithstanding that the flux is arranged circumferentially in any way whatever.

If, therefore, we laminate our spinner longitudinally we are at liberty to short-circuit the extreme ends of our laminations to a circumferential ring, as, for instance, by welding them on to it.

In this way we may construct a substantial and mechanically sound spinner which yet is laminated.

It is interesting to note that these " spinner " types, originated by the late H. A. Mavor, have recently been revived by Messrs Krupp as single-phase railway motors.

PART IV

CHAPTER XXI

GENERAL PRINCIPLES OF THE VARIABLE POLE MACHINE

Speed change by changing the number of poles. Without ignoring the work of early inventors, such as Dahlander, Lindström, and Alexanderson, who have provided means of adapting a single winding to give, for instance, two numbers of poles having a fixed ratio, it is not unfair to say that the idea of using the method of pole changing to produce a machine having anything resembling a gradual variation of speed by small steps between wide limits, would have been regarded a few years ago as chimerical. Yet it is also obvious that if by any means we could render practically available all the numbers of poles which exist within a given range, we
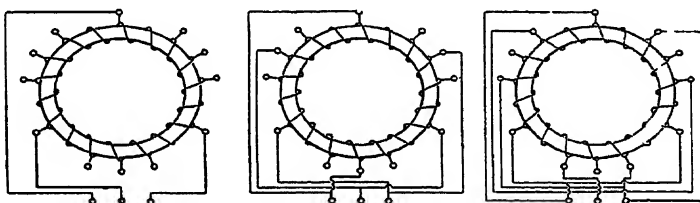
FIG. 153

have a machine with a very high degree of practical usefulness. For instance, on 50 periods between the speeds of 375 r.p.m. and 1,000 r.p.m. there are six numbers of poles available.

One method by which all these numbers of poles might theoretically be rendered available with a single winding has long been known, i.e. the use of a ring winding tapped at the various points necessary to give different numbers of poles, as shown in Fig. 153. It is well known that a ring winding can be used on any number of poles, if a number of equidistant tappings equal to three times the number of pairs of poles are brought out on each speed, equalizer connections being made between tappings 1 to 4 throughout the circumference. To obtain six numbers of poles from the same winding in this way, however, would require a formidable number of tappings, and to connect them together in the various manners required would involve an equally formidable type of switchgear.

Although a ring winding is inconvenient in practice, the results given by such a winding can be closely imitated by the means of a drum winding of either the lap or wave type. Fig. 154 is the

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ordinary diagrammatic representation of a lap winding with tappings J_A, J_B, J_C . . . spaced apart by an amount equal to the slot pitch of the winding. In the case shown the slot pitch is 1—4, that is to say, a coil having one side in slot 1 has its other side in slot 4. Suppose the currents in the winding on opposite sides of
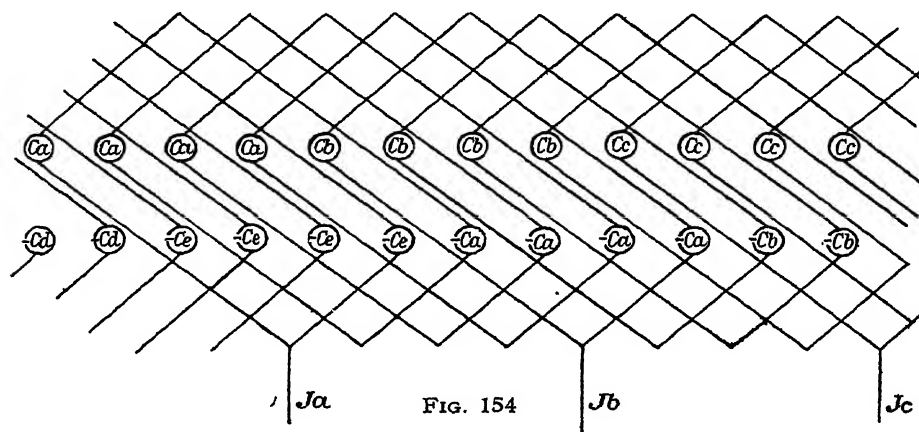
Fig. 154
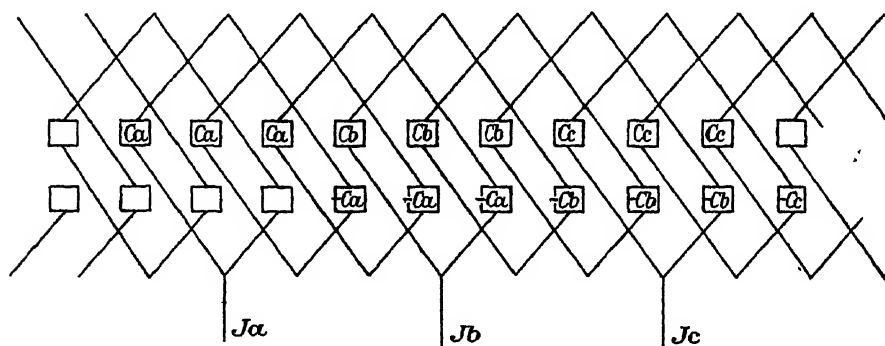
Fig. 155

tapping J_B are $-C_A$ and C_B respectively, those on opposite sides of tapping J_C, $-C_B$ and C_E respectively, and so on. All the slots embraced by coil connected to tapping J_B will have in them a bar carrying current $C - C_B$ and another carrying current C_A, so that together they constitute a zone of current of magnitude $C_B - C_A$. Similarly, the slots within the span of the coil joined to J_C make a zone of current in J_C, $C_C - C_B$, and so on. But the current in J_B is $C_B - C_A$, and the current in J_C, $C_C - C_B$. So that the effect of the winding is to produce zones of current proportional in value to the current in the adjacent tapping. In all windings the zones or bands should preferably be identical.
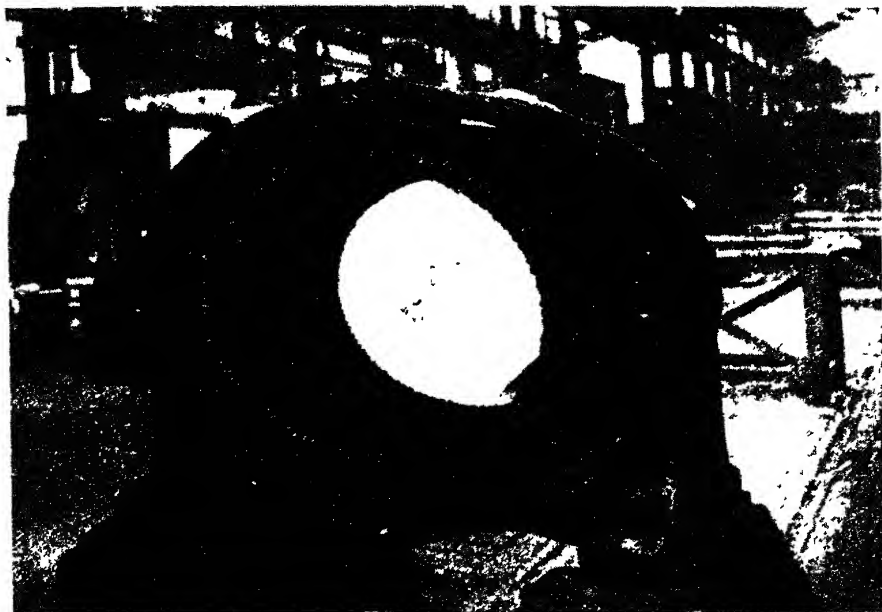
PLATE IV

Stator winding of six-speed motor shown in Frontispiece.
Speeds 1,000, 750, 600, 500, 428, 375
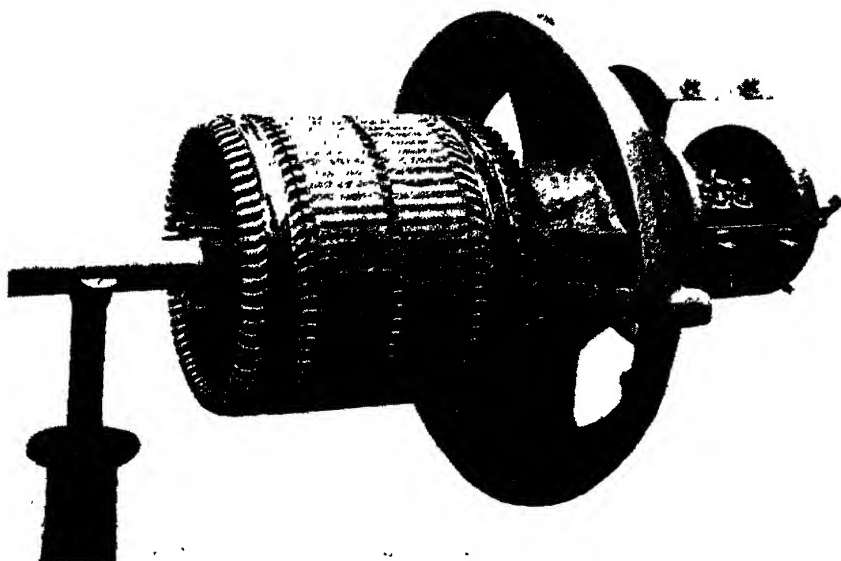
PLATE V

Rotor winding of six-speed motor shown in Frontispiece.
Speeds 1,000, 750, 600, 500, 428, 375

If the pitch of the tappings were greater than the slot pitch the same effect would be produced, only the current zones would be separated by slots containing bars carrying equal and opposite currents. By the method indicated below it is possible to secure a large number of different zones with a relatively small number of tappings.

One method is illustrated by Fig. 155, which shows a 2-layer winding like that of Fig. 154, but with a slot pitch greater than the pitch of the tappings and not an exact multiple of it. The effect of this is to produce within the span of the coil joined to J_A three different bands of current, viz. a band $C_A - C_D$, another $C_E - C_E$, and a third, $C_B - C_E$, so that in all there are twice as many zones as tappings.

In general all the tappings of the winding must be brought out to independent terminals, but where all the numbers of poles it is desired to use fulfil certain conditions it is sometimes possible to reduce the number of terminals.

For instance, if for each of the numbers of poles with which the motor is intended to work certain points in the winding are equipotential points, they may be joined by equalizers and only one terminal provided for each set.

Similarly, if for each of the numbers of poles there are groups of bars carrying equal currents, the bars of these groups may be united into a wave winding in place of the lap winding. It will sometimes occur that the same distribution of currents is repeated several times round the circumference of the machine, so that the bars can be connected by a wave winding having a short pitch less than the pole pitch at one end, and a long pitch spanning several pole pitches at the other.

The least number of independent bars or groups or bands required in a machine which is to work with each of several given numbers of poles is the product of the number of phases in the shortest pole pitch used, and the maximum number of poles used, divided by the G.C.M. of all the numbers of pairs of poles used. To secure satisfactory working it is well to have not less than two phases in the shortest pole pitch, and the number of phases in each larger pole pitch will then be more than two, though it may not be a whole number. The winding will present a number of identical sections equal to the G.C.M. of all the numbers of pairs of poles.

As it is important that this matter should be perfectly clear, it may be of advantage to regard it from another point of view.

Fig. 156 shows three sketches of a drum-wound coil lying (*a*) in a field of 6 poles, (*b*) in a field of 10 poles, and (*c*) in a field of 16 poles, the pitch of the coil being approximately one-tenth of the circumference. The sine curve shown in the figure may be taken to represent the values of the air-gap density at a particular instant, and, if so, the ordinates to the curve will represent on a suitable scale

the voltage induced in the left-hand conductor. Similarly, the ordinate CD represents that induced in the right-hand conductor. The curves have in every case been so drawn that the left-hand conductor lies at the point of maximum density. The total E.M.F. induced in the coil will in each case be $(AB + CD)$. It will be seen that in curve (b) (Fig. 156), representing the 10-pole condition, the E.M.F. induced in the coil is a maximum, since CD as well as AB is a maximum. In the curve corresponding to 6 poles, the pitch of

(a) 6 poles

(b) 10 poles
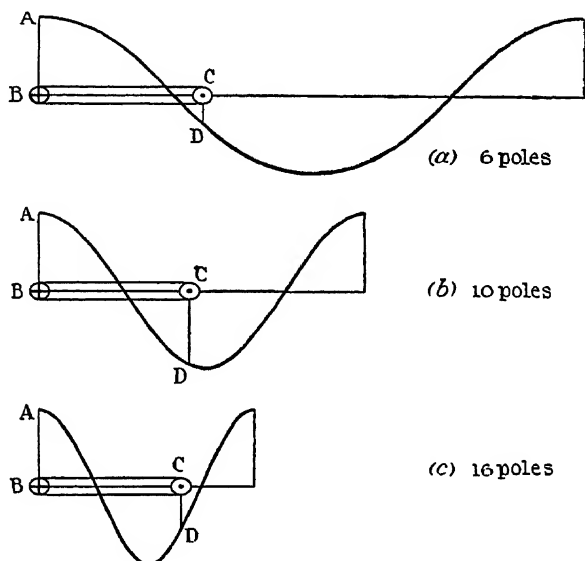
(c) 16 poles

FIG. 156

the coil is considerably less than the pole pitch, and, therefore, CD is a good deal less than AB, but, nevertheless, the sum of the two is only reduced to about 70 per cent of its previous value. Similarly, in curve (c), representing 16 poles, the pitch of the coil is a good deal more than the pole pitch, and, therefore, CD is again reduced, but again the sum of AB and CD is only about 70 per cent of its full value. It is, in fact, in order to prevent too great a difference between the pitch of the coil and the pole pitch, that it is usual to limit the speed range of these motors to a value not exceeding 3—1.

A further difficulty arises from the fact that the voltage induced in any section of the machine consisting of, say, two, three or more coils connected in series having a given number of turns is proportional to the rate at which the magnetic lines cut the conductors, and this rate is, of course, proportional to the speed of the flux wave, which is practically the same as that of the revolving element.

Consequently, at the highest speed of the motor, the flux wave is cutting the conductors at a very high rate, and, if the E.M.F. applied to any section is fixed, only a small flux will be required to produce it, owing to the high rate of cutting.

On the low speed of the motor, on the contrary, the flux wave is cutting the conductors at a low rate, and, consequently, if the E.M.F. across the sections remains the same as before, a large flux will be necessary to produce it.

Hence the flux of the motor will tend automatically to fall off in proportion to the rise of the speed in much the same way as it
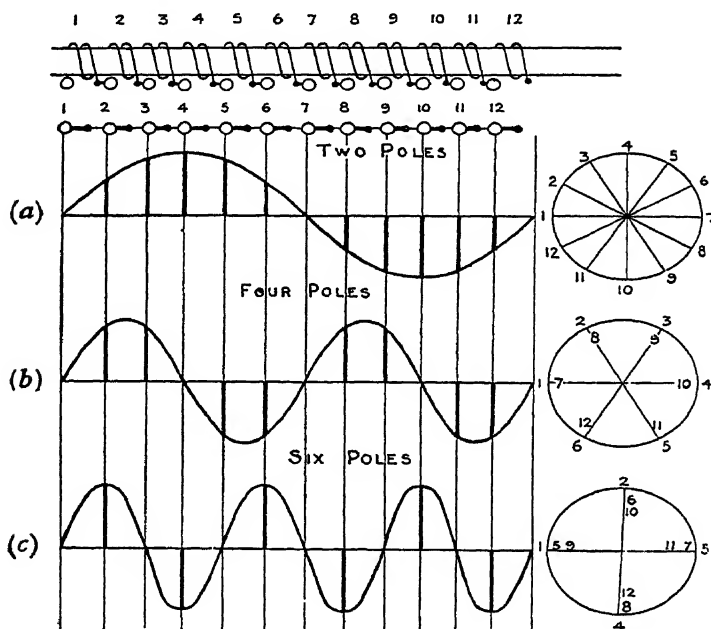
FIG. 157

does in the direct-current motor, and, hence, such a motor having a constant voltage per section, will tend to have a constant horse-power characteristic, which is not at all what is demanded in practice.

Means must, therefore, be provided to increase the voltage across each section more or less in proportion to the speed. These difficulties may be summarized as follows—

1. The difficulty of producing a winding having a moderate number of terminals and capable of simple switching.

2. The difficulty of varying the voltage across a section containing a fixed number of turns, so as to keep the flux approximately constant with increasing speed without the use of costly apparatus.

Method of overcoming these difficulties. In order to overcome the necessity for a very large number of terminals, a method of pole changing entirely different from that described above may be adopted. This method is shown in Figs. 157 and 158. Still considering a ring-wound machine, assume it to be divided into 12 sections, shown developed in Fig. 157.

At this point it may be convenient to recall the diagrammatic
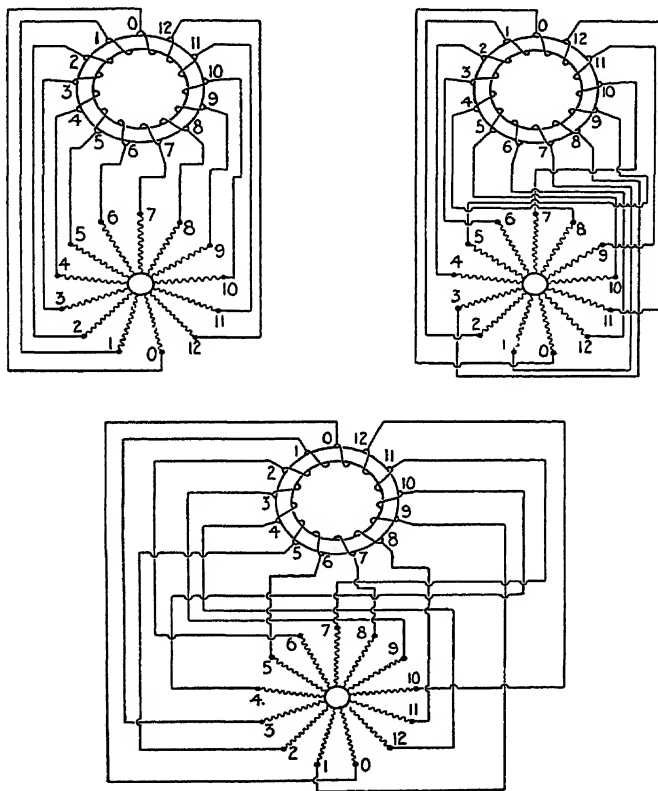
Fig. 158

method of Fig. 115 for denoting alternating-current windings developed in order to simplify winding diagrams. It will be seen that the ring winding shown at the top of Fig. 157 is divided into 12 sections, each section being shown as consisting of two turns. The beginning of each section is denoted by a white circle and the end by a smaller blackened circle, the extremities of the various sections being connected in series in the usual manner. An obvious simplification for diagrammatic purposes is to omit the drawing of the winding and merely retain the two terminals of each section, joining them by a straight line in order to show that they belong to

the same section. This diagrammatic method of denoting the winding is shown immediately below the drawing of the ring in Fig. 157, the sections being numbered in order round the circumference.

Imagine the 12 sections of the ring winding shown in Fig. 157 to be carrying currents differing in phase from one another by 60°, the current vectors being shown on the right of Fig. 157 (*b*), the current in section 1 having a phase 0°, that in section 2 having a phase 60°, that in section 3, 120°, and so on. These vectors when projected on a vertical axis in the usual way give, of course, the instantaneous currents in the different sections, which may be plotted as shown on the left of Fig. 157 (*b*). We thus obtain a series of ordinates the extremities of which lie on a sine curve,
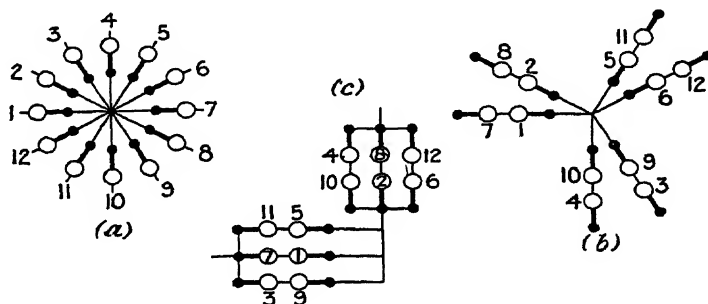
FIG. 159

having two positive and two negative maxima, i.e. four poles on the circumference. If, however, we so reconnect these sections that the phase difference between adjacent sections is 30° and plot out the ordinates as in Fig. 157 (*a*), we obtain another sine curve having one positive and one negative maximum, i.e. two poles, while if we make the phase difference between the sections 90°, as in Fig. 157 (*c*), we obtain a curve having three positive and three negative maxima, i.e. 6 poles.

In Fig. 158 are shown the connections whereby this change of phases may be accomplished for the case of 13 phases, which is practically more convenient in some respects, since by using a prime number of phases we prevent connections from any two phases of the machine being made to one phase of the convertor. As will be seen, it involves the use of 13 terminals on the motor (still shown as a ring winding) and some means of producing a number of phases (viz. 13) equal to the number of sections in the motor winding. Since 13-phase distribution is never used, this involves some means of converting from the usual 3-phase supply to the number of phases required by the motor and a new piece of apparatus, the phase transformer or convertor, is required with this method of control.

The theory of this method of pole changing may be further elucidated by the use of the method of studying a rotating field explained by Hellmund (*Transactions of the American Institute of Electrical Engineers*, 1908, " Graphical Treatment of the Rotating Field "). Figs. 160 and 161 show how to plot out the flux density for one number of poles, say, for the 14-pole arrangement, assuming 31 phases and 93 slots. The phase difference between adjacent groups of bars in the 14-pole arrangement will obviously be $\frac{7}{31} \times 360°$. In Fig. 160, *CD* is a vector representing in magnitude and phase of any one current band due to the upper layer of the
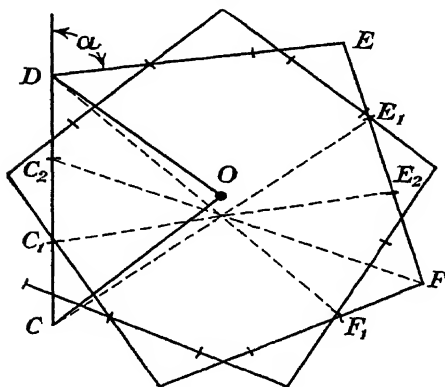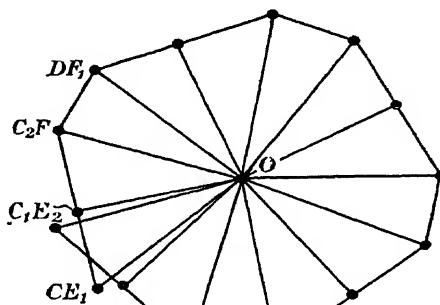
FIG 160 FIG 161

winding alone, due, that is, to the upper conductors in three neighbouring slots, the three equal parts into which the vector is divided indicating the ampere-conductors due to the respective slots. *DE*, set off at an angle of $\frac{7}{31} \times 360°$, similarly represents the ampere-bars or magnetomotive force of the next band of the upper layer. The magnetomotive force of the whole winding can be plotted in similar manner. The figure shows a part only of the polygon which will be obtained. For the polygon does not close when the sum of its exterior angles *a* amount to approximately 360°—in fact, because the phase number is prime, not until the exterior angles total $7 \times 360°$. The ends of the vectors lie on a circle. A radial line from the centre of this circle to any point in the polygon represents vectorially the magnitude and phase of the flux density in the air-gap due to the upper layer of the winding alone at the corresponding point of the circumference of the machine. Thus *OC* represents the flux density in the teeth at, say, the right of slot 1, *OD* that on the right of slot 4, and so on. The lower layer of the winding, however, has so far been disregarded. Suppose the pitch it is desired to find

is 1—8. Consideration of a winding diagram, such as those in Figs. 154 and 155, will show that the lower layer of conductors in slots 1, 2, and 3 will be carrying currents equal and opposite to those in the upper layer of conductors in slots 8, 9, 10, or, if the pitch is 10, the lower layer currents will be equal and opposite to the upper layer of slots 10, 11, 12. Taking 8 as the pitch the flux density beside slot 1 will be the difference of the vectors OC and OE_1. This difference is represented vectorially by CE_1, which, therefore, represents the resultant flux density beside slot 1 due to the whole winding.

In Fig. 161, vectors $O - CE_1$, $O - C_1E_2$, $O - C_2F$, $O - DF_1$, etc., are set out from a centre O equal and parallel to the vectors CE_1, C_1E_2, C_2F, DF_1, etc., of Fig. 161, and the junction of their ends gives a second polygon of which the radii represent resultant flux density. Points on this polygon diametrically opposite represent points distant by a pole pitch in the actual machine. Because the pitch chosen is not an exact multiple of the tapping pitch, the polygon of Fig. 161 has twice as many sides as that of Fig. 160, being similar to it but with the corners cut off.

It will be seen from this figure that the mesh sections are intermediate in phase between the star sections, e.g. section 2 is intermediate in phase between sections 1 and 3, and so on ; thus, although the winding connected as in Fig. 162 requires only four terminals. the E.M.F.'s across the various sections are nevertheless in eight distinct phases.

Again, it was found in most cases that while this method of connection might suffice to double the number of phases on a particular number of poles, yet, in order to produce the same result on another number, the whole or many of the sections must be disconnected from one another and reconnected in a different order. Thus it would seem that we have only simplified the transformation in order to complicate the switching.
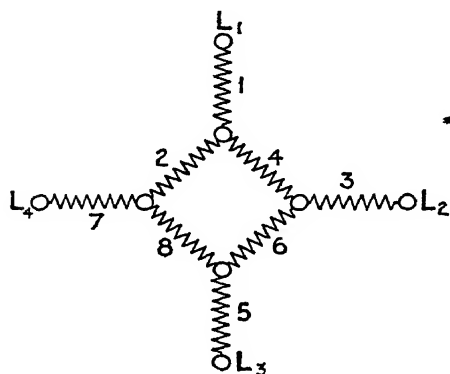
The method of reversal and the star-mesh method may be combined, and thus we may obtain windings in which the E.M.F. across the sections has four times as many phases as that supplied to the terminals.

After a very large amount of research, extending over a considerable period of time, a winding was found in which this result could be accomplished, not on one number of poles only, but over a large series of numbers covering a speed range of 3—1 or more, without reconnecting the winding in any way when changing the numbers of poles. This winding is shown in Fig. 163 for the case of 36 sections, 18 of which are connected in mesh and an equal number connected in star to the angles of the mesh, alternate sections in the mesh being mutually reversed and alternate sections in the star being also mutually reversed, the mesh sections alternating with the star sections around the circumference of the machine.

The winding shown in Fig. 163 (containing only 18 terminals) can be used on all numbers of poles from 6 to 16, and is, with a slight further modification, also capable of being used on further numbers of poles from 20 to 26.

This connection, therefore, gives us not only a winding of a very simple character with a moderate number of terminals (18) capable of reconnection in a simple manner for a large number of different numbers of poles, but it also gives us the characteristic which was shown above to be desirable, viz. that of having an approximately constant flux on all numbers of poles. It is difficult to explain the
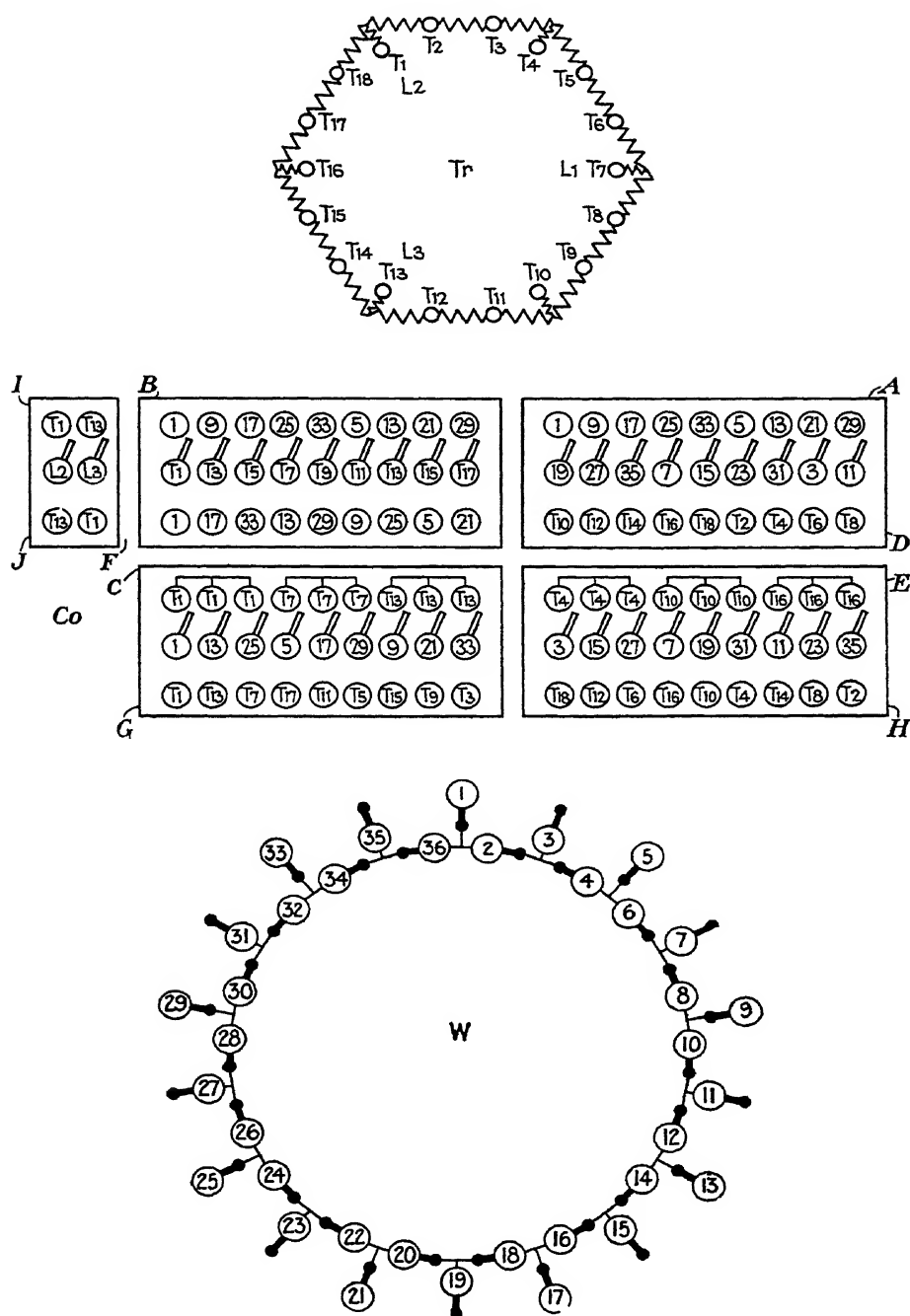
Fig. 162

FIG. 163

cause of this in a simple manner, but one may say briefly that it is due to the fact that the number of sections in series between terminals having a given phase difference of, say, 120°, becomes gradually greater and greater as we increase the number of poles, or decrease the speed, so that as the rate of rotation of the flux wave gets less and less, due to the decreasing speed of the motor, the voltage across any section having a given number of turns also becomes less and less approximately in the same proportion, and

14 Pole.
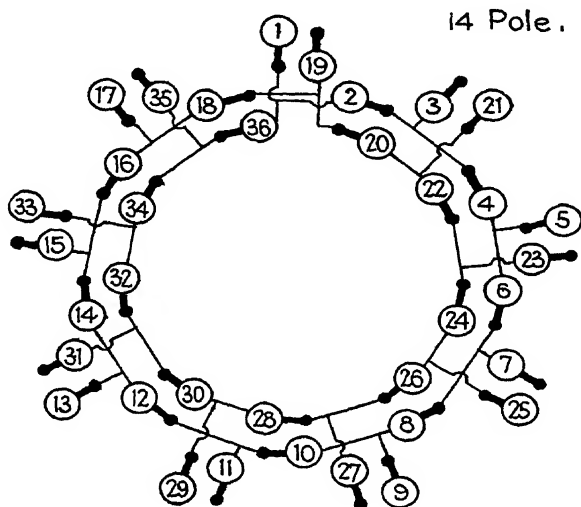
FIG. 164

thus about the same flux is needed to balance this voltage at all speeds.

For reasons of economy in the number of supply phases needed, and also where a winding is to serve as a secondary on one pole number in addition to acting as a primary on another, it is often an advantage if several terminals of the winding are connected to the same phase of supply. The number of terminals connected together is the common factor of n and $\left(\dfrac{n}{2} - p\right)$, where n is the number of star sections and p the number of pole pairs. From the characteristics of the winding already set forth, it is apparent that the number of sections must be divisible by 4, i.e. n must be even. If $\dfrac{n}{2}$ is odd, then whenever p is also odd $\left(\dfrac{n}{2} - p\right)$ is even and will have a common factor with n of at least 2, i.e. on odd numbers of pole pairs the terminals will be connected at least in pairs. There are then sometimes four times as many different phases in the sections

of the winding as there are in the supply. If $\frac{n}{2}$ is even, then the terminals will be connected in pairs on even numbers of pole pairs.

As an example of the use of this type of winding for variable pole working, Fig. 163 shows a winding W of 36 sections, together with the transformer T_r and a controller C_o necessary to enable it when supplied from 3-phase mains to produce 6, 8, 10, 12, 14, or 16 poles. The supply mains are connected to the alternate angle

10 Pole.
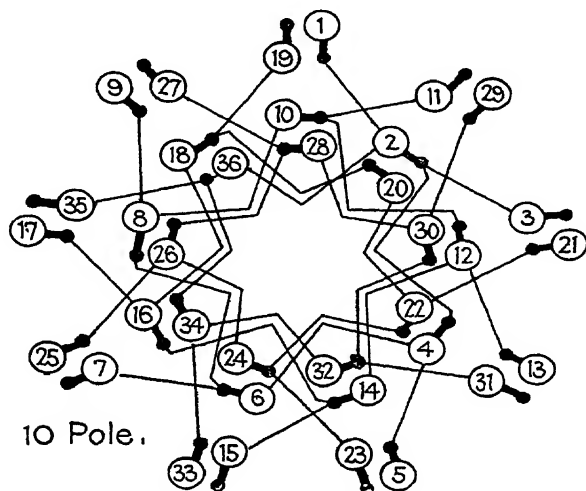
FIG. 165

L_1, L_2, L_3, of the transformer. The controller consists of four 9-pole 2-way switches, and one 2-pole 2-way switch. The contacts and knife blades of the switches are connected to the tappings of the transformer or the sections of the winding whose numbers they bear, the connections being omitted for simplicity. The 2-pole switch serves to reverse the order of connection of the supply mains to the transformer, and in conjunction with the other four switches brings about the changes in the phase of the sections shown in Figs. 164 to 169, in which figures the direction in which each section is drawn represents its phase as above explained. The two positions of the respective switches are indicated by the letters *A—J* respectively. If *A*, *C*, and *I* alone are closed the winding has the phases shown in Fig. 166, it has 6 parallels and produces 6 poles. If *B*, *D*, and *J* are closed the winding is connected as in Fig. 168 *and gives 8 poles. If A, B, and I are closed, the winding is connected as in Fig.* 165, has two parallels and produces 10 poles. If *C*, *E*, and *J* are closed the winding has the phases shown in Fig. 167, has three parallels and produces 12 poles. If *A*, *F*, and *I* are closed the

winding has the phases indicated in Fig. 164, has two parallels and produces 14 poles. Finally, if switches G, H, and I are closed the phases of the winding are those shown in Fig. 169, and the winding produces 16 poles. Examination of the figures will show that in all of them the winding retains the connections between its sections shown in Fig. 163. It is only the connections to the supply that are changed.

The following rule is of great importance in understanding the relations between the phases on different pole numbers.
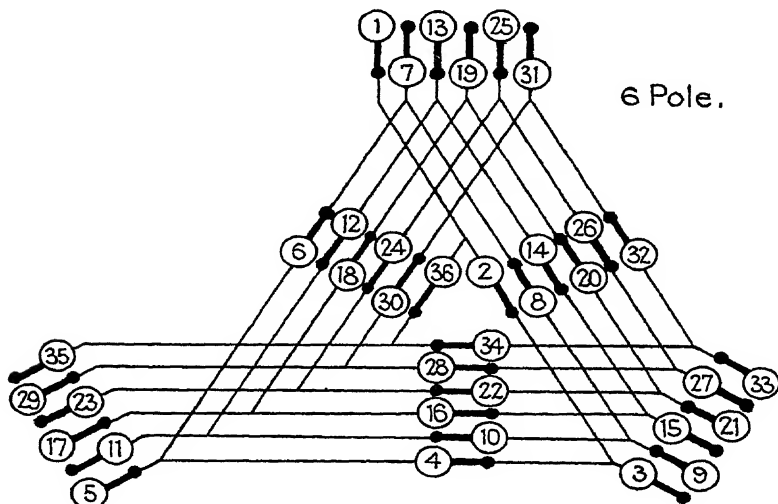
FIG. 166

If the winding comprises r equally spaced groups of $P + Q$ sections each, the relative phases of the sections in each group will be the same on P pole pairs and Q pole pairs. The relation between their actual phases may be expressed by saying that for Q pole pairs as compared with P pole pairs each group is "reversed" round one of a set of r equally spaced axes, of which the first, round which group 1 is reversed, passes through section 1 of that group, and those round which the remaining groups are reversed are to be taken in the same order as the groups, i.e. group 2 reverses about an axis differing $\frac{\pi}{r}$ in position in space from the first axis, group 3 about one differing $\frac{2\pi}{r}$, group n round one differing $\frac{(n-1)\pi}{r}$. By the statement that a group of sections is "reversed" about a particular axis in space in changing from P pole pairs to Q pole pairs, the relative phases of its sections remaining unaltered, is meant that any section distant by a given amount in a clockwise

direction on P pole pairs has the same phase as a section distant by the same amount in a counter-clockwise direction on Q pole pairs.

Thus concisely stated it is not very easy to grasp, and it will be desirable to devote further space to elucidating it.

If the winding comprises r equally spaced groups of $P + Q$ sections each, the relative phases of the sections in each group will be the same on P pole pairs and Q pole pairs.

Fig. 163 shows a winding of 36 sections. Consider the phase changes as between the 8-pole connection (Fig. 168) and the 10-pole connection (Fig. 165). Here we have $P + Q = 4 + 5 = 9$. In a winding of 36 sections there are 4 groups of 9 sections ; $r = 4$. The groups are to be equally spaced : thus group 1 consists of section numbers 1, 5, 9, 13, etc. ; group 2 of section numbers 2, 6, 10, 14, etc., and so on.

12 Pole.

FIG. 167

Now we have to consider the relation of the phases of the sections in these groups on 8 and 10 poles. Then simply write down the phases of the sections.
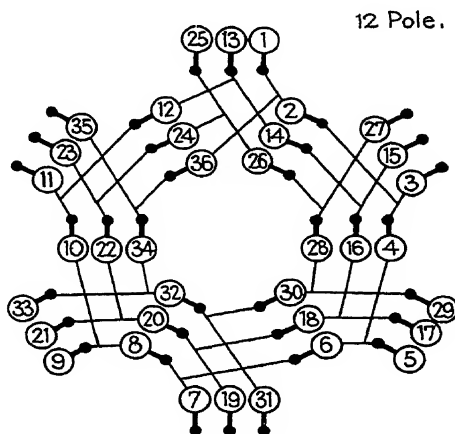
Section Number	Phase on 8 poles	Phase on 10 poles	Section Number	Phase on 8 poles	Phase on 10 poles
1	*0*	*0*	19	**0**	**180**
2	40	50	20	40	230
3	**80**	**100**	21	*80*	*280*
4	120	150	22	120	330
5	*160*	*200*	23	**160**	**20**
6	200	250	24	200	70
7	**240**	**300**	25	*240*	*120*
8	280	350	26	280	170
9	*320*	*40*	27	**320**	**220**
10	0	90	28	0	270
11	**40**	**110**	29	*40*	*320*
12	80	190	30	80	10
13	*120*	*240*	31	**120**	**60**
14	160	290	32	160	110
15	**200**	**340**	33	**200**	*160*
16	240	30	34	240	210
17	*280*	*80*	35	**280**	**260**
18	320	130	36	320	310

The phases of the sections of group 1 are in italics. They are, on 8 poles, 0°, 160°, 320°, 120°, and so on. On 10 poles they are also 0°, 160°, 320°, 120°, and so on. But it will be noted that in putting down the phases on 8 poles the sections are also in order clockwise, section 1, 5, 9, 13, etc., and to find the corresponding phase on 10 poles we take the sections in anti-clockwise order, 1, 35, 29, 25, and so on. The *actual* phases happen to be the same ·n the two cases. But the rule says only that *the* RELATIVE *phases of the sections in each group will be the same on P pole pairs and*
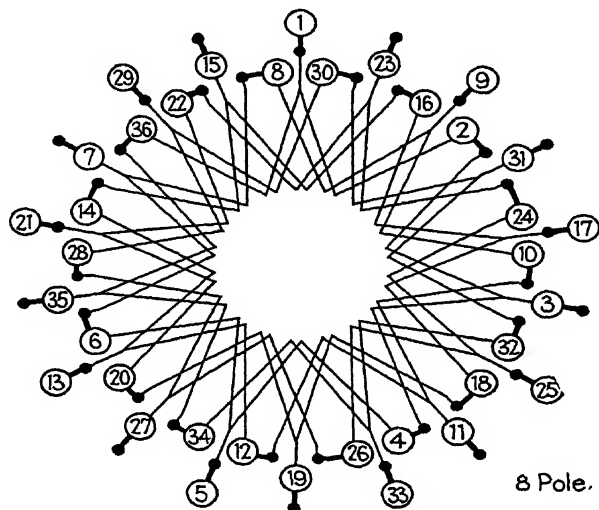
FIG. 168

Q pole pairs. Try another group. Group 3 is in heavy type. The phases of the sections of group 3 on 8 poles are 80, 240, 40, 200, and so on.

The phases of the sections of group 3 on 10 poles are 100, 260, 60, 220, and so on.

It will be noted the sections have again been taken in opposite order on 10 poles. It will also be noted that the RELATIVE phases of the sections are the same on both pole numbers; the actual phases differ by 20°.

The relation between their actual phases may be expressed by saying that, for Q pole pairs as compared with P pole pairs, each group is " reversed " round one of a set of r equally spaced axes.

Is this the case? The axis *round which group* 1 *is " reversed " passes through section* 1 *of that group*, so that the phase of section 1 remains the same. Now what does " reversed " mean? It means *that any section distant by a given amount in a clockwise direction on P pole pairs has the same phase as a section distant by the same*

amount in a counter-clockwise direction on Q pole pairs. Take section 5. Its distance from section 1, i.e. from the " axis " is $\dfrac{4 \times 360°}{36}$ $= 40°$ in a clockwise direction. On P ($= 4$) pole pairs it has the phase of 160°. This is to be " the same phase as a section distant by the same amount in a counter-clockwise direction on Q pole pairs." Which section is 40° distant from the axis in a counter-clockwise direction ? Section 33. What is its phase on Q ($= 5$) pole pairs ? 160°.
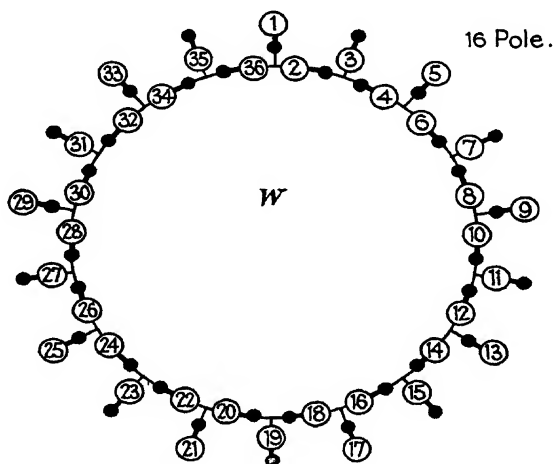
Fig. 169

Take another group. The axes *round which the remaining groups are reversed are to be taken in the same order as the groups*, i.e. *group* 2 *reverses about an axis differing* $\dfrac{\pi}{r}$ *in position in space from the first axis, group* 3 *about one differing* $\dfrac{2\pi}{r}$, and so on. Take group 3. The axis about which it is to be reversed is $\dfrac{2\pi}{4} = 90°$ from the axis about which group 1 is " reversed " i.e. is 90° from section 1. Consider section 7, which is a section of group 3. Its angular position is $\dfrac{6}{36} \times 360° = 60°$ clockwise from section 1. It is, therefore, 330° measured clockwise from the axis about which group 3 is to be " reversed." Its phase on P ($= 4$) pole pairs is 240°. What section is 330° from the axis measured COUNTER-clockwise ? Obviously, the section which is 30° from the axis measured clockwise, i.e. the section which is 120° from section 1 ; that is section number 13. The phase of section 13 on Q ($= 5$) pole pairs is 240°.

The first portion of the rule is easily proved. It has been pointed out in Chapter XV above that, if we have two fields of P pole pairs and Q pole pairs respectively, they will induce E.M.F.'s in any conductor, which will coincide in phase at $P + Q$ points, and hence, the relative phases of the E.M.F.'s in the sections of such a group of $P + Q$ sections must be the same on P pole pairs as on Q pole pairs, forming a balanced $P + Q$ phase system if we assume that P and Q have no common factor. Hence, suppose that the phases of the E.M.F.'s in one section or conductor (say section 1) are brought into coincidence as can always be done, then if both fields revolve in the *same* direction any section distant by a given amount from section 1, in a clockwise direction, has the same phase on P pole pairs as a section distant by the same amount in a counter-clockwise direction has on Q pole pairs. But the proof of the remainder of the rule is so complex that it has to be omitted. What has already been said should be sufficient to make the meaning and accuracy of the rule clear.
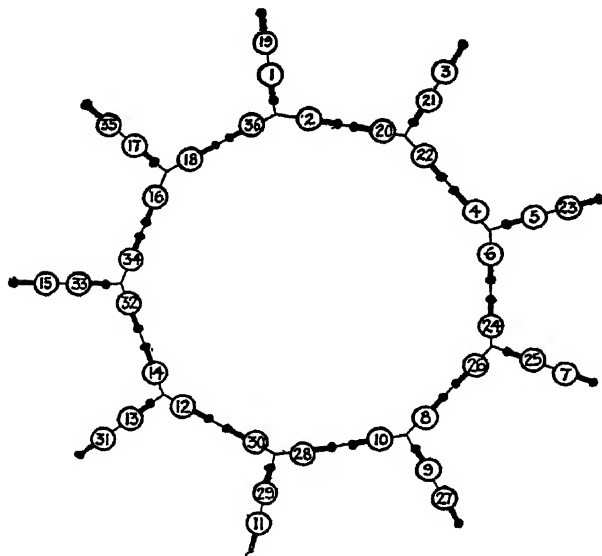
FIG. 170

A further simplification is possible where, for instance, only odd numbers of pairs of poles are required, and, consequently, diametrically opposite sections are always opposite in phase on every number of poles which a machine is required to give, and, therefore, may be permanently connected in series, mutually reversed.

The winding of such a machine is shown in Fig. 170, and, as will be seen, has only 9 terminals, capable of operating on 2, 6, 10, and 14 poles, and, with a slight further modification, on 22 and 26 poles. This winding is, probably, the simplest known multi-speed winding. It requires no more than 9 phases, while the original winding may require 18 phases.

CHAPTER XXIII

THE PHASE CONVERTOR

HAVING described how the difficulties connected with designing a multi-speed winding have been overcome by means involving the use of a phase transformer or convertor, it will next be desirable to describe how this apparatus is constructed in practice. In certain cases, e.g. where a single motor is operated from a generator which serves no other purpose, as in ship propulsion, no such transformer is needed, as there is no difficulty in winding the generator for the number of phases required, but in the majority of cases the machine, if it is to be of any practical use whatever, must operate from a power supply having, say, not more than three phases, and, hence, it is of vital importance to the practical success of the apparatus that this
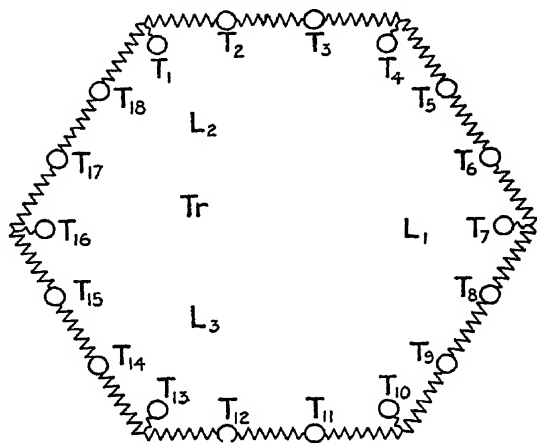
FIG. 171

phase transformer should be not merely technically satisfactory, but should also be capable of being built at a very moderate price.

By restricting the number of phases to multiples of three, the transformers to be excited from a 3-phase circuit may be simplified very materially, and it will be convenient to describe the transformer as used in connection with the winding previously referred to. This transformer does not differ externally in any way from the standard 3-phase core type transformer, built in the usual way with three exactly similar limbs. The connections, however, are as shown in Fig. 171, in which all the windings shown by lines parallel to one another are considered to be wound on the same limb of the transformer. Since the windings shown in this figure are drawn parallel to three straight lines at 120°, it follows that only three such limbs are needed.

If we take a 3-phase transformer having two secondary sections on each limb, and connect these sections in the manner shown, which may be called a hexagon connection, we obtain an apparatus from which 6-phase currents may be derived by bringing out

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terminals from the corners of the hexagon. If, however, we tap the sides of the hexagon in the manner shown. winding on the three limbs, in addition, six small auxiliary sections to be connected to the corners of the hexagon, it is possible without further elaboration to obtain no less than 18 phases, as has been amply proved in practice, the tappings shown in the diagram lying in a circle.

So far, we have regarded the hexagon winding with tappings as being the secondary, and have assumed that there is a primary winding which carries the 3-phase currents. But if no voltage
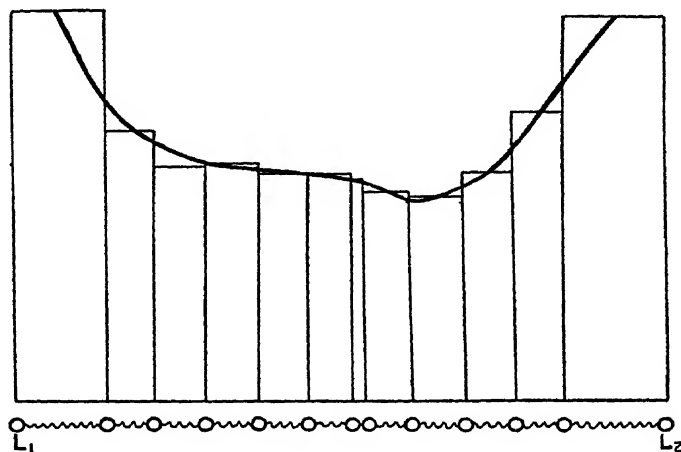
FIG. 172

transformation is needed, as in this case, a separate primary is not needed, for the primary terminals may be connected direct to alternate corners of the hexagon, or even, in order to save extra terminals, to the extremities L_1, L_2, L_3 of three of the small auxiliary sections wound on the transformer and corresponding to the secondary terminals T_1, T_7, T_{13}. This latter plan is adopted to avoid the necessity of bringing out three extra primary terminals.

By using the apparatus as an auto-transformer in this way we at once abolish the whole of the primary windings, and, therefore, reduce the amount of copper on the transformer to half, or, in other words, double the rating of a transformer of a given size.

We do more than this, in fact. because the primary and secondary currents flowing in opposite directions in the same windings give a resultant which is very materially less than the secondary current alone. This is exemplified in Fig. 172, which shows the currents in the sections of the 30-phase transformer, which formed part of the original experimental machine which was constructed to test the present method of speed variation in its earliest form, L_1, L_2 being the 3-phase terminals.

It will be seen that the current in the neighbourhood of the 3-phase terminals is much greater than at points intermediate between them, and, in fact, falls off to less than one-half at a point midway between the terminals.

Fig. 173 shows a similar curve for more recent 9-phase and 18-phase transformers. The same reduction in current per section at a point intermediate between the 3-phase terminals T_1, T^0, is noticed here, and the analogy between the reduction of current in these transformers and the similar reduction which takes place in the armature of a rotary convertor in sections intermediate between the tapping points may be referred to.

As a result of this further reduction a phase transformer, to transform a given amount of power, say, from 3 to 9 or 18 phases, will require a size of transformer of only between 30 to 40 per cent of the rating of the transformer which will be required to convert the same amount of power, say, from one voltage to another.

Still another economy which has the effect of still further reducing their capacity can be effected in the use of these phase convertors. In

Fig. 173

a constant torque motor giving a horse-power proportional to the speed, the amount of power taken will also be approximately proportional to the speed. For instance, the power taken at 750 r.p.m. will be only 0·75 times the power taken at 1,000 r.p.m., and if we can arrange the winding of our motor so that it is directly connected to the line when operating at 1,000 r.p.m., the phase convertor not being used on that speed but coming into operation for the first time only at 750 r.p.m., we shall be able to reduce its capacity to 0·75 times the, value which would be needed to supply the power required at 1,000 r.p.m., making it only 22 to 30 per cent of the capacity required to transform the whole power.

Curves, such as that of Fig. 173, may also be calculated by means of a vector diagram. For the case of a 3-phase primary and a secondary number of phases, also divisible by three, and
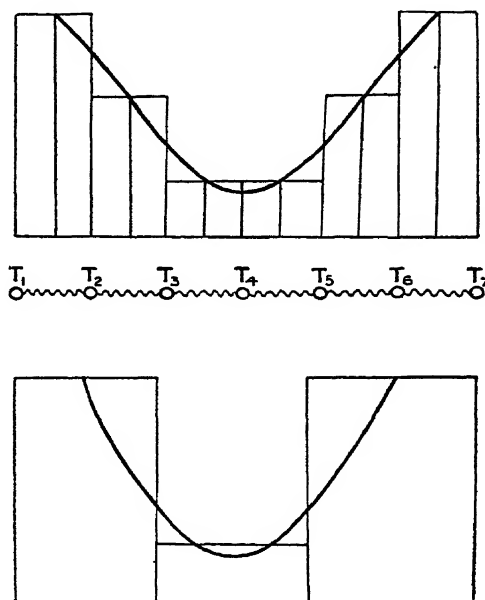
symmetrically placed with respect to those of the primary, the following rule[1] has been developed.

Draw an equilateral triangle ABC (Fig. 174), each of whose sides is divided into three parts, AD, DE, EB, and similarly for the other two sides. Let the three central sections ED, etc., be proportional to the primary currents I_1, I_2, I_3, supposed equal and balanced. The six outer sections are all equal and of length:

$$x = I_1 \frac{\left(3\sqrt{3}E_1 - 2n\,E_2 \sin \dfrac{\pi}{n}\right)}{\left(2nE_2 \sin \dfrac{\pi}{n}\right)}$$

where E_1 = primary star volts $\dfrac{\text{(line volts)}}{\left(\quad \sqrt{3}\quad\right)}$

E_2 = secondary star volts (radius of circle of secondary voltage vectors OT_1, OT_2, OT_3, OT_4, etc.)

n = number of secondary phases.

From the points E and H set up perpendiculars intersecting in L from D and G perpendiculars intersecting in N. From L as centre, draw an arc of a circle passing through E and H. From M as centre, draw an arc of a circle passing through G and D. From N as centre, draw an arc of a circle passing through F and I. Each of these arcs will span 120°, or one-third of the circumference. Divide each of these arcs into $n/3$ parts (Fig. 176). Find the centre of the circle circumscribed to the triangle. From the centre draw radii to the points obtained, by dividing the three arcs into $n/3$ parts. These radii measure in magnitude and phase the currents in the consecutive sections of the winding.

In order to understand clearly the meaning and application of the diagram, let us apply it to the simple case in which $n = 3$, and the delta connected coils of the primary are simply tapped in the middle.

In this case the formula gives us

$$E_2 = \tfrac{1}{2}E_1$$
$$x = I_1$$

and the current diagram is as in Fig. 175.

In this simple case it is not hard to show that the current diagram is derived from the potential diagram by the application of Kirchhoff's law that, " the sum of the currents flowing towards a tapping point is zero." Let P, Q, R, S, T, U be the tapping points.

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[1] This rule assumes that the phase relation between the primary current and voltage is the same as that of the secondary, and neglects losses and leakage.
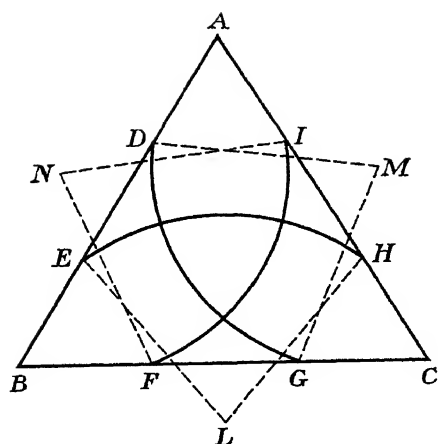
FIG. 174
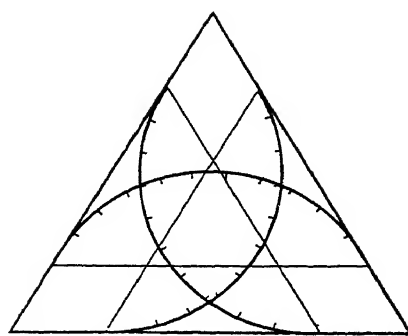
FIG. 176
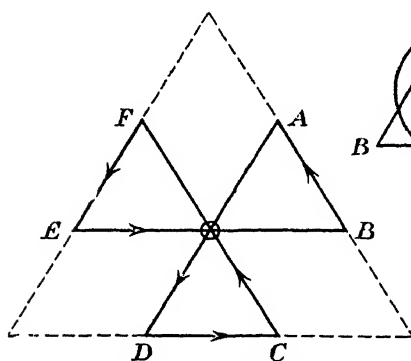
Current Diagram.

FIG. 177

Potential Diagram.
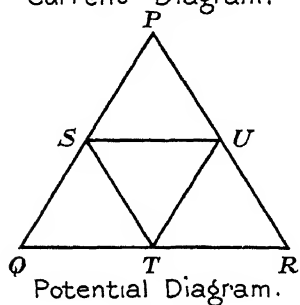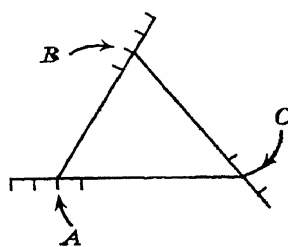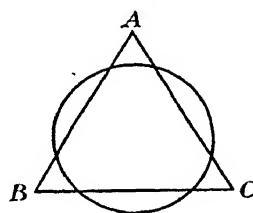
FIG. 175
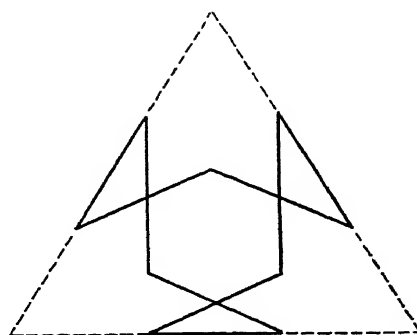
FIG. 178

Let DC be the current inflowing at P
             CF ,,   ,.   outflowing at U
             FE ,,   ,,   inflowing at R
             EB ,,   ,   outflowing at T
             BA ,,   ,,   inflowing at Q
             AD ,,   ,.   outflowing at S

Assume, for instance, a purely wattless load, both primary and secondary currents lagging 90°.

Then OC will be the current in PU
             OF ,,   ,,   ,. UR
             OE ,,   ,,   ,, TR
             OB ,.   .,   ,. QT
             OA ,,   ,.   ,, QS
             OD ,,   ,,   ,, SP

For at P, $DC + CO + OD = 0$
At U, $CF + FO + OC = 0$
   R, $FE + EO + OF = 0$
   T, $EB + BO + OE = 0$
   Q, $BA + AO + OB = 0$
   S, $AO + DO + OA = 0$.

Hence the current diagram satisfies Kirchhoff's law at all the tapping points, and it is clear that no other diagram could do so.

From this simple case a more general rule is now apparent for forming the current diagram, which is now applicable even to unsymmetrical figures.

" Set off the currents flowing in at all tapping points in order, so as to form a closed polygonal figure. Radii from the centre[1] of this figure to its vertices give the current in successive sections."

In forming this polygon we must remember that at the instants in which currents are flowing *into* the primary tappings they are flowing *out of* the secondary tappings.

Similarly, the general rule first given may be applied to the case where $n = 6$, giving Fig. 178. The formula gives $E_2 = \dfrac{E_1}{\sqrt{3}}$ $x = \tfrac{1}{2}I_1$.

As applied to a 3-phase — 30-phase transformation the formula gives $E_2 = \dfrac{E_1}{\sqrt{3}}$ $x = \cdot4651_1$, and results in Fig. 176.

It is clear from the above diagrams, and the general rule given, that the current distribution within such a phase convertor depends *only* on the currents at the tapping points both primary and secondary, and not in any way on the connections of the phase

[1] The " centre " of an unsymmetrical figure is a point such that the vector sum of all radii drawn from it to the figure is zero."

convertor. We can, in fact, draw the diagram complete as soon as we know the primary and secondary currents and their phase relation without any knowledge of the arrangement of the phase convertor. All phase convertors, in which primary and secondary currents flow through a number of taps in windings forming a closed circuit, have the same currents in the sections intermediate between corresponding taps.

It is sometimes, for instance, desirable, in such a phase convertor as that shown in Fig. 177, to feed-in the primary current at points other than that corresponding to the vertices of the potential triangle or to vary the primary feeding in point, to vary the secondary star voltage. This does not in any way alter the current distribution *relative to the feeding-in points*, although, of course, as we alter the feeding-in points the current in any given section will change. It is, therefore, possible to wind the different sections of the transformer with windings of varying cross-section proportional to the maximum currents which they are likely to be called on to carry. These currents may be approximately determined by the above method of calculation. If the coils are proportioned in this manner, those coils nearest the tapping points will be of far heavier cross-section than any of the others, or in the case where the primary tapping points are variable, all those coils which may be near the tapping point in any of its positions.

PLATE VII

Rear view (showing phase convertor) of controller
shown in Plate VI

PLATE VI

Phase convertor (three to eighteen phases)
for machine shown in Plates I, IV, and V

CHAPTER XXIV

THE PRACTICAL DESIGN OF POLE CHANGING APPARATUS

The controller. The only part of the equipment still to be described is the controller needed to effect the changes of connection between the phase transformer and the motor.

Different types of controller are, of course, required for different speed combinations. It is very easy to obtain combinations giving rise to very complicated circuits, but if care is taken to use the speed
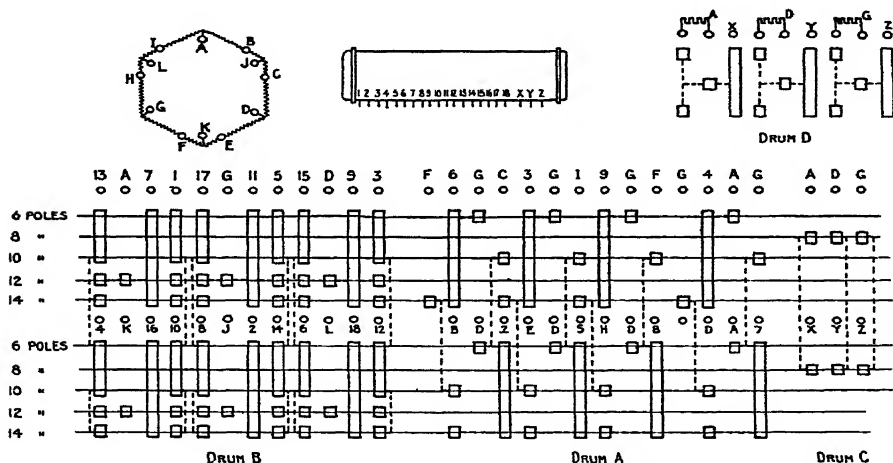
FIG. 179

combinations best adapted to known types of winding and to choose the types of winding best adapted for the speeds required, the following rule for 3-phase circuits may be enunciated—

A 3-phase multi-speed motor, for any number of speeds up to six, will require three times as many terminals as there are speeds.

That is, a multi-speed motor with a single winding on each member may be built for any number of speeds up to six, requiring no more terminals than if each speed were produced by a different winding. This does not mean that any arbitrary selection of numbers of poles can be built with no more terminals than these, but that by suitably choosing the numbers of poles within the range of 3—1, these results may be obtained.

For instance, the winding described above (see Fig. 170) is usually arranged for 6, 10, and 14 poles, and requires 9 terminals, thereby obeying the rule, but a winding arranged for, say, 8, 10, and 14

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poles, would not obey the rule, although windings for 6, 8, 12, or 6, 10, and 12, could be designed to obey it.

There is thus an amply sufficient selection of combinations for practical purposes, although there are a few abnormal combinations which fall outside the rule. Further research is rapidly lessening the number of these combinations.

A series of combinations adjusted with great care so as, on the one hand, to lend themselves to simple control, and on the other to
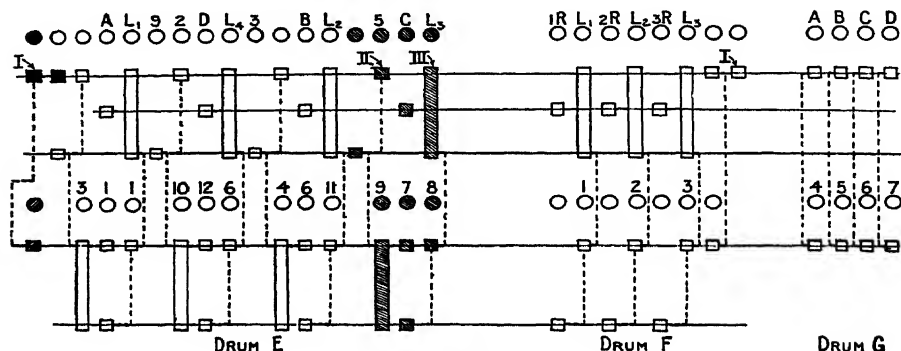
Fig. 180

cover most requirements, is shown in Fig. 181. covering speed ranges on 50 periods as follows—

> 1500/1000/750 . Drum F
> 1000/750/500 : Drum F
> 750/500/375 : Drum F
> 750/600/500 : Drums $F + G$
> 1000/600/428 : Drum A
> 1000/750/600/428 : Drums $A + C$
> 1000/750/600/500/428 : Drums $A + B + C$

It will no doubt be agreed that these combinations cover practically everything that is needed, and the control of all of them is sufficiently simple to be dealt with by a drum controller. Figs. 179 and 180, in fact, show diagrams of the drum controllers which are used to give these combinations, and it will be seen that all, except that for the 5-speed machine, are simple pieces of apparatus requiring only a single drum. The 5-speed machine is usually built with two drums geared together, and forms the only case in which this is necessary, while in Fig. 181 has been assembled a series of speed/torque curves giving one example of each speed combination.

An important point in connection with these machines is, of course, the cost as compared with other means of producing the

same results. In Fig. 182 is given a curve, for different horse-powers, for machines of this type as compared with a slip-ring induction motor with its control gear. This curve ignores entirely the enormous economy of current secured by an apparatus of approximately constant efficiency on all speeds.

These controllers are suitable for small to medium sized motors, up to 200 h.p. The larger sizes, say, from 200 h.p. to 800 h.p., are rather beyond the scope of a drum controller, and, consequently,

FIG. 181

an apparatus of a different type is employed. In Fig. 183 is shown such a large controller, from which it will be seen that the controller consists of a slate panel bearing what are essentially four, nine-bladed 2-way switches with vertical axes. The switches are of the finger type as in drum controllers, and bear on studs mounted in the panel. They can be clearly seen in the lower half of the figure, while on the right and left of these switches are the terminals for the phase transformer and motor, respectively. The switches are opened and closed in the proper sequence by means of cams which appear immediately above the switches. For each speed, two of the switches make contact, thereby connecting the 18 terminals of the motor to the line or the phase transformer, in the different ways appropriate to different speeds.

It is found, by taking advantage of the following fact, that the switching may be simplified. One half of the connections for one number of poles, say 8, with the machine running clockwise, are identical with half the connections for another number of poles, say 10, with the motor running counter-clockwise. It conduces to simplicity of switching, therefore, if we make no change in these

connections in changing from 8 poles to 10 poles, but merely interchange two of the 3-phase lines, by means of the two contactors shown in the upper half of the panel which perform a quadruple function, as detailed below.

1. They serve to reverse the 3-phase connections when this is required, in order to economize switching.

2. They are fitted with overload relays (shown immediately below them), so that they perform the function of a circuit breaker.

3. In changing from one speed to the next in any multi-speed motor, it is necessary to open the circuit which, in most forms of switchgear, gives rise to a certain amount of arcing.

(It is a good practice in all such cases to open the circuit first,
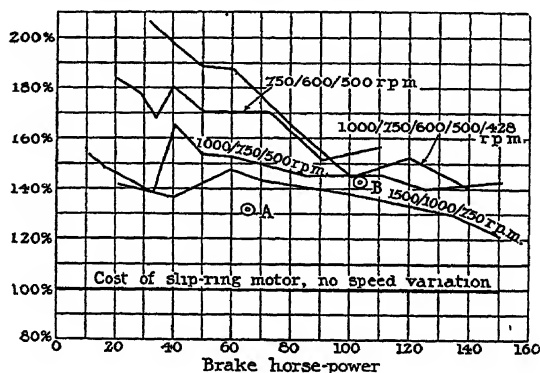
Fig. 182

by a special switch designed with adequate blow-out apparatus, etc., to avoid arcing ; then, while the motor circuits are completely dead, to change the connections, which may, of course, be done without any arcing, because they are not carrying any current ; and, lastly, to close the main circuit again through the switch specially provided for that purpose. The contactors shown in the figure perform this function also.)

4. The small switch on the right serves to open the contactor magnets, and causes them to act as a line switch, isolating the whole of the apparatus from the line as soon as it is opened.

The connections on the back of the board are not unduly complicated, and are carried out by means of bare copper rods about $\frac{5}{16}$ in. diameter, in the standard manner. A mechanical interlock formed by the vertical rods below the contactors is fitted between the contactors and the camshaft by means of an auxiliary shaft geared to the camshaft, in the ratio of 1 to 4.

The operation of this interlock is as follows : On making a slight movement of the controller handle, the contactor magnets are opened. No further movement is possible until both the contactors
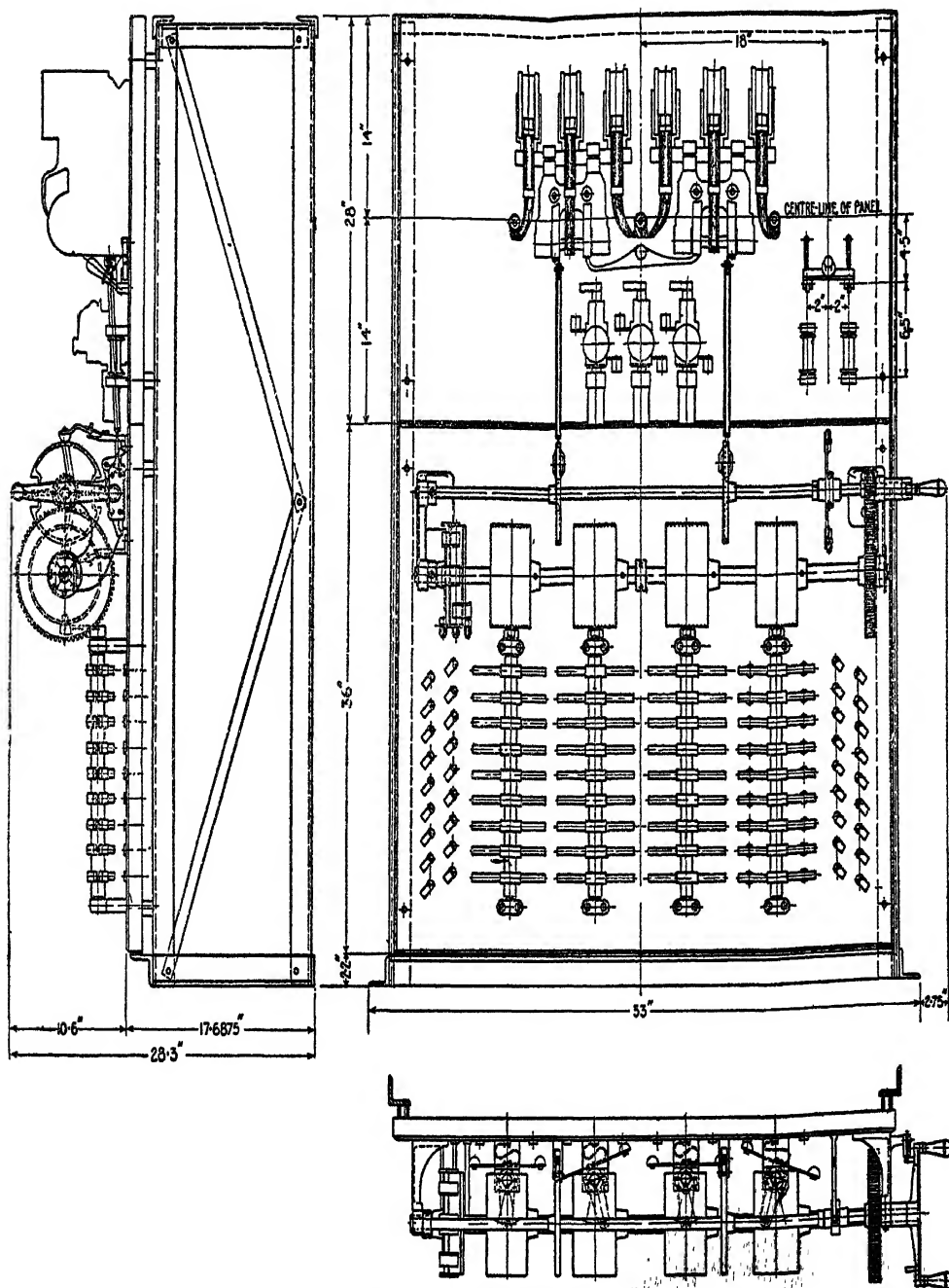
FIG. 183. Arrangement of variable-speed motor control panel

are fully opened. When this has taken place the shaft is freed, and on moving it further the cams begin to move and alter the adjustment of the main switches. When the handle of the controller has moved, say, 180°, then the adjustment has been completed and the magnet circuit of the contactors is re-energized, closing them when the machine continues to operate on the new number of poles. Although these operations take rather a long time to describe, they do not, in practice, take more than one second.

Characteristics of the equipment. We now come to the description of the characteristics of machines of this type which are
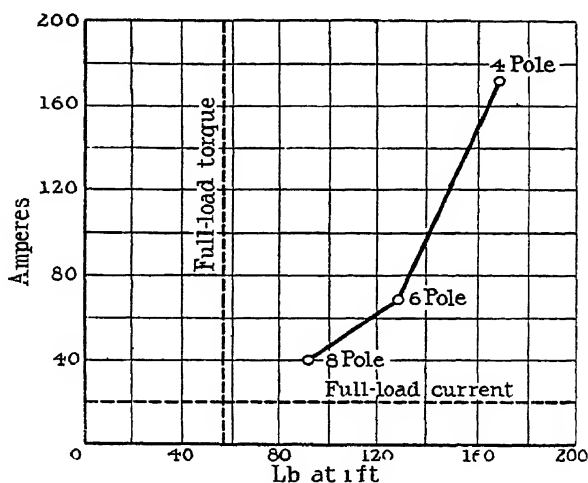
Fig. 184

rendered available by the system of control just described. As has already been pointed out, squirrel-cage motors of the multi-speed type can find a very much greater field of utility than when used on a single speed only, since the chief drawback is almost completely overcome in the multi-speed form. This advantage, of course, relates to the starting

Consider, for instance, a 2-pole and a 4-pole motor, both having the same air-gap density. In the former there will be only two conductors in series, each lying in the middle of the pole face, and the 4-pole motor will have four such conductors. If, therefore, we send the same current through the four conductors as through the two conductors, we shall obtain twice as much torque in the 4-pole machine as in the 2-pole machine, or, alternatively, for the same torque the 4-pole motor will require only half as much current; hence, if we can arrange a motor so that it starts on four poles and runs on two poles, to obtain a given starting torque it will take only half the starting current that it would require if it started on

two poles. This is the origin of the very greatly improved perform-
ance of the squirrel-cage motor when used as a multi-speed machine.
In squirrel-cage motors having a speed range of two to one, or
more, it is always possible to obtain full-load starting torque with
not more than twice full-load current. All machines, of whatever
size, having the same speed range, are capable of this ratio of start-
ing torque to starting current, but if switched straight on to the line
larger machines frequently take much greater current and give
corresponding greater torque. In most cases, therefore, the use of
an auto-transformer is necessary with large machines. This may be
seen from Fig. 184, in which is shown starting torque plotted against
starting current for a 4-, 6-, and 8-pole motor. It will be seen that
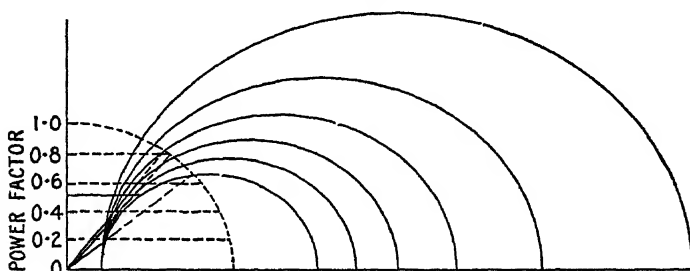
Fig. 185

with the 8-pole connection the machine gives 1·7 times full-load
torque with twice full-load current. With a 6-pole connection it
gives 2·4 times full-load torque with 3·6 times full-load current.
With a 4-pole connection it gives 3 times full-load torque with
8·6 times full-load current.

Another point frequently arising with multi-speed squirrel-cage
motors relates to the current rush on changing speed and the rate
of acceleration from one speed to the next. Both these factors are
under exact control by the use, in series with the motor, of primary
resistance, which, of course, has the effect of reducing the voltage
during the change of speed.

Returning to Fig. 183, in order to make use of this method of
regulating the rate of acceleration between speeds, there is fitted
on the panel by the side of those shown a third contactor which
opens when they open, and which closes, due to the action of a
dashpot, a few seconds after they close. This third contactor, when
closed, bridges three resistances each placed in series with one of
the lines ; hence, directly the contactor opens, these resistances
are placed in circuit, being adjusted to produce any desired rate of
acceleration. They remain in series with the line during the opera-
tion of changing speeds, and are cut out when this operation has
been completed.

In Fig. 196 are shown the current surges produced in changing speed in a squirrel-cage motor arranged for all numbers of poles between 8 and 16, both with and without (dotted curves) a similar device, substituting an auto-transformer for the resistance. It will be seen that by its use the maximum current surge is reduced to about one-half.

In a motor having a wide speed range, the lowest speed steps are very small, and there is seldom any need for such a device. It is on high speeds, however, that it is needed. The figure illustrates the marked reduction in the current surge on these higher speeds
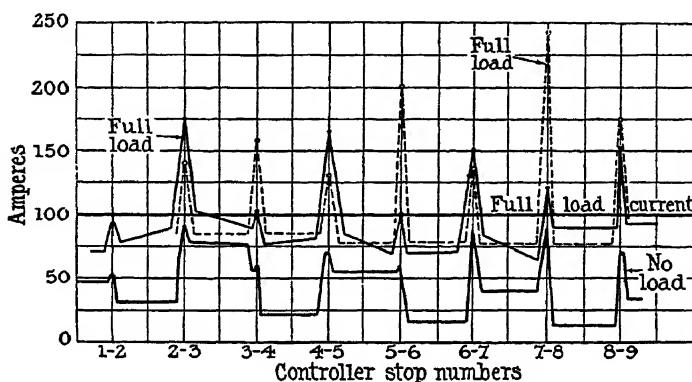
FIG. 186

(shown towards the right of the figure). the device, of course, being adjusted to suit this case.

This method of starting and changing speed can be employed in motors up to several hundred horse-power, and renders the squirrel-cage motor adaptable to a vast variety of purposes for which it has never been found possible to use it before.

To sum up. the sequence of operations to change speed, in the case of a squirrel-cage motor, for instance, would be as follows—

1. The first movement of the controller handle causes the contactor to open, thereby cutting off the supply from the installation and rendering it " dead."

2. A further movement of the controller handle opens the " switch units," and recloses them in the correct combination to suit the next speed, and. as the supply is cut off meanwhile, the change takes place without any sparking.

3 When the controller handle has been moved to the notch corresponding to the speed desired, the contactor closes again and the circuit is re-established. the whole operation of changing speed taking approximately one second and being accomplished by one movement of the controller handle. In Fig. 185 is shown a series

of circle diagrams for such a multi-speed motor operating on all numbers of poles from 6 to 16, and absorbing the same magnetizing current in all cases. These diagrams assume that the leakage coefficient is proportional to the number of poles, thus being twice as great in a 12-pole machine as in a 6-pole one. It will be shown below that this assumption is very nearly fulfilled in this type of machine.

This figure shows clearly the cause of the very great economy in starting current which is found in practice and was illustrated in Fig. 16. It will be seen that with 16 poles the motor takes about one-third of the starting current required with a 6-pole connection. Since the motor gives the same torque at all speeds, the ordinates
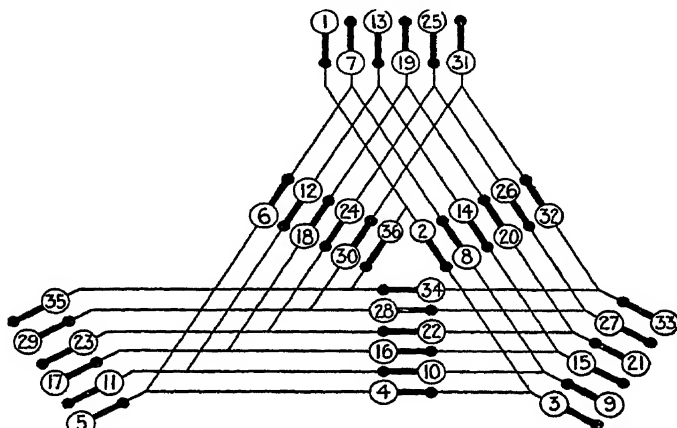
FIG. 187

to each circle represent the torque to a scale such that the torque represented by 1 mm. on the diagram is proportional to the number of poles. Hence, in spite of the small current consumption, the motor can yield a powerful starting torque at the lowest speed.

Slip-ring motors. It is possible to design a rotor winding for a machine having any number of speeds which shall give genuine slip-ring control up to one of these speeds, for instance, the top speed, and operate as a short-circuited secondary winding on all other speeds. One convenient way of effecting this result is by using such a winding as is shown in Fig. 187, but permanently connected so as to be capable of direct connection to the line. It will be seen that, as so connected, the winding requires no more than three terminals and can, therefore, be connected to the usual number of slip-rings.

On all other numbers of poles than six, however, the winding would require more than three terminals to enable it to be connected to any outside circuit, and if these terminals are short-circuited the winding is also internally short-circuited. Hence the

secondary currents which on 6-pole operation flow through the slip-rings, on any other number of poles flow through the internal circuits of the winding and do not pass through the slip-rings at all.

By means of this construction a motor is produced which can be arranged to be accelerated, by means of an ordinary resistance connected across its three slip-rings, from standstill to any one of its six speeds, starting, if desired, with from 2 to $2\frac{1}{2}$ times full-load torque. On reaching the speed at which it is desired to run, a movement of the controller handle changes the number of poles of the
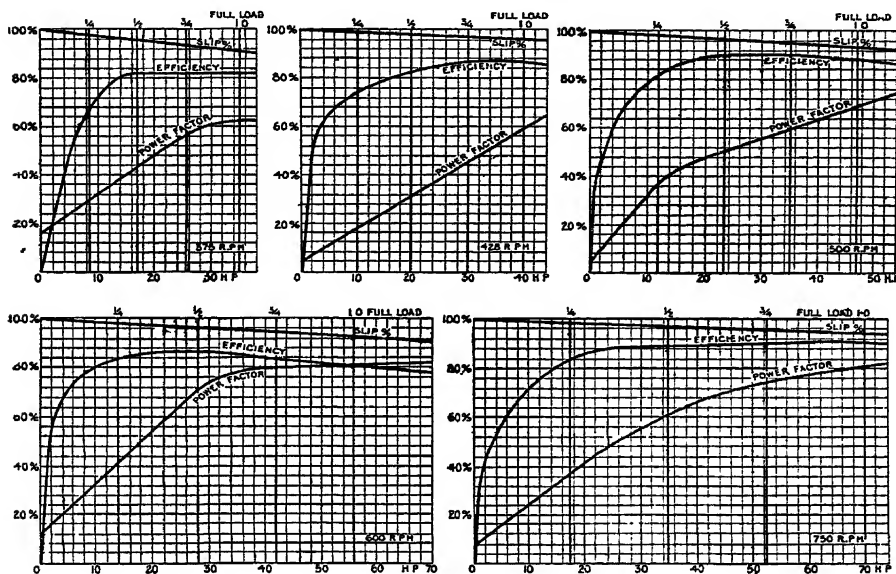
Fig 188

Performance curves with high resistance rotor

motor to the desired value, when the currents in the rotor winding are automatically diverted from the slip-rings and continue to flow in closed circuits in the winding.

The operation of starting the slip-ring motor on any other than the top speed is, therefore, as follows—

1. Set the controller to the starting position which will produce in the motor a number of poles corresponding to the top speed.

2. Gradually close the starting rheostat, when the motor will speed up.

3. When the motor has reached the speed desired, which will, of course, be less than top speed, move the controller from the starting position to the running position corresponding to the speed at which it is running. This will have the effect of changing the number of poles to that corresponding to the controller notch employed, when

the starter is automatically cut out as already described and the motor continues to run with its rotor short-circuited.

4. The starter may now be open-circuited at leisure by the operator.

A tachometer may be provided in order to enable the operator to know the proper moment to change from starting to running position.

In Figs. 188 and 188A are shown some curves of efficiency and power
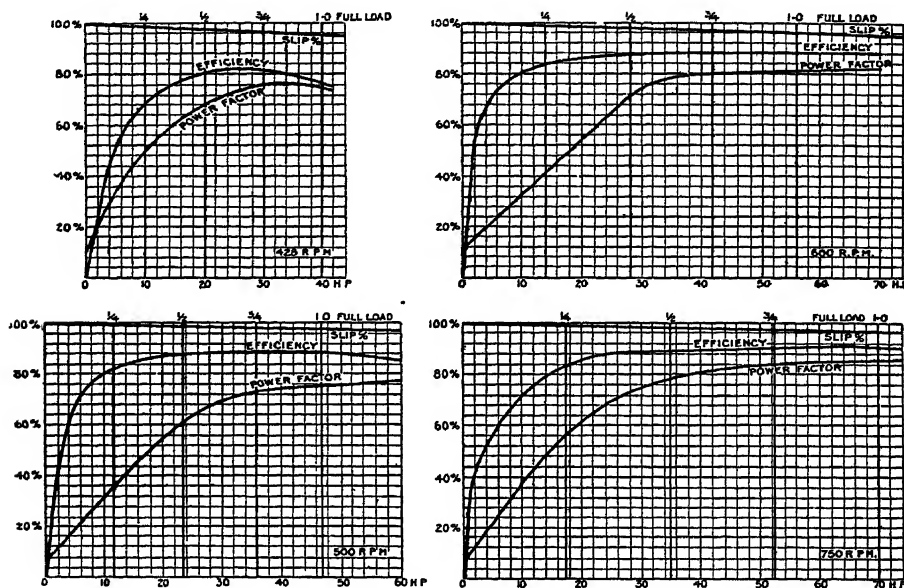
FIG. 188A.

Performance curves with normal low resistance rotor

factor of a 70 h.p. motor arranged to give all numbers of poles from 8 to 16.

Fig. 188 shows curves corresponding to the results obtained when fitted with a high resistance rotor adapted to give the powerful torque required in connection with this motor.

Fig. 188A shows corresponding curves for the same machine when fitted with low resistance rotor.

Comparing these two sets of curves, a result will be noticed which at first appears rather strange, namely, that the efficiencies of the machine with a high resistance rotor and with a low resistance rotor are not very different for the same speed, while the power factor of the machine with high resistance rotor is a good deal less than that of the machine when fitted with low resistance rotor.

This is due to a method which can be employed in the induction motor to enable a high resistance rotor to be employed without loss of overall efficiency. The torque of the motor is, of course, proportional to the product of ampere-conductors by flux density. If, therefore, we compare two motors having, say, flux densities in the ratio of 1·5 to 1, then in order to give the same torque they will require a number of rotor ampere-conductors in the ratio of 1 to 1·5, the machine having the stronger flux density requiring only two-thirds of the current in the rotor bars required by that having the smaller flux density.. Two-thirds of the rotor current means, of course, four-ninths, or less than half, of the rotor I^2R loss and, consequently, if we desire the resistance loss to be the same in both cases, the machine with the stronger flux density may have $2\frac{1}{4}$ times the rotor resistance of that with the weaker flux density.

This method was adopted in the motor illustrated in the above figures, which by means of tappings on the transformer was adapted to operate with either a strong or a weak flux. The strong flux used with the high resistance rotor involves, of course, low power factor. and this is the explanation of the nature of the curves shown. By using stalloy steel punchings the iron loss was reduced to such a small proportion of the whole that the increased iron loss due to the stronger flux produced only a negligible effect on the efficiency.

Thus, by this process the squirrel-cage motor can be adapted to give any desired starting torque, without loss of efficiency, wherever we are prepared to sacrifice power factor to obtain this result, or to use additional power factor compensation devices such as condensers.

Power factor and efficiency. Referring to the curves given above, it may be seen that the characteristics of the machine as regards power factor and efficiency are, broadly speaking, as follows—

The efficiency remains very nearly the same at all speeds, though there is a slight reduction at reduced speeds, due, broadly, to the fact that the losses remain approximately constant at all speeds, while the output, of course, falls off in proportion to the speed. In the larger sizes, this reduction in efficiency will seldom exceed 5 per cent. The efficiency at top speed is, of course, practically the same as in a standard motor. The power factor also varies, as we change speed, to a somewhat greater degree than the efficiency, this variation being due, broadly speaking again, to the fact that the magnetizing current remains the same at all speeds, while the true power input falls off proportionately to the reduction of speed. Again, in the larger sizes this variation will seldom exceed 10 per cent.

The only thing which needs to be borne in mind from the power supply point of view is that these machines absorb a constant amount of wattless current at all speeds, and that this amount is

not greater than that absorbed by a standard machine having the same rating. Perhaps it may serve to make the matter clearer if we compare the equipment with a standard motor driving through a gear-box, a load which requires constant torque at all speeds. At top speed the motor is fully loaded and operates with its best power
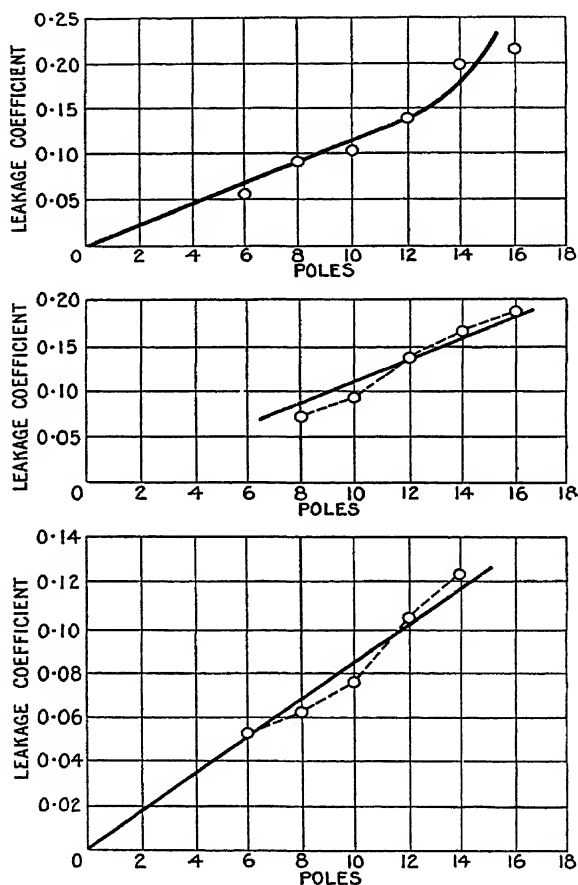
FIG. 189

factor and efficiency. As the speed and the load are reduced by changing gear, the output of the motor falls off and with it the power factor and efficiency to some extent. To almost exactly the same extent do the power factor and efficiency of the equipment described fall off when the load is reduced ; in fact, the controller described above may be regarded as an electrical method of changing gear.

Reference has been made above to the fact that the leakage

coefficient of these machines is practically proportional to the number of poles, and in Fig. 189 are shown some curves of leakage coefficients plotted against the number of poles. These curves illustrate this feature. Such curves, it is believed, will be of considerable interest to designers, and clearly could not be obtained from any other type of motor except one retaining an absolutely identical winding on all numbers of poles. Comparisons have frequently been made between motors having different numbers of poles, but this, of course, is not at all the same thing as in the case of a single multi-speed motor.

PLATE IX

Rear view of controller shown in Plate VI
(showing wiring)

PLATE VIII

Front view of controller for six-speed, three-phase
motor shown in Plates I, IV, and V

CHAPTER XXV

SINGLE-PHASE POLE CHANGING APPARATUS

IN order to design a single-phase winding adapted to any set of two pole numbers, it is necessary first of all to pay attention to a few simple principles.

1. Clearly, if there is a common factor between the two numbers of pole pairs, the circumference of the machine will be divided into identical zones equal in number to the common factor. Hence, since these are identical, we need only consider those cases in which no common factor exists.

2 The minimum number of slots adapted for a single-phase winding, arranged to give two different pole numbers, should be the product of these two pole numbers, though in some cases a multiple of the product of the numbers of pole pairs will be satisfactory, for instance, a winding to give 4 and 6 poles, should have a multiple of 24 slots, and one to give 10 and 6 poles should have a multiple of 15 slots.

Before describing further the principles on which these single-phase windings for two pole numbers are based, it will be convenient to describe in detail one or two simple cases. For instance, take a winding which is to be arranged so that it can operate on either 10 or 6 poles. Assume it wound into 30 slots only, as shown in Fig. 195, for the sake of simplicity. Tabulate, side by side, the direction of the currents in the different slots, both for 10 and 6 poles.

Slot No	10 Poles	6 Poles	Slot No	10 Poles	6 Poles
1	Down	Down	16	Up	Up
2	,,	,,	17	,,	,,
3	,,	,,	18	,,	,,
4	Up	,,	19	Down	,,
5	,,	,,	20	,,	,,
6	,,	Up	21	,,	Down
7	Down	,,	22	Up	,,
8	,,	,,	23	,,	,,
9	,,	,,	24	,,	,,
10	Up	,,	25	Down	,,
11	,,	Down	26	,,	Up
12	,,	,,	27	,,	,,
13	Down	,,	28	Up	,,
14	,,	,,	29	,,	,,
15	,,	,,	30	,,	,,

In Fig. 190 the crosses may be taken to indicate the currents flowing down, and the dots those flowing up. It will be seen that in slots 1, 2, and 3 the current flows down on both 6 and 10 poles, while in slots 4 and 5 it flows up on 10 poles and down on 6 poles. In slot 6 it flows up on 6 and 10 poles, and in slots 7, 8, and 9 it flows up on 6 poles and down on 10 poles. In slot 10 it flows up both on 6 and 10 poles, while in slots 11 and 12 it flows up on 10 poles and down on 6 poles. In slots 13, 14, and 15 it flows down both on 10 and 6 poles. In slots 16 to 30 the current is the same as in slots 1 to 15, but reversed in sign.

Considering first slots 1, 2, 3, 6, 10, and 13, 14, 15, in which the current flows the same way on both pole numbers. Corresponding to these slots are slots 16, 17, 18, 21, 25, 28, 29, 30, in which the current distribution is the same but reversed. Hence it is possible to arrange a winding to fill these slots only. This is shown in Fig. 190 with terminals S_1—S_2.

Now consider slots 4, 5, 7, 8, 9, 11, 12, 19, 20, 22, 23, 24, 26, 27. Again we see that the number of slots in which the current flows up is the same as those in which it flows down.

Hence a second winding can be arranged to fill these slots alone, shown in Fig. 190 with terminals $R1$, $R2$. If this winding is connected in series with the first in one direction, we shall get as a result a 10-pole winding, and if it is connected in the opposite direction we shall get a 6-pole winding.

Hence we shall thus arrive at a winding capable of use as either a 10-pole or 6-pole single-phase winding consisting of two separate windings in series, the change of pole number being effected by reversing one winding with respect to the other.

Now many of the properties of the winding just pointed out are generally true. For instance that, in each of these component windings, there must be as many slots in which the current flows UP as there are slots in which it flows DOWN, follows from the fact that the change pole winding, as a whole, is identical in connection 1, with a winding for R poles say, and in connection 2 with a winding for S poles say. In either the R pole or the S pole winding it is, of course, true that there are as many slots in which the current flows UP as there are those in which it flows DOWN. In changing, therefore, say from the S pole to the R pole condition, we must reverse as many wires in which the current flows UP as wires in which it flows DOWN, or the previous statement could not be true for both pole numbers. Hence it must be true also for each component of the change pole winding.

We may distinguish two different cases among numbers of pole pairs having no common factor.

1. Cases in which both numbers of pole pairs are odd.
2. Cases in which one is even and the other is odd.

Non-
reversing

Reversing

a
b
c
d

e

S_1

R_2

R_1

S_2

a
b
c
d

e

Fig. 190

Non-reversing

Reversing
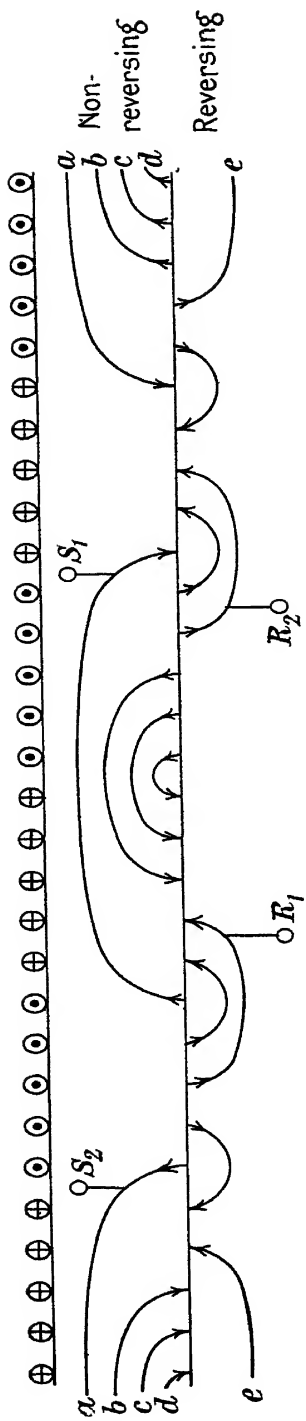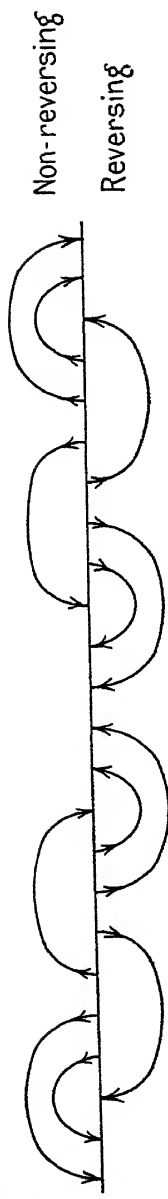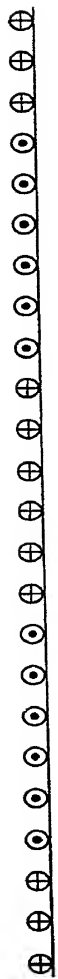
Fig. 191

Since it is always possible to divide both numbers of poles by two, the circumference may always be divided into two zones, each covering one half.

Considering first the case in which both numbers of·pole pairs are odd, which we may call the symmetrical case, the distribution of both the mutually reversible component windings will be the same in these two zones, but opposite in sign. That is to say, considering any slot, no matter which of the two mutually reversible windings it carries, then a diametrically opposite slot will carry a current opposite in direction. Thus a winding of this character may be conveniently carried out by means of coils of diametric pitch.

There is another class of winding which may be called the skew symmetrical class, in which the numbers of pole pairs are odd and even respectively—an example of this is a 4- and 6-pole winding, as shown in Fig. 191. Such a winding may be wound into 24 slots.

Slot No.	4 Poles	6 Poles	Slot No.	4 Poles	6 Poles
1	Up	Up	13	Up	Down
2	,,	,,	14	,,	,,
3	,,	Down	15	,,	Up
4	Down	,,	16	Down	,,
5	,,	,,	17	,,	,,
6	,,	,,	18	,,	,,
7	,,	Up	19	,,	Down
8	,,	,,	20	,,	,,
9	,,	,,	21	,,	,,
10	Up	,,	22	Up	,,
11	,,	Down	23	,,	Up
12	,,	,,	24	,,	,,

In Fig. 191, as before, the dots may be taken to indicate current flowing UP, and the crosses, currents flowing DOWN. Comparing the direction of the currents in these tables, we see that in slots 1, 2, 10, 15, 23, and 24 the current flows UP both on 4 and 6 poles, while in slots 4, 5, 6, 19, 20, and 21 it flows DOWN on both 4 and 6 poles. In slots 7, 8, 9, 16, 17, and 18 it flows UP on 6 poles and DOWN on 4 poles, while in slots 3, 11, 12, 13, 14, and 22 it flows DOWN on 6 poles and UP on 4 poles. Hence, in this case also, we may join slots 1, 2, 4, 5, 6, 10, 15, 19, 20, 21, 23, and 24 into one winding, shown in Fig. 191 as " non-reversing," there being as many slots in which the current flows up as there are slots in which it flows down, and the remaining slots into another winding shown in Fig. 191 as " reversing," the latter being reversed with respect to the former in the change from 4 to 6 poles.

The distribution of these two windings around the circumference is identical, and either winding may be produced from the other by displacing it bodily through half of the circumference. In such

a skew symmetrical winding it is not true that a slot, diametrically opposite to a given slot, belongs to the same winding, and carries an opposite current. On the contrary, there is an axis of symmetry which, in the case of the winding described, passes midway between slots 12 and 13 and slots 24 and 1. If we consider any slot lying at a certain angular distance from the axis in a clockwise direction, and another slot lying at an equal angular distance from this axis in the counter-clockwise direction, it will be found that both these slots belong to the same winding.

Thus we have definitely shown above in a perfectly general manner that each component winding contains as many conductors in which the current flows UP as it does conductors in which it flows DOWN.

It follows, that for any two pole numbers whatsoever, a single-phase winding can be built consisting of only two parts, and capable of changing from one pole number to the other by reversing one of the parts with respect to the other.

Finally, the precise construction of such a winding in a typical case may be described.

Consider a winding arranged in 24 slots so as to be adaptable either for 4 or 6 poles, as shown in Fig. 191. It will consist of 8 coils, four in each mutually reversible component of the winding. Coil 1 will lie in slots 1, 2, 4, and 5 ; coil 2 in slots 6 and 10 , coil 3 in slots 15 and 19 ; and coil 4 in slots 20, 21, 23, and 24.

All these coils are connected in series, and form one of the mutually reversible components of the winding. The second component of the winding is identical in distribution with the first. The description already given will, therefore, apply to it if we add 12 to each of the slot numbers just referred to. It will be seen that corresponding conductors of the two component windings lie at opposite ends of the diameter of the machine, the conductor corresponding to that in slot 1 being that in slot 13.

Another method of securing reversal of one of these windings with respect to the other is the following—

Since in the skew symmetrical type of winding the portions of any winding, for instance, those whose terminals are $S1$, $S2$ lying on one side of the axis of symmetry, are exactly similar to those portions of the same winding lying on the other side of the axis of symmetry, it is possible to connect them in parallel as well as in series.

Precisely the same remark applies to the winding whose terminals are $R1$, $R2$. Owing to this possibility we may connect the two windings in a closed circuit. This closed circuit has four arms, there being a terminal at the extremity of each of these arms. These may be lettered P, Q, R, S.

Branch PQ, for instance, may contain the coils A and B, and the branch SR may contain the coils C and D in parallel with A and B.

Let now the branch QR contain the coils E and F, and PS the coils G and H, then, according as we switch the line on to the terminals QS or PR, the current in coils E, F, G, H will be reversed with respect to A, B, C, and D therefore.

The windings so far described are essentially of the single-phase type, but in certain cases, by making use of two or more such windings wound in the same or alternate slots, a polyphase winding may be produced. We shall confine ourselves to the case of two windings giving rise to a resultant winding adapted for two phases in quadrature. If two such windings are to be used on two distinct pole numbers, they must be in phase quadrature with one another on both pole numbers, and not on one only.

The windings of both phases will be identical in arrangement and in connection, but corresponding coils will be displaced from one another by a fraction of the circumference, say $1/n$th. If the two windings are to be in quadrature on two numbers of pole pairs, R and S say, the following relation must subsist, namely,

$$\frac{360° \times R}{n} = \frac{360° \times S}{n} = k \times 180°$$

k being an arbitrary integer. This relation cannot always be satisfied, but for the case where R and S are both odd, for instance, say $R = 3$ and $S = 5$, it can be satisfied by taking $n = 4$, that is, if the two windings are displaced by a quarter of the circumference. The type of winding in which both the pole pair numbers are odd, is that called above the symmetrical type.

Hence it follows that two windings of the symmetrical type displaced by one-quarter of the circumference form a 2-phase winding, and can be wound to give any pair of pole numbers whatsoever which are both odd.

PART V

CHAPTER XXVI

PHASE ADVANCERS AND POLYPHASE COMMUTATOR GENERATORS

In order to understand clearly the essential principles of the various types of phase advancer, it will be necessary to recapitulate what was said in Chapter X with regard to the general law of magnetization. It was pointed out that the density of magnetic energy in the air-gap of an electrical machine at any point where there is a flux density B is equal to $\dfrac{B^2}{2\mu}$. μ being the permeance of the gap area per square centimetre. The rate of change of this energy density is the magnetizing power. In Fig. 192 is shown the stator and rotor of an induction motor diagrammatically indicated in the same manner as in Chapter X.

Consider the tooth D, intermediate between any two bars. The magnetic density in this area is. say. $B = B_0 \sin pt$, and, hence, the magnetic energy density $W = \dfrac{B_0^2}{2\mu}\sin^2 pt$. The flow of magnetizing power due to the flux in this tooth is $\dfrac{dW}{dt} = \dfrac{B_0 p}{\mu} \sin pt$
$\cos pt = \dfrac{B_0 p}{2\mu} \sin^2 pt.$

This power, therefore, alternates, being an inflow when the flux is rising, and an outflow when it is falling. Can we provide a storage device which will store up this energy when it flows out of the machine, and supply it once more when it flows in ? This can be done in a large number of ways. Kapp has worked out in the Kapp vibrator, a device whereby it is done by storing the magnetic energy as the energy of rotating masses. He provided a small direct-current machine whose armature A is shown connected in shunt across adjacent bars, there being as many separate machines with separately excited fields F as there are teeth in our elementary case.

The rule governing the operation of the Kapp vibrator is : Magnetic energy + kinetic energy of revolving armature = constant. Thus, when the magnetic energy is a maximum, the kinetic energy will be zero, that is, the armature of the vibrator will be stationary. When the magnetic energy is zero, the kinetic energy of the vibrator will be maximum. This rule enables us readily to calculate the maximum angular speed of the armature. If its

moment of inertia is I, and its angular speed S, then its kinetic energy will be $\frac{1}{2}IS^2$. Hence, $\frac{B_o{}^2}{2\mu} = \frac{1}{2}IS_0{}^2$, if S_0 is the maximum angular speed of the armature. In practice the number of different independent machines in a Kapp vibrator is reduced to three, since it is found that 3-phase windings are sufficient to give good phase compensation.

If it is clear that the Kapp vibrator is a device for storing the magnetic energy liberated from the dynamo-electric machine, at the instant when the flux is decreasing, and delivering it back at the instant when the flux is increasing, the next thing that requires investigation is how this takes place. When the flux is a maximum also in any given tooth, the current flowing round that tooth must be a maximum, and the speed of the vibrator, as we have seen, is zero.
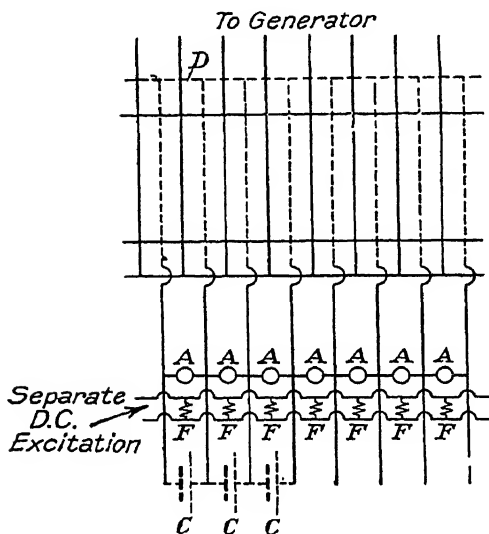
FIG. 192

When the flux in the tooth is zero, the speed of the phase advancer is a maximum and, hence, its voltage is a maximum, while the current required to produce the zero magnetic flux is also zero. In both cases, the magnetizing power which is the product of current $\times$ E.M.F. is zero.

The magnetizing power reaches a maximum when the rate of change of magnetic and kinetic energies is equal. The effect of a current in the armature of the Kapp vibrator, as in any other direct-current machine, is to produce a torque which, as the machine is on no-load, gives rise to an acceleration. At top speed, since the machine is no longer being accelerated, the current must be zero, although the E.M.F. is a maximum. The maximum inflow of current and, therefore, maximum torque of the vibrator and maximum rate of acceleration, occur when the speed and E.M.F. are zero. The most obvious form of phase advancer, of course, is an electrostatic condenser, and in this, as in the vibrator, the current is a maximum when the E.M.F. is zero. In both of these the energy stored is zero when the current is a maximum and the E.M.F. is zero. In the case of a magnetic flux, however, the E.M.F. is zero when the flux and the current are a maximum, and the energy stored, therefore, is a maximum.

To sum up, in the vibrator or condenser, the current is a maximum when the E.M.F. is zero and the energy stored is zero. In the case of a magnetic flux, the exciting current is a maximum when the E.M.F. is zero and the energy stored a maximum. Hence, by combining a magnetic flux on the one hand with a vibrator or condenser on the other, we are able to construct an apparatus such that one portion of it is able to store the energy flowing out of the other portion at all parts of the cycle, and so permit the maintenance of an oscillation.

It is of vital importance to notice that neither the electrostatic condenser nor the Kapp vibrator has any power of generating steady power, and, in fact, must consume it. Consider an induction motor, as shown in Fig. 192, fitted, say, with a Kapp vibrator, and running exactly at synchronous speed, and on no-load. The secondary can receive no induction from the primary owing to the exactly synchronous speed. The Kapp vibrator is incapable of generating the power necessary to drive a current against the resistance of a circuit and, hence, the apparatus cannot work at all. Clearly, when the induction machine is running at synchronous speed and with zero slip, the current required to magnetize it would be continuous. If, therefore, the several machines of the Kapp vibrator could be driven externally at appropriate fixed speeds so as to supply continuous currents to the winding. these currents would be effective in magnetizing the machine, but due solely to the fact that no means of producing from the vibrator an output of steady power exists, it cannot operate under these conditions.

Perhaps it should be pointed out once more that on no-load the only currents in the induction machine are those necessary to magnetize it. If by any means these magnetizing currents could be made to flow in the rotor the stator current will disappear, and we shall get a high power factor on the primary sides. If, however, they cannot be made to flow in the rotor, they must be derived from the stator, and we get the reduced power factor of the ordinary machine.

If, however, the rotor is running below synchronism, with a small amount of slip, then the mean power required to drive the current through the resistance of the secondary circuit can be derived by induction from the stator, and the Kapp vibrator can supply the alternating or magnetizing power, and thus become effective.

Hence, the Kapp vibrator cannot operate on no-load or with very small slips. This applies to the condenser for the same reason, but, of course, the condenser is practically ruled out by the enormous size and cost which would be required for use on slip frequency.

Wall's phase advancer. Another type of apparatus, which has been suggested by Dr. T. F. Wall for use as a phase advancer, is

the secondary cell. A secondary cell is also a means of storing energy, and he has found that it is able to charge and discharge itself following the slow alternations corresponding to slip frequency of one or two cycles per second, if connected as shown at *c, c. c* (Fig. 192). In the secondary cell, as in the condenser, the charging current is a maximum when the E.M.F. is low and the energy stored low, and it becomes less when the E.M.F. is high, and the energy stored high. Hence it is able, to a certain extent, to replace the
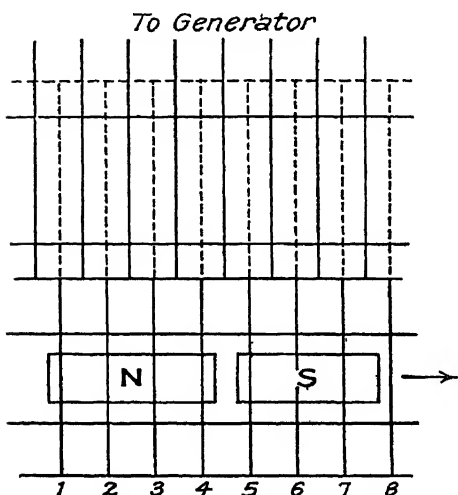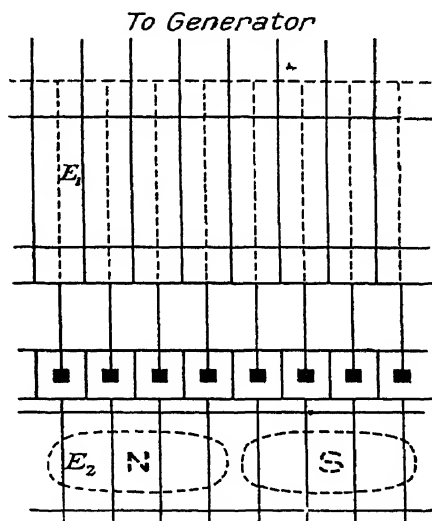
FIG. 193   FIG. 194

condenser or vibrator, and it will be clear from the explanation given of the Kapp vibrator how this can be possible.

It would be perfectly possible to obtain the advantages of the phase advancer by simply connecting the secondary winding of the induction machine to a very low frequency synchronous motor, capable of operating, say, on one or two cycles per second. This is shown in Fig. 193. The principle of its operation would be exactly the same as that of the synchronous motor described in connection with Fig. 49, namely, that the E.M.F. generated in any bar by the flux of the induction machine should be equal and opposite to that generated by the flux of the synchronous motor. By over-exciting the synchronous motor, thereby turning it into what is known as a synchronous condenser, it can operate perfectly well as a phase advancer.

The principle of magnetization, of course, would be that described in connection with Fig. 50, namely, that magnetizing power flows out of the induction machine into the synchronous exciter at those

points in space where the flux density is falling, and flows into the machine from the synchronous exciter at those points where the flux of the induction machine is rising, that is, out of the induction machine along conductors 1, 2, 5, and 6, and in along the remaining conductors.

The Scherbius machine. Another objection to such an arrangement would be the enormous size and cost of a synchronous machine to operate at these extremely low frequencies. The reason of this enormous size and cost, of course, is that if a synchronous machine is to run at an extremely low frequency the speed of rotation itself must be very low. This is overcome in practice by making use of a commutator machine, as shown in Fig. 194, usually known as a Scherbius type of phase advancer. By introducing a commutator we render the speed of rotation of the machine independent of the frequency of the current in it, and, hence, render it possible to build the machine of an economical size. The principle of magnetization does not alter at all. It still remains true that we must have $E_1 + E_2 = 0$, and that power must flow out of the induction machine at points where the flux in it is falling and into it at points where the flux is rising.

The Scherbius phase advancer differs from almost every other type of electrical apparatus, in that it does not require a stator and rotor both carrying different windings. On the contrary, it merely consists of an armature with a commutator carrying an appropriate number of brushes and driven, for instance, by belting from the main machine at any appropriate speed.

The currents flowing from the induction machine set up a field in this armature which is exactly equal and opposite to that of the induction machine itself, in the case illustrated, provided that the speed of the flux relative to the bars of the phase advancer is the same as its speed relative to the bars of the induction machine.

This may be seen more clearly as follows—

Suppose, for instance, that the main induction machine has a synchronous speed of 1,000 revs. per min., and is running at 950 revs. per min. with 5 per cent slip. The speed of the flux relative to the bars of the induction machine will be 50 revs. per min.

Now imagine the phase advancer as stationary. The current flowing through the conductors sets up a certain field which rotates with respect to them with a speed of 50 revs. per min. If we suppose that the flux of the phase advancer is equal to that of the induction machine, then, with the machine stationary, we shall have $E_1 = E_2$, and, consequently, we cannot have $E_1 + E_2 = 0$. Now, owing to the action of the commutator, the distribution of the field of the phase advancer does not depend in any way on its rate of rotation, since, when it is set in rotation, every time a bar moves away from a given brush another bar takes its place. Consequently, the

distribution of the phase advancer flux and its rate of rotation in space depend in no way on the speed of the phase advancer. Now let the phase advancer be set in motion and run up to, say, 50 revs. per min. in the direction of rotation of the flux. It is now running in synchronism with its own flux and, consequently, there will be no E.M.F. induced in its bars.

Let us next suppose that it is run up still further to 100 revs. per min. The phase advancer flux is still unchanged, but the bars are now running at twice synchronism with respect to it, and, hence, the voltage induced in them is exactly reversed as compared with its value at standstill. It is now possible for the equation $E_1 + E_2 = 0$ to be satisfied, since the bars of the phase advancer are moving relatively to its flux in the opposite direction to the bars of the induction machine, relative to the flux in that machine.

A further increase in the speed of the phase advancer above twice synchronism merely serves to reduce its flux.

The speed may, of course, be made many times greater, perhaps ten or twenty times, and in such a case the flux of the phase advancer need only be one-tenth or one-twentieth of that of the main machine. No matter what the speed, the frequency of the currents flowing through the brushes will always remain the same, owing to the presence of the commutator. In practice, of course, complexities are introduced owing to the requirements of commutation, which lead to the use of commutating poles, etc.

Before proceeding to discuss the type of phase advancer which is capable of producing an improvement in power factor at no load as well as at full load, it will be desirable to discuss the polyphase commutator generator in a general way, since this type of phase advancer is simply a special application of the generator.

Consider first, therefore, a polyphase series machine constructed in the manner as shown in Fig. 195 (opposite). In this figure the rotor is shown as consisting of a number of bars joined together at one end by means of an end-ring, each bar being attached to a commutator segment, these being equal in number to the bars. On each segment rests a brush, and to each brush a stator conductor is connected in such a manner that the current flowing in one direction along the rotor bar flows back in an exactly opposite direction along the stator bar, as shown in Fig. 203. Thus, neglecting leakage, the stator and rotor bars connected to a given brush form a non-inductive circuit.

The current having passed through these two bars now returns to the commutator end of the machine by means of a third bar, which is known as the field bar. This may be displaced by any amount from the stator bar just described, which is known as the neutralizing bar. From the field bar the current is taken to the load which is shown in Fig. 195 as a self-induction.

Since the rotor bar and neutralizing bar cancel each other's magnetizing effect, the field is clearly due entirely to the distribution of field bars, and may be displaced in position round the circumference of the machine to any extent, by varying the position of these field bars, that is, by lengthening or shortening the connections *aa*, Fig. 195.

Let B be the flux density at any point in the machine, the flux being, let us suppose, a rotating one revolving with respect to the stator with peripheral speed v_2, and with respect to the rotor with peripheral speed $v_3 = v_2 - v_1$, where v_1 is the peripheral speed of rotation.

Thus, since the rotor bar and neutralizing bar are connected in opposition, the voltage induced in the two considered as a single whole is $(v_2 - v_3) B = v_1 B$. At standstill, when $v_2 = v_3$, clearly this voltage is zero. The voltage induced in the field bar owing to its displacement in space from the rotor and neutralizing bars will differ in phase from these voltages.

Let us consider two cases—

1. In which the field bar is displaced from the neutralizing bar by one-quarter of a wavelength of the flux or 90 electrical degrees, and the E.M.F. in it is, therefore, in quadrature with that in the neutralizing bar. This case is shown in Fig. 195.

2. In which the field bar is not displaced at all with
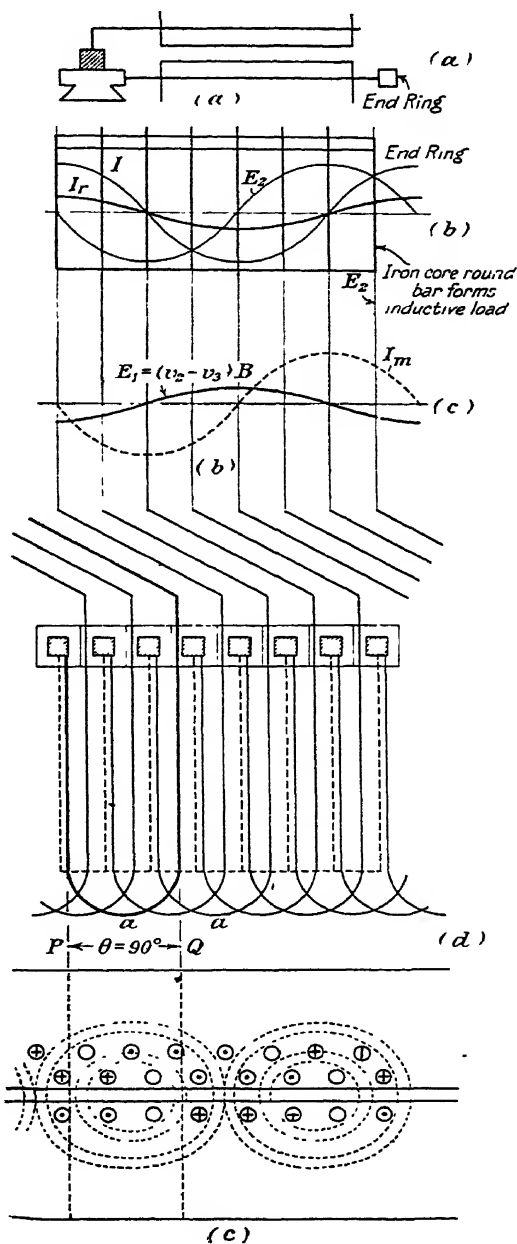
FIG. 195

respect to the neutralizing bar, and, consequently, the E.M.F. in it is equal and opposite to that in the neutralizing bar.   This case is shown in Fig. 196.

In both cases, of course, the magnitude of the E.M.F. in the field bar is v_2B, being the same as that in the neutralizing bar, the only difference in the cases being as regards the phase of these E.M.F.'s.

From the field bars, as shown, the current is conducted to the inductive load, which we may suppose to consist of an alternating flux identical with that in the machine itself, the electromotive distribution in which is shown in Fig. 195b, while that in the machine is shown in Fig. 195c. The distribution of bars in the inductive load is identical with that of the field bars in the machine, and, hence, the flux of the load is also identical as stated, being displaced in space, however, in the diagram, by an amount equal to the displacement of the field bar. Hence the E.M.F. of the load will be $E_2 = v_2B$ in quadrature with the current, being exactly the same as the E.M.F. in the field bars. Summing up the results of Fig. 203, we have the following E.M.F.'s—

(*a*) In the inductive load, $E_2 = v_2B$.
(*b*) In the field bar, $E_3 = v_2B$.

Both these, of course, are in phase with E_2.

In the rotor and neutralizing bars, $E_1 = (v_2 - v_3) B$.

These E.M.F.'s are in quadrature with E_2. In addition to these, we may suppose that there is an E.M.F. Ir due to resistance drop, which will be in quadrature with E_2 and, therefore, in phase with E_1.

In the lower part of Fig. 195 is shown a diagram of the fluxes occurring in the motor due to the field bars. It is clear from this diagram that the air-gap density B is a maximum at a point one-quarter of a wavelength distant from the point at which the current is a maximum. For instance, if the field current is a maximum at the point Q, then the air-gap density B will be at a maximum at the point P in the figure. Hence, if the E.M.F.

$$(v_2 - v_3) B$$

is to reach its maximum at the same time as the current, that is, to be in phase with it, the displacement between field bar and armature bar must be a quarter wavelength.

Summing up in the same manner the results of Fig. 196, the E.M.F. E_2 will be the same as before, and $E_3 = v_2B$ will also be the same. The E.M.F.'s $E_1 = (v_2 - v_3) B$ will still retain the same magnitude, but instead of being in quadrature with E_2 it will be in phase with it. Hence we can write the equation—

$$(v_2 - v_3) B - v_2B - E_2 = 0.$$

From this equation we see that the E.M.F. in the neutralizing bar and the field bar cancel, as is, in fact, perfectly obvious from the figure, and the equation reduces to the form

$$v_3B + E_2 = 0.$$

Since the E.M.F.'s in the two bars on the stator cancel, these bars are clearly unnecessary, and a conductor could be led straight from the commutator brush to the load, so that this case of the commutator generator simplifies itself into the case of the Scherbius phase advancer.

Returning to Fig. 195, the fundamental electromagnetic law, of course, is that the sum of all the E.M.F.'s in the circuit must be zero, and if we have E.M.F.'s in two distinct phases in quadrature with each other, this law involves, as is well known, that the sum of the E.M.F.'s in any given phase must be zero, and the sum of all the E.M.F.'s in quadrature with them must be independently zero. Hence, in the case of Fig. 203, we obtain two distinct equations—

FIG. 196

(1) E.M.F.'s in phase with E_2. This gives $E_2 + v_2B = 0$

(2) E.M.F.'s in quadrature with E_2.
$(v_2 - v_3)B + Ir = 0$. or, $v_1B + Ir = 0$

Since $E_2 = v_2B$, as we saw above, equation (1) may only be satisfied if $v_2 = 0$, that is, if the rate at which the flux wave moves relative to the conductors of the load or of the stator is zero. This, of course, involves that the current shall be continuous and not alternating.

If $v_2 = 0$, equation (2) becomes $Ir - v_3B = 0$, which is the ordinary equation of self-excitation of the direct-current machine.

Coming to Fig. 196, we derive the equation

$$E_2 - v_3B = 0$$

and it was also known that $E_2 = v_2B$. Hence this equation can be satisfied if $v_2 + v_3 = 0$, or, $v_2 = -v_3$.
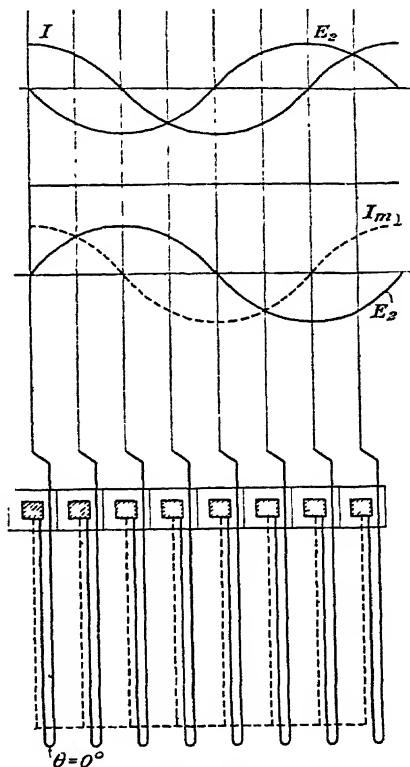
That is to say, that the revolving rotor cuts the flux in one direction at the same rate as the flux cuts the stationary conductors in the load in the other direction, or, in other words, that the rotor runs as much above synchronism as the conductors in the load are below synchronism. This, of course, assumes that there is no resistance drop in the circuit.

Figs. 197 and 198 represent cases of the polyphase commutator generator which are worth a brief examination.

In Fig. 197, the displacement between the neutralizing bar and the field bar is three-quarters of a wavelength instead of one-quarter.
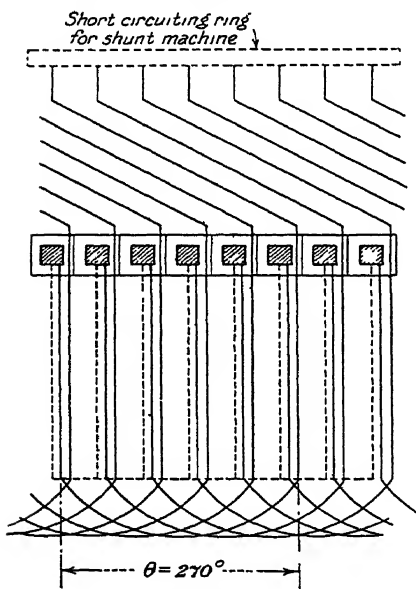
FIG. 197
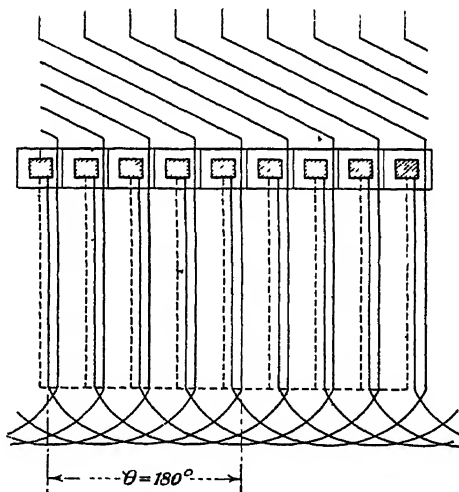
FIG. 198

measured in a particular direction, say clockwise. The effect of this is simply to reverse the E.M.F. $(v_2 - v_3) B = v_1 B$ with respect to E_2 and the field E.M.F. $v_2 B$.

In Fig. 198, the field bar differs from the neutralizing bar by half a wavelength, hence, instead of the E.M.F.'s in the two bars cancelling, they add. This also is merely a way of saying that the E.M.F.

$$(v_2 B - v_3) B = v_1 B$$

is reversed.

The effect of this reversal is as follows—

Whereas in Fig. 195 the machine, as has been shown, is capable of self-excitation as a direct-current generator, in Fig. 197, the field has been reversed, and the machine is incapable of operating as a generator, but can operate as a motor. Whereas in Fig.

196 it was shown that the machine could operate as a negative reactance capable of balancing the E.M.F. of self-induction of an inductive load, yet in Fig. 198 the machine operates as a positive reactance and, hence, is itself equivalent to an inductance of a value depending on the speed.

Thus, to recapitulate for every connection of the field successively differing by a quarter of a wavelength, the machine will act as follows—

(*a*) Negative reactance generates magnetizing power (phase advancer) (Fig. 196).

(*b*) Negative resistance generates mean power (series generator) (Fig. 195).

(*c*) Positive resistance absorbs mean power (series motor) (Fig. 197).

(*d*) Positive reactance absorbs magnetizing power (reversed phase advancer) (Fig. 198) ; and for intermediate connections we get combinations of these effects.

It should be pointed out that while in Fig. 196 the apparatus can supply magnetizing power to a purely inductive load, and, therefore, could conceivably maintain an alternating flux in a circuit which was absolutely devoid of resistance or other means of consuming mean power, yet, in actual fact, the only self-exciting type of generator, so far described, is that of Fig. 195, which is the ordinary series direct-current generator.

For intermediate connections of the circuit, however, it is possible to obtain a type of generator which will be capable both of supplying the magnetizing power required by the alternating flux, and also of giving the mean power required to supply I^2r losses, etc.

It has been pointed out that if the field bars are displaced by one-quarter of a wavelength from the neutralizing bars, so that the angle θ, Fig. 195, measured in electrical degrees is 90°, the machine is capable of operating as a self-exciting direct-current generator.

If the angle θ is less than 90°, however, we have the following E.M.F.'s—

(*a*1) In the inductive load, $E_2 = v_2B$.

(*b*1) In the field, v_2B.

(*c*1) In the armature and neutralizing circuit $(v_2 - v_3) B \cos \theta$. All these are in phase with E_2, or in quadrature with the current. Hence,

$$E_2 - v_2B + (v_2 - v_3)B \cos \theta = 0$$

(*a*2) The Ir drop.

(*b*2) $(v_2 - v_3)B \sin \theta$.

Hence

$$(v_2 - v_3)B \sin \theta + Ir = 0$$

Thus we see that the E.M.F. of the rotor and neutralizing bar combined has components both in phase and quadrature with the current, the component in phase with the current or in quadrature with E_2 being capable of balancing the Ir drop and hence exciting the machine, and that in quadrature with the current capable of balancing the inductive E.M.F. of the load.

Thus the machine becomes self-exciting, not as a direct-current machine, but as an alternating current machine, having a definite frequency of its own.

It is clear that the self-inductive E.M.F. of the load is proportional to the frequency, and, hence, we deduce the rule : *In a self-exciting series polyphase generator, the frequency will be such as to make the self-inductive E.M.F., due to the load and field circuit combined, balance the component of the E.M.F. of the armature and neutralizing circuit which is in quadrature with the current.*
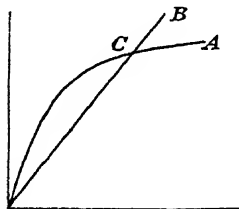
FIG. 199

Considering the component in phase with the current we deduce the rule : *The strength of the field will be such that the component of the E.M.F. of the armature and neutralizing circuit in phase with the current balances the Ir drop and other load E.M.F.'s in the circuit.*

One of the principal applications of the above discussion is to shunt types of polyphase self-exciting machine in which the field bars are connected to a short-circuiting ring, as shown in dotted lines in Fig. 197. In this case the Ir drop referred to in the above rule is simply that of the field circuit, and the self-inductive load is again due entirely to the E.M.F. induced by the flux in the field bars.

In this case these rules simplify as follows—

In the self-exciting shunt polyphase generator, the frequency will be such as to make the self-inductive field E.M.F. balance that component of the E.M.F. in the armature and neutralizing circuit, which is in quadrature with the shunt current, while the second rule becomes

The strength of the field will be such that the component of the E.M.F. in armature and neutralizing circuit in phase with the shunt current balances the Ir drop round the shunt circuit.

This second rule is, of course, identical with the rule for the self-excitation of the direct-current machine, and involves a certain amount of saturation in the magnetic circuit.

It is clear that both the density B and the Ir drop are proportional to the current so long as the iron is unsaturated, and, hence, they cannot become equal for some particular value of the current except in virtue of the saturation of the circuit.

If the saturation curve of the machine be as in curve A, Fig. 199, the Ir drop as a function of the current being as in curve B, then

the machine will excite up to the point C, where these two curves intersect.

We may note also that the frequency is dependent for a given type of winding only on the speed of the machine, and is, therefore, independent of the voltage. For the type shown in Fig. 195, the rate of rotation of the flux will be such that the armature runs as much above synchronism as the field is below synchronism. In fact, the flux will revolve at half the rate of the armature.

This frequency may be adjusted by inserting a further self-induction in the shunt circuit, while the voltage may be adjusted by means of a resistance precisely as in a direct-current machine. We can now recapitulate some of the characteristics of the shunt polyphase generator.

1. The machine is self-exciting if the displacement between the field and armature circuits is intermediate between that shown in Figs. 195 and 196.

2. It gives a frequency which may be varied at constant speed, (*a*) by moving the brushes, that is, varying the angle θ, Fig. 195 ; (*b*) by changing the ratio of armature to field turns : or, (*c*) inserting an external inductance in the field. By any means, in fact, that varies the field reactance.

3. The frequency is almost directly proportional to the speed.

4. The strength of the field is regulated by exactly the same principles as govern the fields of direct-current machines, and may be regulated by a rheostat exactly as in a direct-current machine without affecting the frequency.

5. Hence the voltage and the frequency of the machine are almost entirely independent of one another, and may be regulated separately.

Some further properties of the series machine which are worth noting are as follows—

1. The frequency of a series polyphase generator with a unity power factor load will be invariable and independent of the load. The voltage in such a machine will vary in just the same way as that of a direct-current machine on a similar load.

2. The frequency of a series polyphase machine is inversely proportional to the amount of the inductance in the circuit.

3. If a series polyphase machine is connected with another apparatus giving rise to a definite frequency such as a synchronous machine, then for those positions of the brushes which render it self-exciting it will generate its own natural frequency IN ADDITION to any frequency impressed on it from an outside circuit.

This remark gives us the key to the hunting which sometimes occurs when these self-exciting machines are employed.

It will next be desirable to deal in some detail with the various applications of such a machine. Before doing so, however, the

previous discussion may be summarized by saying that it has been shown that the machine is itself self-exciting for all positions of the brushes lying between two points differing in position by 90 electrical degrees. If we regard the number of poles as variable, then for any given position of the brushes the machine will excite with a number of poles such that the phase displacement between field and armature bar is less than an odd multiple of 90 electrical degrees and more than the next lower even multiple (see page 100). At one extreme point of the range the machine generates direct current, in which case the current is of necessity in phase with the E.M.F., and at the other extreme point of its range the machine generates an E.M.F. which is exactly in quadrature with the current. In positions intermediate between these two extreme points the machine can generate alternating E.M.F. of a definite frequency
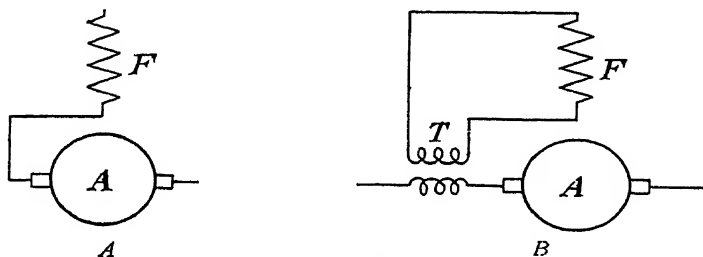
Fig. 200

which has a component in phase with the current to balance the resistance drop in the circuit. In order that any machine may be able to generate alternating current, two conditions must be satisfied—

1. It must be capable of generating an E.M.F. which will balance those E.M.F.'s due to the load which are in quadrature with the current, that is, it must be capable of supplying the reactive power required by this load.

2. It must be capable of supplying an E.M.F. which will balance those E.M.F.'s due to the load which are in phase with the current. that is, it must be capable of supplying the mean power required by this load.

These two conditions are entirely independent—

1. The apparatus may be a generator as regards reactive power, and a motor as regards the mean power. This is the case in all types of compensated motors which absorb mean power and generate reactive power sufficient not only to supply that required by their own reactance, but frequently to supply that needed by the reactances occurring in other portions of the circuit.

2. A machine may be a generator as regards the mean power, and may require to receive reactive power from an external source,

as, for instance, in the induction generator, so that the independence of these two conditions is quite manifest.

The only important case in which such a polyphase commutator machine as already described is used independently, is that of the polyphase commutator motor.

It has been pointed out already that where a polyphase commutator generator can generate, for instance, direct current or some current of definite frequency, it will do so IN ADDITION to carrying current of any frequency impressed upon it from an outside source. It is the existence of currents of two distinct frequencies in the same machine which constitutes the phenomenon of hunting. It is clear that hunting can only be avoided in one of two ways—

1. By making the natural frequency of the machine identical with the impressed frequency.

2. By rendering the machine non-self-exciting, and, as already pointed out, since two independent conditions are required to render it fully self-exciting, this self-excitation can be prevented by seeing that one of these conditions is not satisfied, even though the other may be, or, although it may be the whole object of the apparatus to satisfy it.

When the polyphase commutator machine is used as a motor, the brush setting will usually be such that the machine is non-self-exciting.

Various cases in which polyphase commutator machines are used in practice may conveniently be discussed in turn, so as to make it clear whether hunting is possible and if not, how it is avoided. It will be convenient to include in this discussion the single-phase series motor, as this is one of the simplest cases and throws considerable light on the others.

Case 1. Single-phase Series Motor

Attempts have been made to use single-phase series motors for regenerative purposes on traction systems by reversing the fields and making them into generators, which it was hoped would restore power to the single-phase line and would so brake the train.

These hopes have been disappointed because the series generator generated direct current as well as single-phase current, which is the only type which can usefully be restored to the line. Oscillograms have frequently been published showing these effects. They can be eliminated if, instead of connecting the machine as in Fig. 200 *a*, care is taken that the field of the series machine is coupled to the armature only inductively through a transformer as in Fig. 200 *b*, in which case the generation of direct current is impossible. This is an instance of one of the points explained in the previous discussion.

CASE 2. SHUNT MACHINE

If the terminals of a series motor are short-circuited and leads taken out from the armature it becomes a shunt machine, of which a diagram in the elementary form used throughout this work is shown in Fig. 201. The conditions of self-excitation of a shunt machine have already been worked out. They reduce to two distinct rules, of which the first is—

In the self-exciting shunt polyphase generator, the frequency will be such as to make the self-inductive field E.M.F. balance that component of the E.M.F. in the armature and neutralizing circuit, which is in quadrature with the shunt current, while the second rule becomes—

The strength of the field will be such that the component of the E.M.F. in armature and neutralizing circuit in phase with the shunt current balances the Ir drop round the shunt circuit.

If there is no component of the armature E.M.F. in quadrature with the shunt current, the self-inductive field E.M.F. must be zero, i.e. the machine generates a continuous E.M.F.

Where the position of the brushes is such that the machine tends to generate such a continuous E.M.F., it will produce direct current in addition to carrying any alternating current which may be impressed upon it from an external source. This may be prevented, as with a series machine, by connecting the armature and the field only through a transformer. The existence, of course, of currents of two different frequencies in the same circuit gives rise to powerful synchronizing forces whenever these two frequencies are close together. In other cases, however, where the frequencies are widely different, no such forces are produced. For instance, in the case previously alluded to in which the series machine produced direct current as well as, say, 25 cycles alternating current, it is often impossible to tell from the behaviour of the machine that anything abnormal is occurring, the only clue to the difficulty lying in the abnormal readings of any instruments that may be in circuit.

Now the word " hunting " is usually taken to mean a process of rising and falling of speed, accompanied by a periodic noise, hence such a phenomenon would not ordinarily be called hunting. In the narrow sense of the word, " hunting " must be confined to the case in which the natural frequency of the machine is close to the impressed frequency.

A type of shunt machine in which hunting is impossible is that shown in Fig. 202. In this figure the armature is represented as before by a number of star-connected bars, each connected to a commutator segment on which the brush rests. On the stator are a number of turns, one end of each being connected to a common star point at the end nearest the commutator, while the other ends *AAA* of the same turns are joined by transformers. Leads taken

from the brushes are also joined to the secondaries of the same transformers. If the line is attached to leads *BBB*, then the field circuit will be fed entirely through transformers, and, although the arrangement of the circuits is the same as that of Fig. 201, which would generate continuous current, yet no continuous current can be generated because there is no conductive circuit between the field and the armature, hence, if current of suitable frequency be
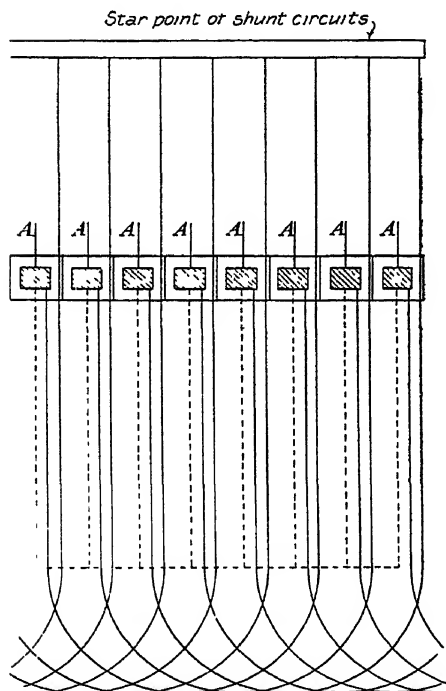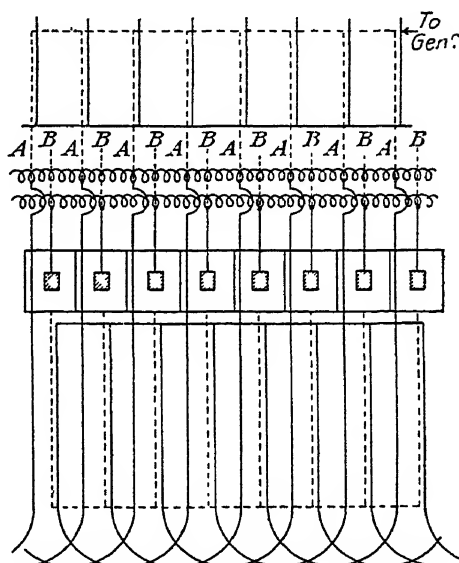
FIG. 201   FIG. 202

impressed on the terminals *BBB*, the motor can operate without self-excitation, and, hence, without hunting.

CASE 3. SHUNT INDUCTION TYPE

As an alternative to this, the line terminals might be connected to *AAA* instead of *BBB*, in which case the armature is fed through the transformers and the line connected into the stator winding. This obviously makes no change as regards the problem of self-excitation. It is, however, the type of machine which is usually employed in such polyphase commutator machines as are used in practice, because it permits of the stator being wound for commercial voltages, while the winding connected to the commutator can be wound for a sufficiently low voltage to ensure good commutation.

Hence we may conclude that in commercial types of polyphase commutator machines no tendency to hunting occurs.

CASE 4. CONDUCTIVE SHUNT TYPE

There are, however, certain conductive forms, such as that shown in Fig. 201, which are little used owing to their not being well adapted to commercial voltages and frequencies, but in which hunting might be possible. These forms should be avoided.

CASE 5. IN CASCADE WITH AN INDUCTION MACHINE FOR THE PURPOSE OF REGULATING SPEED

Returning to Fig. 202, if, instead of the terminals AAA being connected to the line, they are connected to the slip-rings of an induction machine, which is shown in the upper part of the figure, we have the more usual form in which the polyphase commutator machine is used.

When the machine is used as a speed regulator, it acts as a motor below synchronism and as a generator above synchronism, the same brush position corresponding to both these functions. This position, when it acts as a generator, would cause it to generate direct current, and the solution of the possible difficulty which might be caused by this is the same as before, namely, a purely inductive connection between the field and the armature.

In Fig. 203 is shown a diagram of the same type of apparatus in another form. In this figure A represents the large induction machine with slip-rings BBB. These are connected in series with the armature C, and the neutralizing winding NNN of the shunt commutator machine. The same slip-rings BBB are connected to three terminals of a star-connected transformer DDD, the secondary of this transformer EEE being provided with a number of taps for adjusting the voltage. These taps are connected in series with the shunt field FFF, and with another piece of apparatus which will be described later on. It is clear that in this diagram the connection between the armature and the field of the commutator machine is purely inductive, and, hence, self-excitation and, consequently, hunting is rendered impossible. This connection operates quite satisfactorily except in the immediate neighbourhood of synchronism. In the neighbourhood of synchronism the frequency falls to a very low figure, and the transformer ceases to operate. Not only this, but another difficulty makes itself manifest. At frequencies differing considerably from synchronism, the currents in the field circuits will lag approximately 90° behind the E.M.F. applied to them, and the connections of the apparatus are arranged on the assumption that this lag of approximately 90° exists. In the neighbourhood of synchronism the inductance of these coils is reduced to an extremely

low figure, and the current no longer lags 90°, but tends to approximate in phase to the E.M.F. which is applied to the coil. This prevents the proper operation of the apparatus. To correct this a further apparatus, consisting of a small frequency changer having the same number of poles as the large machine, is coupled to the main shaft, which takes current at line frequency through its slip-rings *GGG*, and delivers it at the slip frequency through its commutator *H* to the field *FFF*. By this means this field is separately excited with current of the correct frequency whose phase can be adjusted
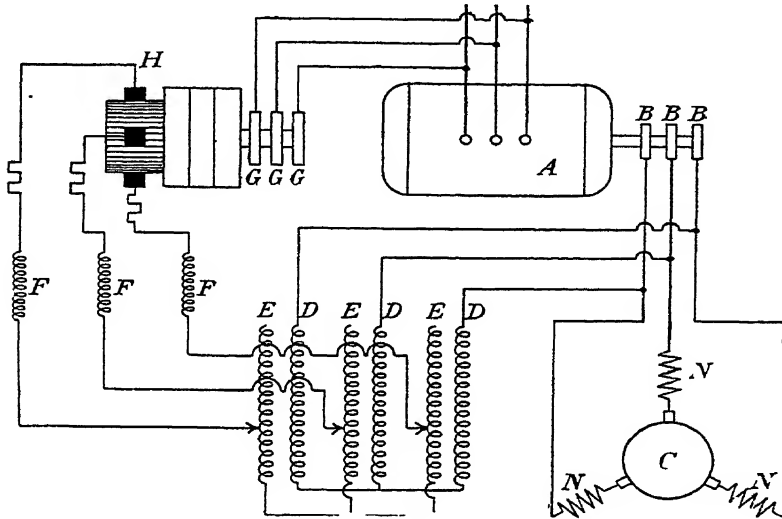
FIG 203

by adjusting the position of the brushes resting on the commutator *H*. This is the so-called " ohmic drop exciter." Clearly, when the commutator machine is separately excited there is again no connection between the armature and the field, and, hence, self-excitation and hunting are impossible.

In some cases the armature and field of this shunt machine are coupled through auto-transformers rather than through transformers having a distinct primary and secondary   This plan is also almost fully effective in preventing self-excitation, since, should there be any remanent magnetism tending to cause a continuous E.M.F. to be generated in the armature, this would produce a current which, instead of building up the field, would be short-circuited through the auto-transformer.

CASE 6. THE PHASE ADVANCER

Coming now to the phase advancer. The very function of the phase advancer is to generate reactive power. Hence the brush

position may most conveniently be made such as to enable it to generate reactive power and no mean power, that is, the brushes may be placed at one extreme limiting position of the self-exciting range. Under these circumstances it is known as the Scherbius phase advancer, and has ·been described above. It is shown in Fig. 196.

If, however, an attempt is made to place the brushes so that the machine can generate mean power as well as reactive power, and regulate the slip as well as the power factor, then it is practically certain that hunting will take place, since the phase advancer now has a natural frequency of its own and will generate current of that frequency which is very little affected by the slip of the main machine. This natural frequency can only agree with the frequency of the main machine for one particular value of the slip, and for all others hunting will take place. Since both the natural and impressed frequencies of the machine are extremely low, inductive couplings are unsatisfactory, and there is no solution to the difficulty except that of placing the brushes in such a position that mean power cannot be generated.

CASE 7. THE FREQUENCY CONVERTOR

In certain cases speed regulation in large induction motors is effected by connecting a frequency convertor in cascade with the slip-ring circuit instead of a shunt polyphase generator, such as that already described. Such a frequency convertor is obtained if to the armature of the polyphase commutator generator are connected a number of slip-rings to which current is led from the ine through suitable transformers. Such apparatus can also hunt unless a precaution is taken similar to that already described, namely, that of adjusting the brushes in a position such that any self-excited current would be neither continuous nor of very low frequency, and then coupling the armature and the field only through transformers.

CHAPTER XXVII

INDUCTION AND COMMUTATOR MACHINES IN CASCADE

WE now come to the combinations of induction and commutator machines in cascade. This opens up a very large subject, as such sets may be used as motors, generators, or convertors, while the commutating element may itself be either a motor, generator, or convertor. Besides this the commutating element may be either primary or secondary. We must, therefore, before we proceed further, effect a sub-classification of our subject. The difference in
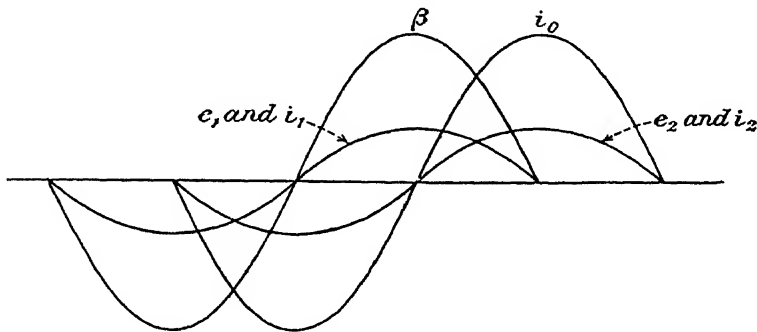
FIG. 204

the arrangement of the set will not be great in general, whether it is used as motor or convertor.

We shall, therefore, base our classification on the type of commutating element employed, whether motor or convertor-

1. Induction motor combined with polyphase commutator motor or generator : (a) series type, (b) shunt type. Use as exciter of motor or generator.

2. Induction motor combined with commutating convertor.

Thus we shall first discuss the induction machine combined with polyphase commutator machines other than convertors. Such apparatus may be used as generator or as motor, and it will be shown that the characteristics of the set are the characteristics of the secondary or commutator machine. The object, therefore, of using the induction machine as well, is to reduce the dimensions of the commutator machine, thereby to a large extent substituting a simple and efficient type for one less simple and less efficient.

We saw above that the series polyphase commutator machine running at constant speed could be adjusted, so that it forms a positive or negative resistance or reactance.

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We shall consider it from this point of view. When adjusted as a generator or motor, the series commutator machine serves to compound the induction motor either differentially or cumulatively.

Consider first the case when the series machine is a motor mechanically coupled to the induction machine. It produces a counter
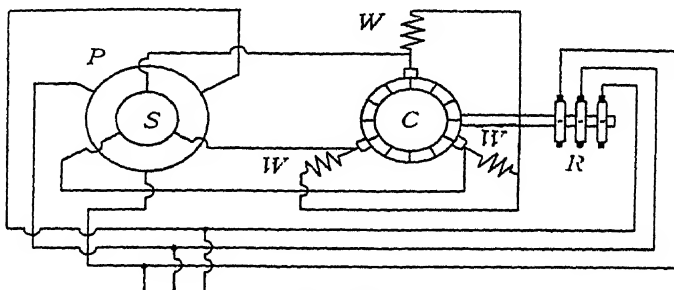
FIG. 205

E.M.F. $e_1 = ir_0k$ opposite in phase to the current and consuming an amount of power $ie_1 = i^2r_0k$. It will help to understand the operation of the machine if we regard this E.M.F. as being a fictitious "resistance drop," which is legitimate since it is proportional to the current.

The counter E.M.F., e_1, is, of course, proportional to the speed k also, and, therefore, the "resistance" interposed by the machine in the secondary circuit of the induction motor falls with the speed. It is clear that the power i^2r_0k given to the series motor appears as mechanical torque i^2r_0 (neglecting for a moment the losses in it) in a motor running at speed k. This torque, of course, is added to the main motor torque.

FIG. 206

Thus our main conclusion is—

By connecting a series polyphase motor into the secondary of an induction motor, we produce an additional slip, corresponding to that due to additional resistance. The additional secondary power due to the additional slip is not lost, but appears as a mechanical torque on the motor shaft, proportional to the square of the motor current and independent of the speed.

In the polyphase shunt machine, the speed is independent of the terminal voltage, if the saturation be neglected and the same ratio be maintained between field and armature volts. It is, however,
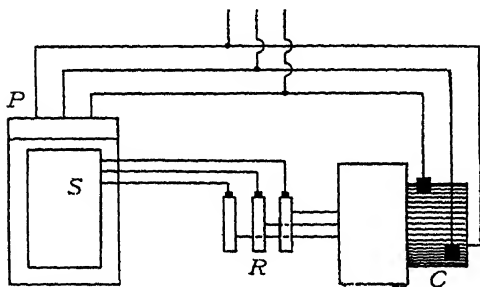
directly proportional to the frequency. If such a machine be cascaded with an induction motor, therefore, the speed of the set will assume an intermediate value below synchronism, since the greater the slip of the induction machine, the higher the speed of the shunt machine.

But in our case the secondary voltage is also proportional to the frequency, and, hence, we conclude—

The field strength of a shunt machine cascaded with an induction motor is independent of the slip, s, since both field voltage and field frequency are proportional to the slip.

Since the field strength is constant, the armature E.M.F. must be directly proportional to the speed, that is, to $(i - s)$. If we suppose that the two are in phase, and that e = secondary voltage of induction machine at standstill (positive below synchronism), e_2 = counter E.M.F. at synchronism in the shunt machine, we have

$$se = \pm (i - s)e_2, \quad s = \frac{e_2}{e \pm e_2}$$

$$(i - s) = \frac{\text{speed}}{\text{synchronism}} = \frac{e}{e \mp e_2}$$

So for this case we get the rule—

$$\frac{\text{No-load speed of set}}{\text{Synchronous speed of set}} = \frac{\substack{\text{Secondary standstill volts of} \\ \text{induction machine}}}{\substack{\text{Synchronous C. E.M.F.} \pm \text{sec} \\ \text{standstill volts of induction machine}}}$$

We must note that this ratio is greater or less than unity, according as the two voltages above assist or oppose one another below synchronism. If they oppose one another below synchronism the no-load speed will be less than synchronous, and if they oppose one another above synchronism it will be greater. The secondary voltage of the induction motor reverses, of course, in passing through synchronism.

By connecting a shunt commutator motor in cascade with an induction motor, we can cause its no-load speed to differ from synchronism either above or below.

Now let us consider the case when e_1 and e_2 are in quadrature.

The most important function of this arrangement is the compensation of magnetization of the main induction motor. In order to understand this, we have only to consider very small slips when the secondary current of the induction motor is sensibly in phase with its E.M.F.

In Fig. 204 let the three sine waves represent—

(*a*) The wave of flux density B.

(*b*) The wave of magnetizing current i_0 at right angles to it in space.

(c) The wave of secondary E.M.F. e_1 and current i_1 in phase with B and in quadrature with i_2.

If now we introduce an E.M.F. e_2 by means of our shunt machine, such that $\frac{e_2}{r} = i_0$ (e_2, being in quadrature with e_1), it will cause a current i_0 to flow in the rotor which will be adequate to magnetize the machine. The magnetizing current i_0 in the stator will disappear, and the no-load current will be reduced to zero —only sufficient remaining to supply the losses. The rotor magnetizing volt-amperes are now $i_0 e_2 = i_0^2 r$. whereas they replace stator magnetizing volt-amperes $i_0 e_e$ where e_e is the terminal E.M.F., an enormous saving. Our final conclusion is therefore—
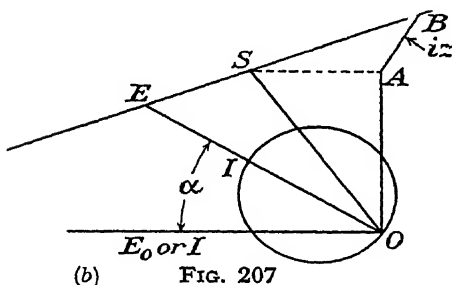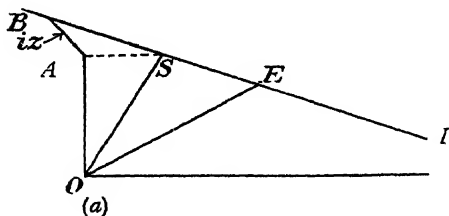
By connecting a shunt commutator machine in cascade with an induction motor, the secondary E.M.F. of the motor and the C. E.M.F. of the machine being in quadrature, we can magnetize the induction motor, raising its power factor to unity or producing a leading current.

(a)

(b)   FIG. 207

1. Shunt Machines.

When using shunt machines to raise or lower the synchronous speed, we obtain the equation

$$i - s = \frac{e}{e + e_2} \text{ or } (i - s) e + (i - s)e_2 = e$$

Multiplying through by the current i we get,

$$(i - s) ei + (i - s) e_2 i = ei.$$

Now ei is equal to the induction motor torque in synchronous watts, and, therefore, $(i - s) ei$ = torque × speed = induction motor H.P. Similarly, $(i - s) e_2 i$ is the shunt motor H.P. Hence, the result may be interpreted—

Algebraic sum of H.P. of both elements = induction motor torque in synchronous watts.

If, for instance, the torque and consequently the input to the induction motor in synchronous kW. is 10 kW., and the machine is running 10 per cent below synchronism, the kW. output will be 9 kW. and, therefore, the shunt machine must have an input of 1 kW. to make up the power put into the induction motor.

Similarly, if the machine is running above synchronism. Hence we conclude

$$\frac{\text{H.P. of shunt machine}}{\text{H.P. of induction machine}} = \text{slip}$$

From the equation,

$$s = \frac{e_2}{e + e_2} = \frac{ie_2}{ie + ie_2}$$

we get

$$\frac{\text{torque of shunt machine}}{\text{total torque of set}} = \text{slip}$$

since $ie_2 =$ torque of shunt machine

$ie =$ torque of induction machine.

The size of the shunt machine necessary for compensating depends entirely on the resistance of the secondary winding. As this is reduced the size of the shunt machine goes down, and in the limiting case may be vanishingly small.

The only difference between series and shunt type machines lies in the characteristics. The rule we declared above for the shunt machine also applies to the series machine running at the same slip. Just as much material is required to produce a given counter E.M.F. in quadrature with the current as in phase with it, hence all machines having the same counter E.M.F. will be the same size.

Fig. 208

We now come to induction machines cascaded with frequency convertors of the commutating type. Such a machine on the commutating side has exactly the same characteristics as a shunt or series machine.

On the collector ring side it will be, of course, connected to the line, the diagram of connections being as in Fig. 205. *P* being the primary winding of the induction machine, *S* the secondary windings, WWW the stator windings of the convertor, *C* its commutator, and *R* its slip-rings. The rotor frequency of the convertor on the collector ring side will be that of the line, the same, in fact, as the stator frequency of the induction machine. The stator frequency of the convertor will be that of the secondary of the induction

machine. The speed of the converter, therefore, which is proportional to the difference between the stator and rotor frequencies, will be either the same as that of the motor or a multiple of it.

In fact, if

f_1 = primary frequency of motor or rotor frequency of convertor,

f_2 = secondary frequency of motor or stator frequency of convertor,

Speed of motor $= k = W (f_1 - f_2)$

Speed of convertor $= k_2 = W_2 (f_1 - f_2)$

$$\frac{W}{W_2} = \frac{k}{k_2}$$

Hence, in a convertor system the secondary power, instead of being converted into mechanical power, which assists or opposes the torque of the main motor, is converted to the line frequency and fed back into the line.
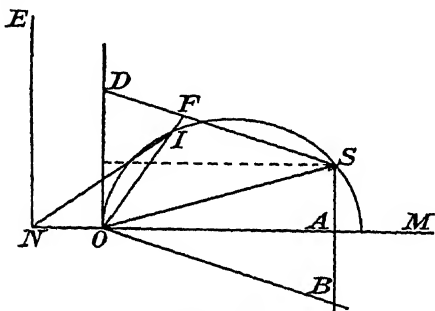
FIG. 209

The characteristics of the system and its diagrams are identical with those developed above.

Such convertor systems have, however, some notable advantages over systems employing motors and generators.

1. The convertors which will be small are not limited to the necessarily slow speeds of the large motor or generator, to which they are auxiliary, when they can be much better designed for higher speeds.

2. The kilowatts converted by the convertor will be the same as those used by the motor or generator previously referred to. However, the apparatus itself will be smaller and require less copper, as is the case in all convertors. Another arrangement is possible, whereby the secondary of the induction machine is connected to the collector rings of the convertor and the commutator end connected to the line (see Fig. 206).

Such a convertor will still run at the same speed as the motor, as the stator frequency is now that of the line, while the rotor frequency is that of the secondary of the main motor. Hence the above arguments still apply.

Such a device, at any rate with shunt convertors, is equivalent to placing the commutator on the main motor, but has the advantage that fewer poles can be used on the convertor to make it go at a higher speed. Also, only that amount of power which it is

necessary to convert may be dealt with, instead of making the whole machine a commutator machine.

Let us now endeavour to deduce the circle diagram of an induction motor cascaded with a polyphase series machine which is coupled to it mechanically. We saw previously that by a suitable brush setting the commutator machine could be arranged to be equivalent to a positive or negative resistance or reactance, or to any combination of these, which may be called the equivalent
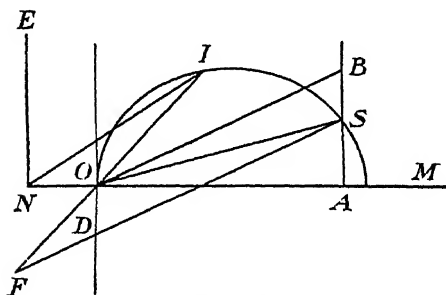
FIG. 210
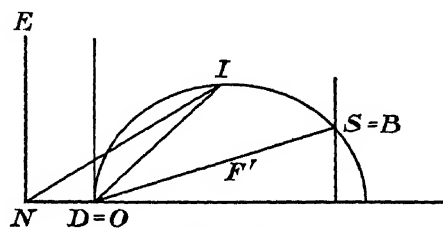
FIG. 211

impedance, and also that the equivalent impedance was proportioned to the speed.

Let s be the slip of the induction motor as a fraction of synchronism and, consequently, $(i - s)$ the speed.

We shall denote the equivalent impedance of the commutator machine by $(i - s) Z$.

In the secondary circuit of the induction machine, we now have,

$$se = i (r + jsx + (i - s) Z)$$

where

  $r =$ true resistance of secondary circuit and commutator machine combined.

  $sx =$ true reactance of secondary circuit, and commutator machine combined, at slip frequency.

  $se =$ E.M.F. induced from primary at slip s.

  $(i - s) Z =$ equivalent impedance of commutator machine as mentioned above.

  $e =$ primary E.M.F.

Let us, in accordance with the usual procedure, endeavour to find the locus of e when i is constant.

The above equation can be written

$$e = i \left(\frac{r + Z}{s} + (jx - Z) \right)$$

1. When $s = i$ we have

$e = i(r + jx) = OS$ in diagram (Fig. 207a), the standstill, E.M.F.

2. When $s = \infty$, we have

$e = i(jx - Z) = OA + AB$ the short circuit E.M.F.

where $OA = i$, jx is the projection of OS on a line perpendicular to OI, $SA = ir$, and $AB = iZ$ can be given any direction or magnitude by suitably adjusting the polyphase machine whose equivalent impedance it is.

3. As we vary s the vector $SA + AB = i(r + Z)$'s varies in length but not in direction. Hence E moves on a straight line. We already know two points on the straight line, viz. OS and OB, corresponding to $s = i$ and $s = \infty$, hence the locus of the E.M.F. vector E is the straight line passing through SB.

By varying Z, this line may be given any direction whatsoever.

The phase difference between current and E.M.F. will be the same whether we consider the current constant or the E.M.F. constant. We must, therefore, draw the constant E.M.F. along the same line as that along which we drew the constant current before (see Fig. 207b).

The equation above shows that when $s = o$, $E = \infty$, on constant current, or, if e is constant, $i = O$.

We saw above that

$$OB = i(jx - Z)$$

and
$$OE = i\left(\frac{r + Z}{s} + (jx - Z)\right)$$

$$OE - OB = EB = i\frac{r + Z}{s}$$

Hence, EB (Fig. 207b) is proportional to $\dfrac{i}{s}$, and, since OB is constant we may also write $\dfrac{EB}{OB}$ proportional to $\dfrac{i}{s}$, and, consequently, $\dfrac{OB}{EB}$ proportional to s. Draw a line SD parallel to OB through S, and one OD parallel to EB through O. Let OE cut SD in F. Draw a line FC parallel to EB. Fig. 208 is Fig. 207b turned clockwise through 90°,

$$\frac{OB}{SB} = \frac{SD}{OD} \text{ and } \frac{OB}{EB} = \frac{OC}{CF} = \frac{DF}{OD}$$

Hence, since OD is constant we see finally that the intercept DF corresponding to any current OI, cut off on a line parallel to OB, is proportional to the slip to such a scale that $DS = I$.

Since the current is zero when $S = o$ or $DF = O$, it follows that OD is tangent to the circle at the origin.

This diagram is identical with that used for the ordinary induction machine, with the exception of the arbitrary vector *AB* introduced by the commutator machine.

It is clear that by varying *AB* we can give the line *EB*, which always, however, passes through the fixed point *S*, any inclination we please.

Let us now consider cases in which *AB* assumes some special values.

1. Let *AB* be parallel to OE_0, i.e. let the commutator machine be arranged as a positive or negative resistance. Then *OD* will be
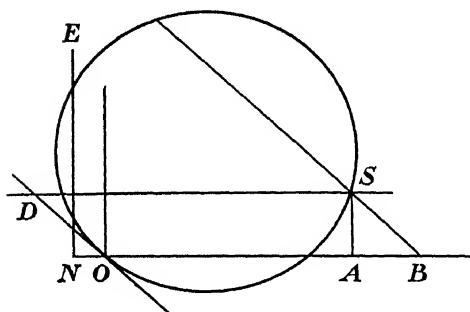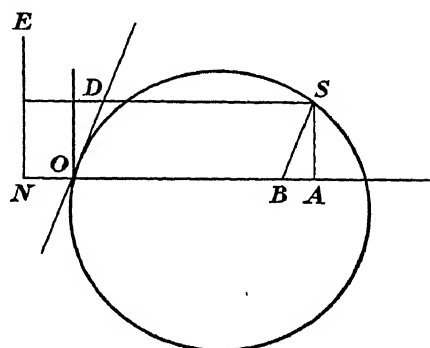
FIG 212   FIG. 213

parallel to OE_0, and the circle will have its centre on *OM* as in the induction motor.

(*a*) Let *AB* be opposite in direction to OE_0. The diagram assumes the appearance shown in Fig. 209. As *SD* (parallel to *OB*) tilts upwards, with increasing *AB*, the slip $\frac{DF}{DS}$ increases for a given current *OI*. That is, the machine is cumulatively compounded.

(*b*) Now let us suppose that *AB* is parallel to *OE* and greater than *AS*. The diagram assumes the form of Fig. 210. *FD* is here opposite in sign to *DS*. Now *DS* always represents a slip of 1 or zero speed. Hence for the motor part of the circle, i.e. that in which the machine consumes power from the line, the slip is negative. That is to say, on no load the machine runs approximately in synchronism, while as the load comes on the speed rises, and the machine is, therefore, differentially compounded, just as we can differentially compound a direct-current motor.

(*c*) In a limiting case.

AB = *AS*, and the diagram assumed the appearance of Fig. 211.

For every value of *OI* the point *F* coincides with the point *D*, and the slip is consequently zero.

The machine is, therefore, synchronous, and if the slip is finite, say *FD*, it follows that the current must be *OS*.

2. Now let us suppose that AB is perpendicular to OE_0, or parallel to OA. (Commutator machine a positive or negative reactance.)

(*a*) Let AB be in the same direction as OA. The diagram assumes the form of Fig. 212.

The centre of the circle is no longer on the line OA, and its radius is enormously increased.

This has the effect of very much improving both the overload capacity and the power factor. The motor may readily be made to have a leading current above a certain load.

(*b*) Now let AB be opposite in direction to OA (Fig. 213). The centre of the circle will now be situated below the axis OA, and, consequently, the overload capacity of the machine as generator will be very greatly increased, while that of the machine as motor is correspondingly diminished.

A limiting case of this adjustment is that in which $AB = OA$, and, consequently, $B = O$ and $ES = OS$. Since the tangent at the origin which is parallel to BS now passes through S, and S is a point on the curve, it follows that in these circumstances the circle must have an infinite radius and, therefore, be identical with the line BS. Under these conditions, therefore, the machine operates at constant power factor.

To sum up the conclusions which we may draw from our circle diagram—

1. A series exciter cannot under any circumstances affect the no-load current or current at synchronous speed.

2. By adjusting it as a positive or negative resistance we may cumulatively or differentially compound our induction motor, so as to produce a full-load slip greater or less than normal, or even zero or negative.

This is easily seen to be possible when we remember that for a given current or torque the slip is proportional to the secondary resistance, and, therefore, becomes great or small, zero or negative, with it.

3. By adjusting it as a positive or negative reactance, we may adjust the power factor and overload capacity of our machines, either as generators or as motors.

When adjusted in this way, these machines are generally spoken of as " phase advancers."

Nothing has hitherto been said of the primary magnetizing current of the induction machine, which, of course, exists. Before any of these diagrams can be completed, therefore, a further line NE parallel to OE and at a distance ON from it must be drawn, where ON represents the primary magnetizing current. If OI represents the secondary current, then NI will represent the primary current of the induction machine.

CHAPTER XXVIII

OPERATION OF A SYNCHRONOUS MOTOR FROM A POLYPHASE SERIES GENERATOR

IN the present chapter we propose to consider the general characteristics of a motor, having a natural frequency of its own when operated from a commutating generator.

For this purpose, we shall choose the synchronous motor as it is the most familiar, though it is necessary to point out that such a motor would have no starting torque.

A system of electric distribution in common use at the present time, particularly for purposes of traction, consists of a number of commutator type motors (usually single-phase series motors) operated from a number of synchronous generators in the power house. In order to obtain a system analogous to this and in which the motors are devoid of commutators, we may operate a synchronous motor from a series commutating generator, either of the single-phase or polyphase type. Such a series commutating generator is capable of generating electrical power in a manner entirely independent of frequency or wave form producing a voltage at every instant which is directly proportional to the current at that instant, if we suppose the magnetic circuit un-saturated.

It will be convenient to consider such a generator as a " negative resistance." We have noted that the voltage in such a generator is directly proportional to the current at every instant.

Now the IR drop due to ohmic resistance is also directly proportional to the current at every instant, but as ohmic resistance consumes power, whereas a generator generates it, the voltage of the generator must be opposite in direction to that due to an ohmic resistance, that is, it must be such a voltage as would be produced by a negative resistance, did such a thing exist.

The inductance of such a generator on moderate frequencies will be considerably less than its apparent negative resistance.

Now, if a synchronous machine operates (as a generator) on a load consisting of ohmic resistance, it will consume mechanical power, and its torque will, therefore, be opposed to the direction of rotation. If E is the E.M.F. of the generator, the current, of course, will be $\dfrac{E}{R}$, R being the resistance of the circuit.

If, however, such a machine be " loaded " with a negative resistance $-R$, the current will be reversed, being $-\dfrac{E}{R}$, i.e. flowing

against the direction of the E.M.F. *E*. Hence, the torque will be reversed and the machine will be a motor.

The frequency of this current will correspond to the frequency of rotation of the synchronous machine, since, as we have seen, our " negative " resistance or series generator is equally ready to generate power at all frequencies.

Hence, in this system as the speed of the synchronous motor varies, the frequency will vary with it, being determined entirely by the motor.

The speed of the motor depends on the mechanical load.

Let us now investigate the characteristics of this type of motor.

The best way to do this appears to be by means of the circle diagram.

In order to draw the circle diagram of such a motor running off the commutator generator, we proceed as follows—

Let $p = 2\pi \times$ frequency in cycles per second be the frequency in radians per second.

$e =$ the motor E.M.F. at unit frequency (1 radian per second).

$pe =$ the motor E.M.F. at frequency p.

$-ir_0 =$ the generator E.M.F. considered as negative resistance.

$ir =$ resistance drop in the circuit.

$ijLp =$ reactance drop in the circuit.

Equating these quantities to zero, we get

$$pe = i\,(r - r_0) + ijLp = 0$$

We may write the equation

$$pe = i\,(r - r_0) + ijLp \text{ as } \frac{e}{i} = \frac{r - r_0}{p} + jL$$

This is represented graphically by the triangle, $O.1B$ (Fig. 214).

It is clear that as p varies, $\frac{e}{i}$ moves on the straight line shown dotted (Fig. 214). If i were constant, of course, e would move on such a straight line. In order to find the locus of i when e is constant we must plot the reciprocal, $\frac{i}{e}$, of $\frac{e}{i}$ for every value of i. It is well known that the reciprocal to a straight line taken in this way is a circle, passing through the origin. Accordingly, the locus of $\frac{i}{e}$ and of i is a circle passing through the origin (Fig. 215).

Now the question is : What circle ? This we may solve as follows.

When $p = \infty$ $\frac{e}{i} = jL$, or $i = \frac{e}{jL}$.

This gives us the relation between i and e for a particular point. i is here at right angles to e, and has the value $\frac{e}{jl}$. It is also a diameter of the circle. We must, therefore, set out e perpendicular to i when
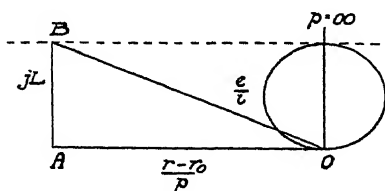
FIG. 214
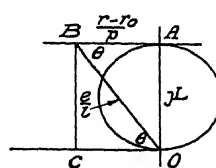
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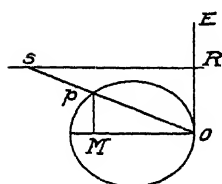
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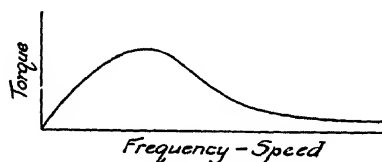
FIG. 217
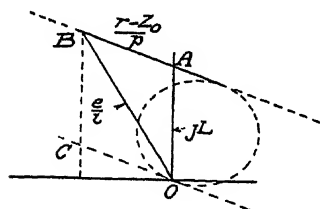
FIG. 218
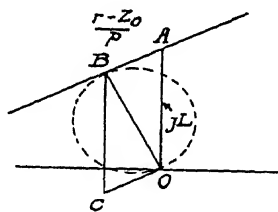
FIG. 219

$p = \infty$, and this determines the relation of e and i for every value of p.

To find the value of i corresponding to any value of p we proceed as follows—

It is clear that $OA/AB = \tan\theta = \dfrac{BC}{OC}$, and is proportional to p. If, therefore, we keep OC constant, the length BC cut off on a fixed line by OB will be a measure of p. Thus we get our final diagram as in Fig. 216, where OP represents the current and RS represents the frequency.

Since the flux of a synchronous motor is constant, we see that the torque is proportional to MP, the projection of the current on OE. Hence, as the frequency varies, the speed torque characteristic will be somewhat as in Fig. 217, rising from zero at standstill to a maximum, and then falling again as in any other series type motor.

This point may be made clearer as follows. The terminal E.M.F. is p times the fixed E.M.F. shown in the diagram. PM is the component of the current in phase with the E.M.F. Therefore, $PM \times OE \times k =$ output $= k \times$ torque. Therefore, torque $= PM \times OE$, or is proportional to PM as OE is constant.

We may readily generalize this diagram for the case in which the generator E.M.F. which we represented before $- ir_0$ is no longer in phase with the current, but makes any desired angle with it.

In this case our E.M.F. triangle is no longer a right-angled triangle, and the locus of $\dfrac{e}{i}$ is as shown in Fig. 218.

Apart from this everything remains the same. The current corresponding to $p = \infty$ is still $i = \dfrac{e}{jL}$, but it is no longer a diameter of the circle. A diameter of the circle passing through the origin, in fact, makes an angle with OA approximately equal to that made by the current with the generator E.M.F.
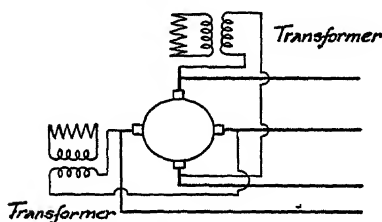
FIG. 220

It is clear, moreover, that the length BC cut off on a fixed line by OB will also measure the frequency if the triangle OCB is similar to the triangle OAB, and that for this similarity to occur OC must be a tangent to the circle at the origin.

It is clear from this that nothing is fundamentally changed. The torque is still zero at starting and varies in the manner shown in Fig. 217. However, the maximum torque may be very greatly increased by this means, this being the chief improvement we can claim. The most serious defect, of course, of such a system is the absence of starting torque. How this may be got over, we shall see later.

Let us now consider a synchronous motor operating from a shunt generator, such as is shown in the diagram (Fig. 220). This generator is fitted with a transformer in the field to prevent self-excitation with direct current, and the brushes are so placed that it is not self-exciting with alternating current. With constant field strength, the voltage of the synchronous motor is proportional to the frequency. And it is clear that if the field winding of such a shunt polyphase generator is supplied with an E.M.F. proportional to the frequency, the flux produced by it will be constant, and the E.M.F. across the armature terminals constant at all motor speeds.

In a word, a constant motor flux gives rise to a constant generator flux to balance it independent of the speed.

The motor, therefore, will run at such a speed that its counter E.M.F. balances the fixed E.M.F. of the generator. It will, therefore,

run at a fixed speed. This speed cannot be adjusted by adjusting the motor field, for any variation will be exactly followed by the generator. It can only be adjusted by varying the generator field, or the number of turns on the motor armature.

Hence, we conclude : (1) That machines operated from a shunt generator have shunt characteristics. (2) That their speed is independent of the motor field. (3) That it may be varied (*a*) by varying the armature turns on the motor, (*b*) by transforming the motor E.M.F. up or down before it is applied to the generator field, (*c*) by any appropriate means of varying the generator field, such as the insertion of self-induction, varying the number of turns, etc. (4) There is no starting torque. It is unnecessary to deduce circle diagrams for these machines as the general nature of the shunt characteristic is well known.

PART VI

CHAPTER XXIX

FUNDAMENTALS OF ELECTRICAL DESIGN

IN the preceding parts of the present volume an attempt has been made, starting from the most abstract point of view, to classify all the possible different kinds of dynamo electric machine and to describe examples of each class, omitting those so well known that a short general description can be of no service. The number of types described runs into several hundreds, and, of course, an unlimited number of further forms may be thought of or even derived from the patent literature. Now it is not the object of the manufacturing engineer to build as many of these forms as possible, but rather as few as possible, provided these few give the best solutions to practical problems. To decide, out of the many possible forms, which are the most suited for practical application, raises the whole question of design.

The subject of design has been exhaustively treated in many able volumes, such as those of Hobart and Miles Walker, etc., and it is not the object of the present work to attempt to duplicate these, which are essentially works of reference. The position of an electrical designer in a manufacturing establishment at the present time is extremely unsatisfactory. His function should obviously be the continual study and improvement of the product as fast as conditions of standardization permit. He should be in a position to make experiments on improved methods of proportioning his apparatus or the use of improved materials or improved types of machine, and this should be his principal function. Yet the drudgery of calculation is such that it is nearly impossible for him to perform this function at all. His whole time is absorbed in " dealing with orders." This arises largely from the peculiar way in which it is customary to deal with calculations. A good example of present-day methods, for instance, is given in Miles Walker's *Specification and Design of Dynamo-Electric Machinery*, in which the author gives a schedule for each type of machine, to be filled up with the particulars relating to it. Many such schedules are given in that work. All these contain a large number of blank spaces, each with its appropriate title, such as—

Flux per pole.
Leakage.
Flux density.

The schedule throws no light on how these quantities are to be calculated, and the designer is expected to memorize all the necessary formulae for doing so, this representing an important part of his professional skill. The result is that no two designers use identical formulae unless they have received exactly the same training, and, consequently, there is always some difficulty in mutual comprehension where several designers of different "schools" meet. As is pointed out in Professor Walker's book, the designer is provided with certain data, these data being sometimes given expressly in the form of specifications, and sometimes being implied by the condition that the machine to be designed must meet standards customary for this particular application, or those set by the manufacturer for his product. Further data are supplied by standard dimensions which must be used in order to accommodate the manufacturer's standard parts. Only very small variations are possible in a machine which is required to satisfy the conditions determined by these data, but the relations between these data and the final dimensions of the machine are rather complexly involved, and the formulae embodying them are too complex for direct use in calculation. The designer, therefore, falls back on what he calls experience, or "judgment." This means that by comparison, often unconscious, of machines satisfying similar conditions, of which he has had past experience, he can tell fairly closely what the dimensions of a new machine should be. Assuming these dimensions, therefore, he checks back to see whether they result in the required performance, this requiring much simpler formulae. The result, in the hands of an expert, will usually be pretty close, and a slight modification in the assumed dimensions will cause the performance to come within the desired limits

For instance, Hobart (*Electric Motors*, Vol. II, p. 347), after describing a design procedure based on output coefficients, says : "From this stage design involves patient calculations of magnetizing current, circle coefficient, losses, stalling load, starting torque, and other characteristics as already outlined in earlier chapters, and the avoidance of pitfalls such as the 'saddle points' discussd in Chapter XI. It is only desired in this chapter to introduce the conception of the output coefficient and to indicate its utility as a means of starting upon a design along reasonable lines." He then goes on to develop a method of roughly estimating the works cost of machines, and then adds (p. 350) : "Of course, it is dangerous to assume that such considerations as suitable stalling load, percentage starting torque, percentage no load current, mechanical designs, etc., will not render impracticable the design with minimum total works cost, but even recognizing that the results are liable to occasionally require serious modification on account of these factors, a good deal is to be learned from a broad study of the field from the

outlined standpoint." The views of Hobart and Miles Walker represent present-day practice very closely. It will be clear that this is not entirely scientific, and if it were possible to calculate the dimensions of the machine directly from the required specifications, without the process of trial and error outlined above, very great advantages would be gained. The greatest of these advantages, in the writer's opinion, would be that the time of the expert designing engineer would be freed from this drudgery of calculation and could be given to his true business, which, as mentioned above, is the continual improvement of his product. A large part of this difficulty arises from inattention to the mechanical details of calculation. Calculations are usually performed by the slide rule, an instrument of extraordinary usefulness, but having several drawbacks—

1. The position of the decimal point is ambiguous, and, consequently, the use of the slide rule must always be accompanied by a more or less subconscious mental calculation in order to ascertain this. In the hands of an expert, this is so habitual as not to be noticed, but the sensation of relief which is felt when the necessity for this is eliminated shows that it creates a certain amount of strain, even though this may not be recognized.

2. The extremely fine divisions of the slide rule give rise to eye strain.

Both these difficulties are overcome by the use of modern calculating machines in which the result comes up in plain figures and with the decimal point definitely indicated. It is true they are less portable and much more expensive.[1] A further advantage they possess is that by their means it is very easy to tabulate complicated formulae, and thus they render the achievement of the objects pointed out above possible. It will be shown in the following chapters that by this means we may separate the use of engineering judgment from the necessity for numerical calculations. Besides the data inherent in the problem, the expert designer uses his experience to satisfy certain further particulars and from these the dimensions of the machine flow by a direct and fairly simple process of calculation. When completed, this calculation can be checked back so as to reproduce data in such a way as to make mathematical errors wellnigh impossible. How this may be done is developed in the following chapters.

It is the possibility of tabulation which enables us to overcome the difficulty of throwing electrical designing into a deductive form, the dimensions of the desired machine being directly derived from the particulars specified.

These particulars are of several types.

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[1] The type used by the writer is the Monroe.

1. *Performance Specification including Horse-power or Kilowatts required—*

Particulars of supply on which it is to operate.

Particulars of enclosure, ventilation, etc.

Particulars of temperature rise.

Particulars of efficiency and power factor.

(These are often merely implied by the necessity of meeting competition.)

2. *Mechanical Requirements.* External and internal diameters and core lengths cannot be determined entirely at the will of the electrical designer, but must be accommodated to a limited number of sizes to suit existing patterns.

3. *Questions of Ventilation and Heating.* These are not susceptible of exact scientific treatment so that empirical data and formulae depending essentially on the designer's skill and experience must be made use of.

4. *Workshop Requirements.* Space-factors and clearances to permit easy and quick manufacture are essentially empirical data given to the designer from outside.

It will be shown that all these specifications may be interpreted by means of a few fundamental data which completely determine the design once they are specified. The skill of the designing engineer is shown by so specifying these fundamental data that they lead to the best possible machine.

From these fundamental data the most satisfactory way to calculate the results required is by means of a schedule, not of the usual type such as that described by Miles Walker, but one in which the whole of the formulae required are plainly stated. When the fundamental data have been entered on such a schedule the calculation of the results required becomes a purely arithmetical problem.

Schedules of this type enable us to make a psychological discovery. Even the expert designing engineer, accustomed all his life to filling in the older type of schedule, and having the necessary formulae entirely by heart, can profit considerably by the new ones, although he fills them in by the use of the slide rule just as he did the older type. The reason of this seems to be that with the older method relying on "judgment," the designer is always subconsciously worrying about whether he has "judged" the various quantities— copper or iron densities, etc.– right, or whether they should not be a little higher or a little lower than he has chosen. With the new schedule, all these quantities are uniquely determined by the formulae, and when the designer has performed his single act of judgment by assigning the fundamental data, the rest is merely arithmetic, and the mind is relieved from the subconscious strain of performing repeated acts of "judgment" however simple.

Where a calculating machine is used instead of the slide rule, further causes of strain are removed as just mentioned. But the chief gain of the method, of course, is that since the filling up of the schedule is merely a question of arithmetic, it need not be done by a skilled engineer, but is well within the capacity of an intelligent lady calculating machine operator, if adequate checking methods such as those given below are used to render arithmetical mistakes impossible.

The designer aims, not merely at producing a machine which will fulfil the specification, but at producing the " best " machine doing so which usually, if the specification is fairly complete as regards technical particulars, power factor, efficiency, etc., so that these cannot be varied much, means that having *minimum* cost or *maximum* output for a given quantity of material. This is, of course, a question that can be dealt with mathematically, as has been done previously in the case of transformers. The simplest type of dynamo-electric machine is the induction motor, and, hence, this will be taken as the subject of the following study, although the results can readily be generalized for a large number of other types of machine.

The first difficulty to be met with is the enormous number of apparently independent variables which, at first sight, seems to make mathematical treatment hopeless. Two expedients may be adopted to reduce these—

1. The number of conductors per slot depends on the voltage, but we may eliminate the necessity for considering different voltages and different numbers of conductors in the slots by expressing all equations in terms of power (kW.) or kilovolt amperes. If we know, in fact, that the machine will be for 400 volts and will have a " threaded " wire winding, the engineer assigns as one of the fundamental data an appropriate space factor (say ·3). The whole of the particulars of the machine can then be worked out including copper losses, iron losses, efficiency, power factor, etc., without knowing the voltage at all, merely assuming that the copper fills three-tenths of the slot area, there being in effect but one conductor per slot. If, on the contrary, we know, as in a squirrel-cage, that there will only be one conductor in fact, we should be justified in assigning beforehand a space factor of, say, ·75 to ·85. This is one point at which real engineering judgment can be manifested. The calculation completed in this way, a short separate calculation will give us the winding we must actually employ.

2. The formulae may be thrown into such a form that they apply to a machine of *unit diameter*, and all quantities such as slot dimensions worked out for a machine of this size. To obtain the slot dimensions, say, of a machine of any other diameter, say, A, we simply multiply those of the unit machine by A. A theory worked

out on this basis may be called a DIMENSIONAL THEORY. If these two expedients are adopted, it will be found that the number of variables is rendered quite manageable.

Units. The object of the present investigation is severely practical, namely, to lead up to a method of calculation in which all unnecessary steps are eliminated, and which can be used with the least possible difficulty for the practical construction of machines. Now, however much we may deplore it, it is a fact that the system of measurement in use both in England and the United States is the inch system, and there is little likelihood of change. Strongly though the present writer would support a change to the metric system were it likely to succeed, there seems no advantage in one department of a manufacturing establishment using centimetres if the rest of it uses inches, and in the following investigations. therefore, inches will be used. Many of the formulae are independent of the system adopted, and in the tables an indication is given of the changes necessary to convert to centimetre units.

Symbolization. For reasons which will be apparent on inspecting the schedules, the number of distinct symbols required is very large indeed. Each column in the schedule must be assigned a different easily legible symbol which has to be embodied in succeed ng formulae, and there are sometimes a great many columns. The plan adopted by the writer, therefore, is to use capital letters, working through the alphabet from A to Z, then using a suffix as A_1, working through the alphabet to Z_1, changing the suffix to 2 as in A_2, and so on. So far it has not been necessary to go beyond Z_8, though many different types of machines have been adapted to this form of calculation. All these calculations have much in common and can, therefore, use the same symbols. Owing to the great multiplicity of symbols it is not possible to use standard letterings as E for E.M.F.'s, I for currents, etc. This was tried at first, but found to introduce restrictions that were very inconvenient.

Several engineers have published papers in which the methods of calculating electrical machines (usually induction motors) have been thrown into a deductive form, but the problem which has hitherto been unsolved, as far as the writer knows, is on what purely rational principle should the flux be determined. Clearly the product Flux × Ampere conductors determines the torque, but how can we determine whether it is best to use a large flux and small number of ampere conductors or vice versa, apart from some empirical rule?

Now in Fig. 221 (*a*) we have a winding adapted to carry so large a flux that there is scarcely any room for slots to carry the conductors. Onthe other hand, in Fig. 221 (*b*) the slots are so large that there is hardly any room for iron to carry the flux. It is clear that neither of these extreme cases is satisfactory, but that there

must exist some case intermediate between these where the product of Flux × Ampere conductors (for a given outside diameter (say unity) and a given inside diameter) will be a *maximum*. The rule that this product shall be a maximum therefore gives us a purely rational method of determining the best proportions of flux and ampere conductors containing no element of empiricism whatever. It is this rule which we shall next proceed to work out.
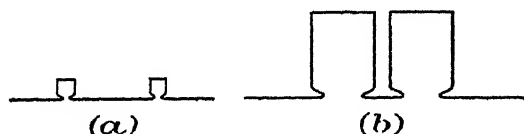
FIG. 221
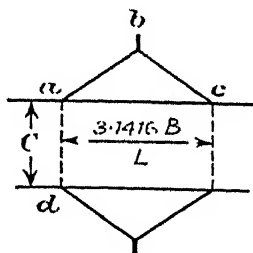
FIG. 222

Fundamental relations.

If B = air-gap diameter

L = number of poles

then $\dfrac{3 \cdot 1416 B}{L}$ = distance spanned by 1 pole.

Very close results are obtained by taking the length of one end-connection from slot to slot as

$$\frac{1 \cdot 75 \times 3 \cdot 1416 B}{L} = \frac{5 \cdot 5 B}{L} \quad \text{(Fig. 222)}$$

The total length of one conductor may be taken as—

$$\text{Length single conductor} = M3 = C + \frac{5 \cdot 5 B}{L}$$

Magnetic density in teeth. This is limited by saturation and, since tooth saturation should be avoided in induction motors, it is permissible to take it as an absolute constant, since it need never be less than the highest value that avoids saturation and should not be more. The effect of varying it will be investigated later.

The *average* tooth density in inch units will be taken as

$$\cdot 07 \text{ megalines per sq. in.}$$

It is necessary to adopt a single unit for flux, and this will be the megaline in terms of which all magnetic quantities will be expressed. Maximum tooth densities will be $1 \cdot 57$ times the above figures (assuming a sine wave).

Copper loss 60° C.

$$= \cdot 8 \times 10^{-6} \text{ (amps. per sq. in.)}^2 \times \text{volume of copper (cub. in.)}$$

A further set of variables we desire to eliminate are those relating to the *method of connection of the windings*, whether star or mesh, series or parallel, etc.

We shall consider the machine to be drum-wound, the winding being made up of a number of identical sections, these, of course, being interconnected among themselves in some appropriate way, of which a very large number are possible.

If in working out windings, etc., we take as a *fundamental datum* to be specified by the engineer not the terminal voltage, but the
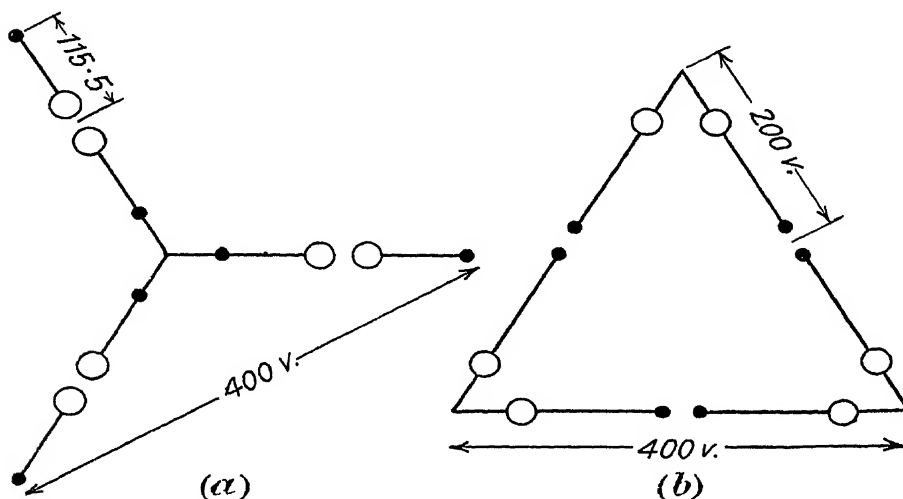
FIG. 223

volts per section, then the method of interconnection of the sections has no influence on the formulae.

For instance, if in one machine we had 6 sections connected in star with 400 volts across the terminals, and in another the same sections connected in mesh, then, according to this plan, the engineer would in the first case specify

$$\text{Volts per section} = N = 115.5$$

and in the second

$$\text{Volts per section} = N = 200$$

and if this figure is given we can work out the schedule without knowing how the sections are connected at all. By building up different circuits from a number of sections all having given volts per section, clearly several different terminal voltages can be obtained. (Fig. 223.)

If N = volts per section, L = number of poles, P cycles per second

O = chord factor × breadth factor

H = total megalines (M.L.) round circumference

= flux per pole (megalines) × number of poles

(note that the flux H is defined in this manner throughout our investigations).

$$\text{Turns per section} = \frac{N \times L \times 100}{4 \cdot 44 \times O \times P \times H}$$

This will be our fundamental voltage formula.

Heating Constants. A purely rational theory of heating is at present impossible, and we have, therefore, to rely on experience, using, however, some calculated figures known as Heating Constants as a guide. One of the most useful of these is the *watts per square inch of copper loss in the end-connections per unit of peripheral speed*, calculated as follows—

Work out the watts copper loss in the whole stator winding, the length of each conductor being calculated by the rule above. Divide by the length of this conductor, and we have the loss in 1 in. of the whole stator winding measured axially. Divide again by the air-gap circumference squared, multiplied by the revolutions per minute (Z_1) $(3\cdot1416^2 B^2) \times Z_1$ of the machine, and we have very approximately the watts per square inch per unit peripheral speed on the inner side of the stator winding where the conductors are most tightly packed together and where this figure is consequently highest. Thus by calculating it at this point we err on the safe side. This figure, viz.—

$$\frac{P_3}{3\cdot1416^2 B^2 \times Z_1 \times M_3}$$

will be taken in what follows as a figure of merit with respect to heating. In our designs we endeavour to give this figure a value dictated by experience (possibly aided by further empirical formulae which each engineer will employ according to his own views).

The Dimensional Theory of the Induction Motor

Part I—The Stator

Let

Primary copper loss	= P_3	Volts per section	N
Number of stator slots	= I	Number of poles	L
Conductors per slot	= S	Cycles per second	P
Area each conductor	= G_1	Chord factor	
Length each conductor	= M_3	× breadth factor	O
Current per conductor	= J_1	Total megalines	H
Amps. per sq. in.	= $\dfrac{J_1}{G_1}$	Number of sections	R
		Stator space factor	D_1
		R.p.m.	Z_1

Vol. copper $= S \times I \times G_1 \times M_3$

$$P_3 = \cdot 8 \times 10^{-6} \left(\frac{J_1}{G_1}\right)^2 \times S \times I \times G_1 \times M_3$$

Solving for J_1

$$J_1 = 1118\sqrt{\frac{P_3 G_1}{S \times I \times M_3}} \text{ (inch units)}$$

From the fundamental voltage formula

Total conductors $= S \times I = \dfrac{2 \times N \times L \times 100 \times R}{4 \cdot 44 \times O \times P \times H} = Z' \text{say}$

$$NR = \frac{2 \cdot 22 \times Z' \times O \times P \times H}{L \times 100}$$

$NRJ_1 =$ volts per section $\times$ amps. per section $\times$ number of sections $=$ voltamperes input.

(a)   *(b)*
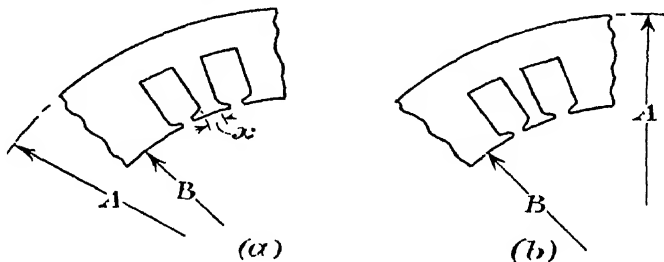
Fig. 224

Note, if the volts change in phase from section to section there will be an equal change in the phase of the current, so the product NJ_1 will remain the same.

$$NRJ_1 = \frac{2 \cdot 22 \times 1118 \times O \times P \times H \times Z'}{L \times 100} \sqrt{\frac{P_3 \times G_1}{Z' \times M_3}} \text{ (inch units)}$$

Put $\dfrac{2 \cdot 22 \times 1118 \times O \times P}{L \times 100} = r \qquad = \dfrac{24 \cdot 8 \times O \times P}{L}$

V.A. input $= rH\sqrt{\dfrac{P_3 \times G_1 \times Z'}{M_3}}$

Now $\dfrac{G_1 \times Z'}{D_1} =$ total slot area

V.A. input $= rH\sqrt{\dfrac{P_3 \times D_1 \times \text{total slot area}}{M_3}}$

Now there are two types of slot to be considered—

(a) The parallel slot (Fig. 224 (a)).
(b) The parallel tooth or taper slot (Fig. 224 (b)).

Taking the parallel slot first, and assuming the average tooth density to have the fixed value ·07 at the air-gap surface (the point x).

Let A = external diameter stator plate
B = air-gap diameter stator plate
F = net core length $F = ·9 (C - D \times E)$
C = gross core length
D = number of ducts
E = width of ducts
T = permissible core density

Width all teeth at air-gap $= \dfrac{H}{·07 \times F}$

Width all slots $= 3·1416\, B - \dfrac{H}{·07 \times F}$

Core depth $= \dfrac{H}{2 \times F \times T \times L}$ (This may easily be checked)

Slot depth $= U = \tfrac{1}{2}(A - B) - \dfrac{H}{2 \times F \times T \times L}$

Area all slots

$$\left\{ \tfrac{1}{2}(A - B) - \frac{H}{2 \times F \times T \times L} \right\} \left\{ 3·1416 B - \frac{H}{·07 \times F} \right\}$$

V.A. input

$$= rH \sqrt{\frac{P_3 D_1}{M_3}} \left\{ \tfrac{1}{2}(A - B) - \frac{H}{2 \times F \times T \times L} \right\} \left\{ 3·1416\, B - \frac{H}{·07 \times F} \right\}$$

and thus the input is expressed directly as a function of H and the main dimensions of the machine.

Our next step must be to express this function in a form in which the actual dimensions of the machine appear as a mere multiplier.

It may be expressed as follows—

V.A. input

$$= rFA^2 \frac{H}{FA} \sqrt{\frac{P_3 \times D_1}{M_3}} \left\{ \tfrac{1}{2}\left(1 - \frac{B}{A}\right) - \frac{H}{2F \times A \times T \times L} \right\}$$
$$\left\{ 3·1416 \frac{B}{A} - \frac{H}{·07 \times F \times A} \right\}$$

or

$$= rFA^2 H_o \sqrt{\frac{P_3 D_1}{M_3}} \left\{ \tfrac{1}{2}(1 - R_o) - \frac{H_o}{2W_o} \right\} \left\{ 3·1416 R_o - \frac{H_o}{·07} \right\}$$

if we put

$$R_o = \frac{B}{A} \qquad H_o = \frac{H}{F \times A} \qquad W_o = T \times L.$$

Now $\quad \dfrac{P_3}{M_3} = \dfrac{P_3 \times 3 \cdot 1416^2 B^2 \times Z_1}{3 \cdot 1416^2 B^2 \times M_3 \times Z_1} = H_{16} \times 3 \cdot 1416^2 B^2 \times Z_1$

where H_{16} = heating constant $= \dfrac{P_3}{3 \cdot 1416^2 B^2 \times Z_1 \times M_3}$

Substituting, placing $3 \cdot 1416 \, R_o A$ outside the radical sign and inserting the value of r.

V.A. input

$$\frac{78 \times O \times P}{L} FA^3 \times R_o \times H_o \sqrt{\frac{H_{16} D_1 Z_1 \left\{ \tfrac{1}{2}\left(1 - R_o\right) - \dfrac{H_o}{2W_o}\right\}}{\left\{3 \cdot 1416 \, R_o - \dfrac{H_o}{07}\right\}}}$$

Now $\dfrac{P}{L} = \dfrac{\text{synchr. r.p.m.}}{120} = \dfrac{Z_1}{120}$

Substituting and putting

$$S_o = H_o \sqrt{\left\{ \tfrac{1}{2}(1 - R_o) - \dfrac{H_o}{2W_o}\right\} \left\{3 \cdot 1416 R_o - \dfrac{H_o}{07}\right\}}$$

we get the final relation (inch units).

V.A. input
$$= \cdot 65 \times FA^3 \times Z_1^{1 \cdot 5} \times O \sqrt{H_{16} D_1} \times S_o R_o$$

and have thus expressed the voltamperes input by purely rational means in terms of the quantity FA^3 and a formula containing only terms independent of the actual dimensions of the machine.

But the expression S_o contains three variables, H_o, R_o, and W_o.

It will next be necessary to investigate what are the relations between them which make this quantity a maximum for any given value of R_o. Now clearly, when it is a maximum or minimum its square will also be a maximum or minimum, and this fact will enable us to eliminate the radical sign.

Let

$$y = H_o^2 \left\{ \tfrac{1}{2}(1 - R_o) - \frac{H_o}{2W_o}\right\} \left\{3 \cdot 1416 R_o - \frac{H_o}{\cdot 07}\right\}$$

$$\frac{dy}{dH_o} = 2H_o \left\{ \tfrac{1}{2}(1 - R_o) - \frac{H_o}{2W_o}\right\} \left\{3 \cdot 1416 R_o - \frac{H_o}{\cdot 07}\right\}$$

$$- \frac{H_o^2}{2W_o}\left\{3 \cdot 1416 R_o - \frac{H_o}{\cdot 07}\right\} - \frac{H_o^2 I_o}{\cdot 07}\left\{ \tfrac{1}{2}(1 - R_o) - \frac{H_o}{2W_o}\right\} = 0$$

Cancelling the common factor H_o and re-arranging

$$\frac{4H_o^2}{2 \times 07 \times W_o} + 3{\cdot}1416R_o(1 - R_o) - \tfrac{3}{2}H_o \left\{ \frac{1 - R_o}{{\cdot}07} + \frac{3{\cdot}1416R_o}{W_o} \right\} = 0$$

Solving for H_o we get the two roots of the quadratic, one of which does not correspond to practical conditions.

$$H_o = \frac{\dfrac{\tfrac{3}{2}\left\{ \dfrac{1 - R_o}{{\cdot}07} + \dfrac{3{\cdot}1416R_o}{W_o} \right\}}{8}}{2 \times {\cdot}07 \times W_o}$$

$$\pm \frac{\sqrt{\dfrac{\tfrac{9}{4}\left\{ \dfrac{1 - R_o}{{\cdot}07} + \dfrac{3{\cdot}1416R_o}{W_o} \right\}^2 - \dfrac{16 \times 3{\cdot}1416R_o(1 - R_o)}{2 \times {\cdot}07 \times W_o}}{8}}}{2 \times {\cdot}07 \times W_o}$$

H_o as a function of R_o and W_o, taking the other sign of the quadratic, is tabulated in Table II.

In a machine with parallel stator slots and a specified value for the number of poles $\times$ core density, and a given ratio R_o of air-gap to outside diameter, it gives us what has never hitherto been possible, a definite rule for determining the flux so as to give maximum input, which other things such as efficiency, etc., being equal means maximum output.

It is the deduction of this formula which justifies us in asserting that a *dimensional* theory of the dynamo-electric machine is possible, viz. that the whole machine may be worked out for a standard external diameter, viz. unity, and then the particulars of the final machine obtained by enlarging or reducing to scale.

It will be seen that it constitutes a purely rational formula which differs from others in the following respects—

1. It gives voltamperes input instead of watts output clearly more closely related to the losses and heating.

2. It contains (A) the external diameter of stator plate instead of (B) the air-gap diameter. It seems reasonable that those parts outside the air-gap should have some influence on the input.

3. It asserts that the input is proportional to FA^3 instead of FA^2 as usually stated. (This has also been suggested very successfully by Mr. T. Tanaka, *The Basis of Dynamo Design*, Maruzen Co., Ltd., Tokyo, 1920.)

4 It shows the exact effect on input of the heating constant H_{16}, the chord factor, etc., O, and the space factor $D1$.

5. It shows that the input is proportional to the 1·5th power

instead of the first power of the speed, thus taking account of the better ventilation at higher speeds.

Slots with Normal Density One-third Up. Another type of slot which requires investigating is that which corresponds to the case in which the normal density occurs, not at the tooth root (the narrowest part of the tooth), but at a radius exceeding this by one-third of the tooth length. This is important because of the approximate magnetic theorem that the ampere turns required by the tooth will be the ampere turns per unit length corresponding to the density one-third up from the narrowest part multiplied by the tooth length. Assuming as before, tooth density (a constant)

$$= \cdot 07 \, ML \text{ per sq. in.}$$

Width all teeth
$$= \frac{H}{\cdot 07 \times F}$$

Mean width all slots
$$= 3 \cdot 1416 \left(B + \tfrac{2}{3} U \right) - \frac{H}{\cdot 07 \times F}$$

core depth
$$= \frac{H}{2 \times F} \cdot \frac{1}{T \times L}$$

$$U = \tfrac{1}{2}(A - B) - \frac{H}{2 \times F \times T \times L}$$

$$\tfrac{2}{3} U = \tfrac{1}{2} \left\{ A - (B + \tfrac{2}{3} U) \right\} - \frac{H}{2F \times T \times L}$$

Area all slots

$$= \tfrac{2}{3} \left[\tfrac{1}{2} \left\{ A - (B + \tfrac{2}{3} U) \right\} \frac{H}{2F \times T \times l} \right] \left[3 \cdot 1416 \left(B + \tfrac{2}{3} U \right) - \frac{H}{\cdot 07 \times F} \right]$$

Put $B^1 = B + \tfrac{2}{3} U$.

Area all slots

$$= \tfrac{2}{3} \left[\tfrac{1}{2}(A - B^1) \quad \frac{H}{2F \times T \times l} \right] \left[3 \cdot 1416 B^1 - \frac{H}{\cdot 07 \times F} \right]$$

That is the same equations as before with the diameter one-third up (instead of one-half up as in the parallel tooth type), substituted for the air-gap diameter and the constant $\tfrac{2}{3}$ as a multiplier.

Taper Slots. There is another type of slot in fairly common use known as the taper slot (or parallel tooth type), and it is desirable that the above calculations should be extended to cover these. The first part of the calculation remains as before, giving

V.A. input
$$= rH \sqrt{\frac{P_3 \times D_1 \times \text{total slot area}}{M_3}}$$

Assuming tooth density (a constant) $= \cdot 07$ M.L. per sq. in.

Width all teeth $= \dfrac{H}{\cdot 07 \times F}$

Mean width slot $= 3 \cdot 1416 \, (B + U) - \dfrac{H}{\cdot 07 \times F}$

Core depth $= \dfrac{H}{2 \times F \times T \times L}$ (As before)

$\tfrac{1}{2} U \qquad = \tfrac{1}{2} \left\{ (A - (B + U)) \right\} - \dfrac{H}{2 \times F \times T \times L}$

Area all slots

$$= 2 \times \left[\tfrac{1}{2} \left\{ A - (B + U) \right\} - \dfrac{H}{2 \times F \times T \times L} \right]$$

$$\left[3 \cdot 1416 \, (B + U) - \dfrac{H}{\cdot 07 \times F} \right]$$

Put $B' = B + U$, the mean slot diameter, and we have

Area all slots

$$= 2 \times \left[\tfrac{1}{2} (A - B') - \dfrac{H}{2 \times F \times T \times L} \right] \left[3 \cdot 1416 \, B' - \dfrac{H}{\cdot 07 \times F} \right]$$

That is the same equations as before, with the mean slot diameter substituted for the air-gap diameter and the constant 2 as a multiplier.

Thus the whole of the preceding investigations become directly applicable, and the same flux is required for a given mean slot diameter, as was before required for the same air-gap diameter. The slot area, however, is doubled, and hence the V.A. for a given loss is multiplied by $\sqrt{2}$. Such machines will be characterized by a smaller air-gap diameter than parallel slot machines, and this affects the rotor slot.

Parallel Tooth. Working out the output formula in detail we get

V.A. input

$$= 2 \times \cdot 65 \times F A^3 \times Z_1^{1 \cdot 5} \times O \times \sqrt{H_{16} D_1} \times S_o R_o$$

or putting $F = xA$

$$A = \cdot 836 \sqrt[4]{\dfrac{\text{voltamperes input}}{\cdot 65 \times x \times O \times S_o R_o \times \sqrt{H_{16}} \times D_1 \times Z_1^3}}$$

remembering that R_o is now to be taken at the mean slot diameter.

Normal density one-third up. Working out the formula for A in the same way we get

$$A = \cdot 9 \sqrt[4]{\frac{\text{voltamperes input}}{\cdot 644 \times x \times O \times S_o R_o \times \sqrt{H_{16} D_1 \times Z_1^3}}}$$

remembering that R_0 is now to be taken one-third up from the gap surface.

Now the last type of slot is the one which is by far the most frequently employed, especially in small machines. It appears to offer us a saving of 10 per cent in external diameter over that first investigated at the expense of a slot half as deep again and a smaller rotor. It is less advantageous where the output is limited by leakage and, consequently, deep slots are no advantage, or where there is insufficient rotor slot space so that a reduction of its diameter is undesirable.

The parallel tooth type of slot goes somewhat farther in the same direction, giving a further saving in diameter of $\cdot 833 - \cdot 9 = \cdot 925$ or, say, $7\frac{1}{2}$ per cent. It is subject to the same limitations as the last type to an even greater degree.

Core Length. It was shown that

v = voltamperes input = $s \times FA^3$ where

$s = \cdot 65 \times$ synchr. r.p.m.$^{1 \cdot 5} \times O \sqrt{H_{16} D_1} \times S_o \times R_o$

This formula permits the deduction of a variety of rules for determining the core length so as to use a minimum of material.

Unlike transformers, the dimensions of rotating machinery cannot be determined very closely by pure calculation, since they have to be accommodated to a comparatively small number of mechanical parts, which are all that it is possible for the manufacturer to stock. Hence, a rather rough rule is of just as much practical service as a more elaborate one.

The rule which will be adopted, therefore, will be one which reduces the length of each conductor to a minimum.

Assuming no ducts

$$\text{Length conductor} = C + \frac{5 \cdot 5 B}{L} = 1 \cdot 11 F + \frac{5 \cdot 5 B}{L}$$

For given values of R_o and S_o the slot area and, therefore, the cross-section of copper will be proportional to A.

Let cross-section of copper $= kA$

$$y = \text{vol. copper} = kA\left(1\cdot11F + \frac{5\cdot5B}{L}\right) \quad \text{(for full pitch windings)}$$

$$= kA\left(\frac{v}{sA^3} + \frac{5\cdot5R_oA}{L}\right) = \frac{1\cdot1kv}{sA^2} + \frac{5\cdot5R_okA^2}{L}$$

$$\frac{dy}{dA} = \frac{11R_okA}{L} - \frac{1\cdot1 \times 2kv}{sA^3} = 0$$

$$\therefore \quad \frac{11R_ok}{L} = \frac{2\cdot2kv}{sA^4}$$

$$A^4 = \frac{\cdot5v \times L}{s \times R_o}$$

Let x $\quad = \dfrac{F}{A}$, then $FA^3 = xA^4 = \dfrac{v}{s} = \dfrac{\cdot5v \times L \times v}{s \times R_o}$

$$\frac{5 \times L \times x}{R_o} = 1$$

Finally, $\quad\quad \dfrac{F}{A} = x = \dfrac{R_o}{\cdot5 \times L}$

If we give F/A this value the weight of copper will be a minimum. Where ducts are used (these are becoming old-fashioned) a figure of, say, ·57 may be substituted for ·5 in the above formula.

Now in terms of H_o, R_o, and W_o, it is shown above that

$$\text{Width slot} \quad = A \times \frac{1}{1}\left(3\cdot1416\,R_o - \frac{H_o}{07}\right). \text{ and the quantity.}$$

$$\mathbf{\mathcal{Z}_o W_o} = \left(3\cdot1416\,R_o - \frac{H_o}{07}\right)$$

$$\text{Slot depth} \quad = A \times \left\{\tfrac{1}{2}(1 - R_o) - \frac{H_o}{2W_o}\right\} \text{ and the quantity.}$$

$$\mathbf{U_o X_o} = \tfrac{1}{2}(1 - R_o) - \frac{H_o}{2W_o}$$

Thus, from the tables we obtain directly the value of the flux and the stator slot dimensions merely by multiplying by FA or by A respectively. Many designers will be disposed to look with suspicion on a method professing to lay down a definite rule for determining these quantities where they have been accustomed to trust their " judgment." This was the writer's view when the above investigation was first carried out. In order to test the rule the various designs published by Hobart (*Electric Motors,*

3rd Ed., Vol. II, Chaps. XV, XVI, XVII, XVIII) were compared with the results obtained from the tables in the following manner—

From the data given by Hobart the values of R_o and W_o for all his examples were calculated. From these values the flux and the slot dimensions which should have been used according to the tables were worked out, and in the table below these are compared with the values which actually *were* used. The striking parallelism of these two sets of figures is very evident, and it is clear that the designers of these machines by their trial-and-error methods were attempting to reach the same goal which is exactly reached by the formula. That is, they were trying to obtain a maximum input consistent with given densities, etc. A converse way of putting it would be, that for a given rating they were trying to obtain the lightest specific loading both of iron and copper. Thus the formula accurately interprets what the designer formerly aimed at subconsciously.

Ref. Letter	Flux		Slot Depth		Slot Width	
	Actual	By Calc	Actual	By Calc	Actual	By Calc
	m/m	m/m	m/m	m/m	m/m	m/m
B_o	2·8	3·08	30	25·4	10	8·3
C_o	16	13·2	21·6	32·4	14·2	15
B_1	2·74	2·98	31·8	28	15·2	15·2
D_1	6·12	7·32	36	28·5	14	11·75
E_1	10·9	9·6	20	27·6	12·5	11
A_2	11·9	11·7	97·2	28·5	12·2	11
B_2	14·8	16·1	45	52	16·3	13·5
C_2	17·8	17·5	25	27·8	10·5	12·2
D_2	40	35	52	60	13	18·6
E_2	27	21·5	39·7	68·3	21·6	23·7
A_3	34·6	30·4	48	59·5	27	30·8
B_3	83	69	51	72	21	25·1

Part II—The Rotor

Since the flux is already determined by the stator equation, a rather different treatment will be appropriate for the rotor.

As with the stator, two types of slot require consideration—the parallel-sided slot, and the " taper " slot, i.e. that corresponding to a parallel tooth.

The tooth width is determined by the stator flux H.

If H_2 = number of rotor slots

$$\text{Width each tooth} = \frac{H}{\cdot07 \times F \times H_2}$$

Draw a circle of diameter B and two adjacent radii, meeting the circumference at xy (Fig. 226).

Let the distance x_1y_1, measured circumferentially, be the slot pitch $\dfrac{3\cdot1416B}{H_2}$.

Draw lines as PR, ST, one on each side of the radii Ov, Ov, and at distances from it equal to half a tooth width $\cdot\dfrac{H}{07 \times F \times H_2 \times 2}$.

The lines PR, QR will meet at a point R, and the figure P, Q, R represents the maximum size of slot possible consistent with a parallel tooth of a given width. (Fig. 226.)

Such a slot will usually be far larger than could be employed in practice, and quite unsuited to carry a winding. The lower part in particular being so narrow as to be practically useless.
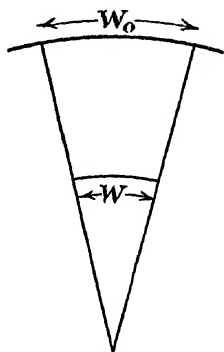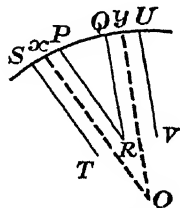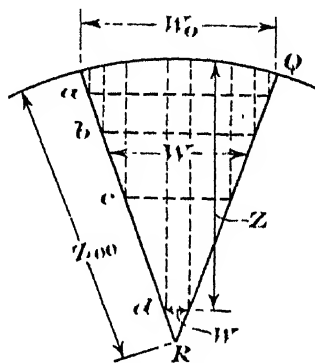
FIG. 225   FIG. 226   FIG. 227

Since rotor windings, moreover, are so frequently made of strip, a parallel-sided slot is more usually required.

Slot I. A parallel-sided slot means a tooth of varying width. If the *minimum* tooth width is to be unaltered, clearly the parallel slot must lie *wholly within* the figure PQR of Fig. 226.

Many different parallel slots can be drawn within the figure PQR, such as those with a corner at a, b, c, d (Fig. 227), of which it is clear that some are larger in area than others. For instance, those with corners at a and d clearly have smaller areas than those with corners at b and c.

Our first problem, therefore, is to find *the slot of maximum area* lying wholly within PQR.

If W = slot width

$\quad W_0$ = slot width at gap diameter = PQ

$$= \left(3\cdot1416B - \frac{H}{0\cdot7 \times F}\right)\frac{1}{H_2}$$

$\quad Z$ = slot depth

$\quad Z_m$ = maximum possible slot depth = PR (Fig. 226)

$$\left(\tfrac{1}{2}B - \frac{H}{6\cdot2832 \times \cdot07 \times F}\right) = \frac{1}{6\cdot2832}\left(3\cdot1416B - \frac{H}{\cdot07 \times F}\right)$$

then by inspection $\dfrac{W}{W_o} = \dfrac{Z_m - Z}{Z_m}$ or $Z_m - \dfrac{WZ_m}{W_o} = Z$

Area of slot

$$= WZ = A' = W\left(Z_m - \frac{WZ_m}{W_o}\right) = WZ_m - \frac{W^2Z_m}{W_o}$$

$$\frac{dA'}{dW} = Z_m - 2W\frac{Z_m}{W_o} = 0$$

giving finally $W = \tfrac{1}{2}W_o$

for maximum area

and, consequently, $Z = \tfrac{1}{2}Z_m$

The following relation will be noted

$$W_o = \frac{6\cdot2832 \times Z_m}{H_{22}} \quad \text{or } W \quad \frac{Z_m \cdot 3\cdot1416}{H_2}$$

Thus the slot of maximum area is the slot of half the maximum possible depth.

Its area is $\tfrac{1}{2}W_o \times \tfrac{1}{2}Z_m = \dfrac{1}{3\cdot1416 \times 8 \times H_2}\left(3\cdot1416B - \dfrac{H}{\cdot07 \times F}\right)^2$

Slot II. Even such a slot is often too deep, since workshop considerations are of very great importance in choosing the rotor slots.

A type of slot often more practical is the following—

With a parallel slot and, therefore, variable width tooth, a good approximate rule in calculating the ampere turns is to work out the magnetic densities at a point one-third of the tooth length distant from the narrowest cross-section, and then calculate the ampere turns on the assumption that the saturation has this value throughout the length of the tooth.

The type of slot we are about to consider has the *same area*,

$\dfrac{1}{3\cdot1416 \times 8 \times H_2}\left(3\cdot1416B - \dfrac{H}{\cdot07 \times F}\right)^2$ as that last discussed, but

is wider and shallower since the cross-section of all teeth at a

point " one-third up " is $\dfrac{H}{\cdot07 \times F}$

$$\text{Slot area} = \frac{1}{3 \cdot 1416 \times 8 \times H^2}\left(3 \cdot 1416 B - \frac{H}{\cdot 07 \times F}\right)^2$$

$$= \frac{3 \cdot 1416}{2H_2} Z_{mo}^2 = W \times Z$$

$$3 \cdot 1416\left(B - \frac{4Z}{3}\right) - WH_2 = \frac{H}{\cdot 07 \times F} \text{ by hypothesis}$$

$$\left(3 \cdot 1416 B - \frac{H}{\cdot 07 \times F}\right) - \frac{3 \cdot 1416 \times 4}{3} \times Z = WH_2$$

$$6 \cdot 2832 \times Z_m - \frac{3 \cdot 1416 \times 4}{3} \times Z = WH_2$$

From previous equation $W \times H_2 \times Z = \dfrac{3 \cdot 1416}{2} Z_m^2$

Substituting for WH_2

$$\left(6 \cdot 2832 Z_m - \frac{3 \cdot 1416 \times 4}{3} \times Z\right)Z - \frac{3 \cdot 1416}{2} Z_m^2 = 0$$

or $\qquad \dfrac{3 \cdot 1416 \times 4}{3} Z^2 - 6 \cdot 2832 Z_m Z + \dfrac{3 \cdot 1416}{2} Z_m^2 = 0$

Solving this quadratic

$$Z = \frac{3}{3 \cdot 1416 \times 4} Z_m \left(1 \pm \sqrt{\tfrac{1}{2}}\right)$$

$$Z = \cdot 32 Z_m \text{ or } 1 \cdot 18 Z_m$$

Taking $Z = \cdot 32 Z_m$ we get

$$\cdot 32 \times W \times Z_m = \frac{3 \cdot 1416}{2H_2} Z_m^2$$

or $\qquad W = \dfrac{3 \cdot 1416}{2H_2} \dfrac{Z_m}{\cdot 32} = 4 \cdot 9 \dfrac{Z_m}{H_2}$

Thus this slot is 55 per cent wider than the last and correspondingly shallower, thus being of a more practical form in many cases.

Note. If we put $\dfrac{H}{F \times A} = H_o$ and $\dfrac{B}{A} = R_o$, we have

$$\text{area all slots} = \frac{A^2}{3 \cdot 1416 \times 8}\left(3 \cdot 1416\frac{B}{A} - \frac{H}{\cdot 07 \times F \times A}\right)^2$$

$$= \frac{A^2}{3 \cdot 1416 \times 8}\left(3 \cdot 1416 R_o - \frac{H_o}{\cdot 07}\right)^2$$

Writing $S_o = H_o\sqrt{\left\{\tfrac{1}{2}(1 - R_o) - \dfrac{H_o}{2W_o}\right\}\left\{3 \cdot 1416 R_o - \dfrac{H_o}{07}\right\}}$

it will be seen, referring to the discussion of the stator, that

$$S_o = H_o \times \sqrt{\text{area all stator slots when } A = 1}$$

If we put

$$T_o = \frac{H_o}{\sqrt{8 \times 3 \cdot 1416}}\left(3 \cdot 1416 R_o - \frac{H_o}{07}\right)$$

$$= H_o\sqrt{\text{area of all rotor slots when } A = 1}$$

Then $\qquad \dfrac{S_o}{T_o} = \sqrt{\dfrac{\text{area all stator slots}}{\text{area all rotor slots}}}$

Slot III. Occasionally, especially where wire wound rotors are employed, the amount of winding space given by slots I and II may be insufficient.

In this case a taper slot (parallel tooth) of the same depth as slot I may be employed.

Since $2W = W_o$, the mean width of such a slot is

$$\tfrac{1}{2}(W + W_o) = \tfrac{1}{2}(W + 2W) = 1\tfrac{1}{2}W$$

and its area, therefore, $1\tfrac{1}{2}WZ$ instead of WZ as in slot I.

Thus the area of slot III is $1\tfrac{1}{2}$ times slot I.

It should be noted that what has been done is to work out formulae for three different kinds of slot, so that the designing engineer may use his judgment as to which type is most adapted for a particular machine. It is here, viz. in *drawing up the design schedule*, that his experience can be used to the best advantage, not in the more mechanical work of making the actual computations.

But a very considerable reduction of the slot depth may make very little change in its area. Consider the effect on the area of reducing the depth Z to pZ $(p < 1)$.

We had

Slot I
$$\frac{W}{W_o} = \frac{Z_m - Z}{Z_m}$$

and $W = \tfrac{1}{2}W_o$

and $Z = \tfrac{1}{2}Z_m$ for max. area.

If $Z = \tfrac{1}{2}pZ_m$ $W = (1 - \tfrac{1}{2}p)\,W_o$

$(1 - \tfrac{1}{2}p)\,W_o \times \tfrac{1}{2}pZ_m = \tfrac{1}{2}W_o\tfrac{1}{2}Z_m \times p\,(2 - p)$

Now the area of slot II was $\tfrac{1}{2}W_o \times \tfrac{1}{2}Z_m$. Thus—

The effect of reducing the slot depth from Z to pZ is to reduce the area in the ratio $p(2 - p) : 1$.

Slot·III. In this case as before.

If $Z = \tfrac{1}{2}pZ_m$ $W = (1 - \tfrac{1}{2}p)\,W_o$

Mean width slot $= \tfrac{1}{2}(W + W_o) = \tfrac{1}{2}W_o\{(2 - \tfrac{1}{2}p)$

Area

$$\tfrac{1}{2}pZ_m \times \tfrac{1}{2}W_o\,(2 - \tfrac{1}{2}p) = 1\tfrac{1}{2} \times \tfrac{1}{2}W_o \times \tfrac{1}{2}Z_m \times \frac{p\,(4 - p)}{3}$$

Now the area of slot III was $1\tfrac{1}{2} \times \tfrac{1}{2}W_o \times \tfrac{1}{2}Z_m$. Thus

The effect of reducing the slot depth from Z to pZ is to reduce the area in the ratio $\dfrac{p(4 - p)}{3}$

The values of these two functions are tabulated below.

In some cases, for instance, in the direct current machine the revolving armature alternator or the rotary convertor (to which, however, the investigation below does not apply without modification) we should aim at obtaining a maximum *rotor* input instead of a stator input.

In carrying out this investigation it will be assumed that slots of type I or type II are employed.

Take the case of the revolving armature alternator as presenting the greatest analogy with the induction motor considered above.

Let

Permissible Rotor Loss	$= P_4$	Volts per section	$= E_2$
Number of Rotor Slots	$= H_2$	Number of poles	$= L_2$
Conductors per slot	$= J_2$	Cycles per second	$= P$
Area each conductor	$= S_2$	Chord factor	
Length each conductor	$= M_4$	$\times$ breadth factor	$= F_2$
Current per conductor	$= C_2$	Total Megalines	$= H_r$
		No. of sections	$= D_2$
		Rotor space factor	$= P_2$

As with the stator

$$C_2 = 1118\sqrt{\frac{P_4 \times S_2}{H_2 \times J_2 \times M_4}}$$

From the fundamental voltage formula

Total conductors $= J_2 \times H_2 = Z''$

$$= \frac{2 \times E_2 \times I_2 \times 100 \times D_2}{4\cdot44 \times F_2 \times H_r \times P}$$

Voltamperes input

$$E_2 \times C_2 \times D_2$$

$$= \frac{1118 \times 4\cdot44 \times F_2 \times H_r \times P}{2 \times L_2 \times 100}\sqrt{\frac{P_4 \times S_2 \times H_2 \times J_2}{M_4}}$$

$$\frac{S_2 \times H_2 \times J_2}{P_2} = \text{total rotor slot area} = H_2 \times Y_2 \text{ say.}$$

Put $\dfrac{1118 \times 4\cdot44 \times F_2 \times P}{2 \times L_2 \times 100} = r_2$

V.A. input $= r_2 H_r \sqrt{\dfrac{P_4 \times P_2 \times \text{total rotor slot area}}{M_4}}$

Choosing Slot II above as being the most practical and taking the average tooth density as t, while B' is the diameter " one third up " the slot.

Effective width all teeth $= \dfrac{H_r}{t \times F}$

Width all slots $3\!\cdot\!1416 \, B' - \dfrac{H_r}{t \times F} = WH_2$

From previous investigation

$$WH_2 = \frac{4\!\cdot\!9}{\cdot 32} Z = 15\!\cdot\!3Z$$

Area all slots $= \dfrac{1}{15\!\cdot\!3} \left(3\!\cdot\!1416 \, B' - \dfrac{H_r}{t \times F} \right)^2 = WH_2 Z$

V.A. input $= r_2 H_r \sqrt{\dfrac{P_4 \times P_2}{M_4 \times 15\!\cdot\!3}} \left(3\!\cdot\!1416 \, B' - \dfrac{H_r}{t \times F} \right)$

Put B_{oo} $= \dfrac{B'}{B}$ and $H_{or} = \dfrac{H_r}{F \times B}$ and we get

V.A. $= r_2 H_{or} \sqrt{\dfrac{P_1 \times P_2}{M_4 \times 15\!\cdot\!3}} \times$

$$\left(3\!\cdot\!1416 \, B_{oo} \times \frac{H_{or}}{t} \right) \times FB^2$$

Now put $\quad H_{26} =$ rotor heating constant

$$= \frac{P_4}{M_4 \times 3\!\cdot\!1416^2 B^2 \times Z_1}$$

$$\frac{P_4}{M_4} = H_{26} \times 3\!\cdot\!1416^2 B^2 \times Z_1$$

Substituting these values, putting in the value of r and remembering that $\dfrac{P}{L_2} = \dfrac{Z_1}{120}$

V.A. $= \dfrac{1118 \times 2\!\cdot\!22}{120 \times 100 \times \sqrt{15\!\cdot\!3} \times 3\!\cdot\!1416} \times$

$$\sqrt{H_{26} \times P_2} \times F_2 \times FB^3 \times Z_1^{1\cdot 5} \times H_{or} \left(3\!\cdot\!1416 \, B_{oo} - \frac{H_{or}}{t} \right)$$

$$= \cdot 0168 \, FB^3 \, Z_1^{1\cdot 5} \times$$

$$\sqrt{H_{26} \times P_2} \times F_2 \times H_{or} \left(3\!\cdot\!1416 \, B_{oo} - \frac{H_{or}}{t} \right)$$

It is clear that the function $S_{oo} = H_{or} \left(3\!\cdot\!1416 \, B_{oo} - \dfrac{H_{or}}{t} \right)$

corresponds to $R_o S_o$ in the stator investigation, and for the same reason should be a maximum

$$\frac{S_{oo}}{H_{or}} = 3{\cdot}1416\,B_{oo} - \frac{2H_{or}}{t} = 0$$

$$H_{or} = \frac{3{\cdot}1416 \times t}{2} \times B_{oo}. \qquad \frac{H_{or}}{t} = 1{\cdot}5708\,B_{oo}$$

(If for instance we take $t = {\cdot}07$ as in the stator, we get the simple result.

$$H_{or} = {\cdot}11\,B_{oo})$$

Using this relation (for any value of t)

Let $\qquad \dfrac{Z_{oo}}{H_2} = \dfrac{W}{B}$ and $U_{oo} = \dfrac{Z}{B}$

Then $\qquad \dfrac{Z_{oo}}{H_2} = \left\{ 3{\cdot}1416\,B_{oo} - \dfrac{H_{or}}{t} \right\} \dfrac{1}{H_2}$

$$U_{oo} = \frac{1}{15{\cdot}3} \left(3{\cdot}1416\,B_{oo} - \frac{H_{or}}{t} \right)$$

Putting in the value of $\dfrac{H_{or}}{t}$ in terms of B_{oo}

$$\frac{Z_{oo}}{H_2} = 1{\cdot}5708\,B_{oo}$$

$$U_{oo} = {\cdot}1025\,B_{oo}$$

Thus the three quantities H_{or}, U_{oo}, and $\dfrac{Z_{oo}}{H_2}$ can very simply be tabulated in terms of B_{oo} alone or the simple formulae for them can be incorporated in a design schedule. For the maximal condition we have

$$H_{or} \left(3{\cdot}1416\,B_{oo} - \frac{H_{or}}{t} \right) = \frac{3{\cdot}1416^2}{4} B_{oo}^2 t$$

and

$$\text{V.A.} = {\cdot}0414\,FB^3 Z^{1{\cdot}5} \sqrt{H_{26} \times P_2} \times F_{\angle} \times B_{oo}^2 t$$

as the final output formula.

Assuming suitable values for B_{oo} and t, this serves to calculate B and the above formulae then give the slot dimensions. The rule for determining $\dfrac{F}{B}$ is identically that given for $\dfrac{F}{A}$ above, this investigation applying unchanged.

CHAPTER XXX

CALCULATION OF AN INDUCTION MOTOR

It is not expected that the detailed schedules developed in this chapter will be used exactly as they are by expert designers who will naturally prefer their own. They are intended to refute the fallacious conception that electrical designing cannot be reduced to rule because repeated acts of " engineering judgment " have to be performed, by showing one way (with, by inference, many alternatives) in which all these " acts of judgment " may be concentrated at the outset instead of dispersed through the calculation. They are also intended for students. While they exhibit the writer's practice, he is well aware that no two designers think alike.

SHEET I. DATA SHEET

The first sheet of the calculation schedule is the data sheet, which collects together the data on which the rest of the calculation is based. These may be divided into several classes.

1. Constructional data—
   L = number of poles.
   I = stator slots ⎫ the leakage depends very largely on
   H_2 = rotor slots ⎬ the choice of these.
   R = number of stator groups.
   R_2 = number of rotor groups.
   I_6 = net air-gap.
   S_L = slot opening.
   L_2 = factor for determining length of secondary end-connections.
   $D \times E$ = number of ducts × width duct.

2. Data depending on the Specification—
   N = stator volts per section.
   E_2 = rotor volts per section.
   Z_1 = revolutions per minute.
   $H.P.$ = horse-power of the machine.

3. Data drawn from workshop practice—
   D_1 = stator space factor
   F_1 = rotor space factor.

4. Estimates which will be checked by the calculation—
   K = estimated efficiency.
   L_1 = estimated power factor.

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DATA SHEET

Calc. Model K.C.3   Date . . . .

Gap Dia. / Ext. Dia.	Poles	Est. Effy	Est. P F	Poles × Core Density	Ch. F. × Br. Co.	Heating Const	Stator Space Factor	Stator Slots	Rotor Slots	Stator V. per Section	Stator Groups	Rotor V. per Section	Rotor Space Factor	Rotor Groups
R_0	L	K	L_1	W_0	O	H_{16}	D_1	I	H_2	N	R	E_2	F_7	R_2
·7	4	·85	·87	·4 ⎫	·96 ⎫	·00005	·3	48 ⎫	36 ⎫	57·5	12	27·5	·3	12
·7	6	·9	·92	·4 ⎬ ·48	·96 ⎬ ·96×·83	·00005	·3	72 ⎬ 96	96 ⎬ 120	38·7	18			

DATA SHEET (contd)

Net Air-gap	Stator Slot Opening	Sat. Factor	Rotor End Factor	Iron Quality Factor	Rotor Space Factor	Rotor ch. F.	Horse-Power	R.p.m	Rotor Depth Ratio	Ducts No.	Ducts Width	Fric. and Windage Factor	Line Volts	Stator Parallels	Rotor Parallels
I_6	S_1	S_t	L_2	q	F_7	O_2	H.P.	Z_1	p	D	E	S_3	V	w	w_2
·02	·125	1·2	4	1·0	·3	1·0	5	1500	‒·65	0	0	·015	400		
						·83	500	1000		0	0				

LEAKAGE DATA

		Wound		Sq. Cage	
K_1 open slots	.	1·33 ☐		1·2 ☐	
K_1 semi-closed slots	.	1·66 ☑		1·5 ☐	
K_2 one coil per slot	.	1·21 ☑		·825 ☐	
K_2 two coils per slot	.	1·02 ☐		·69 ☐	

HEATING DATA

$H_{16} = $ ·0001 for forced ventilation   ☐

$H_{16} = $ ·00005 to ·00006 ⎬ for natural ventilation   ☑

Place a tick opposite the figures to be used

ROTOR SLOT

Type I ☑
Type II ☐
Type III ☐

5. Data which represent decisions made by the designer based on previous experience of what values lead to the most satisfactory design—

R_o = ratio of gap diameter to external diameter (this decision is guided by reference to the tables).

W_o = core density × number of poles.

O = chord factor × breadth factor.

H_{16} = heating constant.

S_t = saturation factor (this may be calculated, but it is often not worth while doing so).

q = quality factor in iron losses depending on the grade used. (A value of q should be chosen so as to bring the calculated iron losses as near to the tested value as possible.)

S_3 = friction and windage factor (this also should be empirically chosen).

On the same sheet are entered the constants required for the leakage calculation for different types of stator and rotor winding. It is necessary to decide which of these to use according to the type of winding desired. In the sheet on the opposite page, those to be used are ticked. It is also convenient to note the most usual values of the heating constant.

Besides the data given, it will usually be necessary to determine the external diameter and core length to suit existing patterns. The formula gives an ideal value, and the nearest practical value (which may not be very near) must be chosen, say, from a manufacturer's list. An unskilled calculator should refer to the expert designer at this point. Further data are given by the wire table.

When these data have been determined, the rest of the calculation follows a predetermined course and only one machine can result for good or evil. The calculation shows what the characteristics will be, and, if the data in sheet I are unskilfully chosen, these characteristics may be inferior. If so, an alteration must be made in the data and the calculation repeated. But by isolating, as is here done, the factors on which results depend, a wise choice is very much facilitated.

The factor having the greatest influence on design is R_o. Where the design has real elements of novelty, it may be desirable to work out in parallel columns the effects of systematically varying R_o through a certain range.

It is sometimes convenient, largely for mechanical reasons and to minimize leakage, to take the stator slot depth W_1r, different from that given by the formula. The schedule is so designed that W_1r may be derived from *data*, instead of calculated, if so desired.

Core Length / Ext. Dia.	Output Func.	V.A. Input	(Ext Dia.)⁴	Ext. Dia.	Net Core Length	Ext. Dia.	Core Length		Ducts	Width Ducts	Net Core Length
x	S_0	T	$A_1{}^4$	A_1	C_1	A	C	B	D	E	F
$\dfrac{R_0}{\cdot 5 \times L}$ (·57 if Ducts)	Tab. V	$\dfrac{H.P \times 746}{K \times L_1}$	$\dfrac{T}{\cdot 664 \times x \times 0 \times S_0 \times R_0 \times \sqrt{H_{16} \times D_1 Z^3}}$	$\sqrt[4]{A_1{}^4}$	$1 \cdot 1 x A_1$	Nearest practicable	Nearest practicable	$R_0 A$	data	data	$\cdot 9(C-DE)$
$\dfrac{\cdot 7}{\cdot 5 \times 4}$	·01932	$\dfrac{5 \times 746}{\cdot 87 \times \cdot 85}$	$\dfrac{5050}{\cdot 664 \times \cdot 35 \times \cdot 96 \times \cdot 01932 y \cdot 7 \times \sqrt{\cdot 00005 \times \cdot 3 \times 1000^3}}$	9·25	3·25	10	3·5	7	0	0	3·16
·35		5050	7400								
$\dfrac{\cdot 7}{\cdot 5 \times 6}$	·01932	$\dfrac{500 \times 746}{\cdot 9 \times \cdot 92}$	$\dfrac{450,000}{\cdot 664 \times \cdot 23 \times \cdot 96 \times \cdot 01932 \times \cdot 7 \times \sqrt{\cdot 00005 \times \cdot 3 \times 1000^3}}$	36·9	8·5	38	8·75	26·7	0	0	7·9
·23		450,000	$1 \cdot 86 \times 10^6$								

Sheet II. Determination of the Leading Dimensions and the Flux

The procedure is as follows—

First calculate the ratio $\dfrac{\text{core length}}{\text{ext. diam.}}$ from the formula deduced above, then, taking S_0 from the table and calculating the estimated volt-amperes input from the *H.P.* and estimated efficiency and power factor, substitute these values in the input formula. From this (*a*) the external diameter and (*b*) the core length may be calculated.

The nearest practicable values to this may then be taken from a manufacturer's list of pattern dimensions, either so as to allow some margin, or so as to give the same A^3F. Quite wide departures from the ideal core length may be permitted

In actual practice it is seldom that this preliminary calculation will be needed, as in most cases the external diameter and core length will be chosen as follows—

A manufacturer's list will be at hand, giving standard machines of approximately the required horse-power and speed. Referring to this list we can at once see that the proposed design must be in " frame size so-and-so."

But this preliminary calculation is useful—

1. When it is desired to *revise* a list and ascertain whether it really represents the greatest economy possible.

2. When two different types of machine are to be compared as regards their practical merits. It is then desirable that calculations of both should be made on rigidly uniform principles, empirical decisions being reduced to a minimum.

Sheet III. Determination of the Leading Dimensions and the Flux (*contd.*)

Having determined the external diameter and core length, we proceed to determine the flux and slot dimensions. These may be derived from the tables as shown on the calculation sheet, or may be decided empirically. While those having little experience of their use, tend to distrust the tables and to rely on their own " judgment," yet the longer one's experience of the tables becomes, the more one recognizes how great a help they are. If the tables are used, the two columns headed " Revised gap diameter " and " Revised slot width " become mere checks, since, if correctly worked, they give the same results as the tables.

In some cases the dimensions derived from the tables give too high leakage (the second example calculated is chosen to give an example of this) and it becomes necessary to cut down the slot depth to correct this. This is practically the only remnant of

SHEET III

Slot Depth	Depth Factor	Flux Factor	Flux	Core Depth	Revised Gap Diameter	Revised Slot Width
WI_r	U_o	H_o	H	C_r	B_r	W_r
$V_o \times A$ or Data	Tab. III	Tab. II	$H_o \times F \times A$	$\dfrac{H}{2 \times W_o \times F}$	$A - 2(C_r + WI_r)$	$\dfrac{1}{I}\left(3{\cdot}1416\,B_r - \dfrac{H}{{\cdot}07 \times F}\right)$
·663	0663	067	$067 \times 10 \times 3{\cdot}16$ $2{\cdot}12$	$\dfrac{2{\cdot}12}{2 \times {\cdot}4 \times 3{\cdot}16}$ ·837	$10 - 2({\cdot}837 + {\cdot}663)$ 7·0	$\dfrac{1}{48}\left(3{\cdot}1416 \times 7 - \dfrac{2{\cdot}12}{{\cdot}07 \times 3{\cdot}16}\right)$ ·258
0663×38 2·52	0663	067	$067 \times 38 \times 7{\cdot}9$ 20	$\dfrac{20}{2 \times {\cdot}4 \times 7{\cdot}9}$ 3·17	$38 - 2(3{\cdot}17 \times 2{\cdot}52)$ 26·65	$\dfrac{1}{72}\left(3{\cdot}1416 \times 26{\cdot}65 - \dfrac{20}{{\cdot}07 \times 7{\cdot}9}\right)$ ·664
1.5 (data)			24	$\dfrac{24}{2 \times {\cdot}48 \times 7{\cdot}9}$ 3·17	$38 - 2(3{\cdot}17 + 1{\cdot}5)$ 28·66	$\dfrac{1}{96}\left(3{\cdot}1416 \times 28{\cdot}66 - \dfrac{24}{{\cdot}07 \times 7{\cdot}9}\right)$ ·49

empiricism left in the present method of calculation. It arises because the number of variables in the leakage function is too great to permit of ready tabulation, so that we cannot predict the leakage from tables prior to calculating dimensions as we can other features of the machine.

In the second example, then, we first calculate strictly from the tables giving a deep slot (2·52 in.), and go forward with the calculation till we reach the leakage. This being too high, we reduce the slot depth to 1·5 (raising W_0 to ·48 and H to 24), and recalculate, obtaining the desired result. It should be noted that the flux H is in megalines and is the total flux = flux per pole × number of poles.

Sheet IV. Rotor Slots

Sheet 4 gives formulae for the calculation of three different types of rotor slot.

Type I. Relatively wide and shallow, characterized by the fact that the magnetic density at a point one-third up from the tooth root is the same as the stator tooth density at the narrowest part of the stator teeth.

This will be by far the most usual slot to be employed.

Type II. Deep and narrow. This has the same area as Type I, but the magnetic density at the tooth root is the same as the stator tooth density (at the narrowest part of the stator teeth).

Type III. Parallel tooth. Where there is a large number of round conductors per slot this type may be employed as having a slot area 50 per cent greater than slots I or II, and a magnetic tooth density equal to that of the stator teeth.

Types I and II are rectangular, and suitable where strip windings are to be used.

In the case of the second example worked, it is necessary to reduce the slot depth in the ratio of ·65 to 1·0 in order to keep the leakage down. This final revision will be found in the last three columns.

Sheets V and VI. Winding Calculations

The procedure followed is to calculate the turns per section from the given volts per section and the fundamental voltage formula, and from this to determine the conductors per slot given the number of slots and sections. This will frequently not be exactly a whole number, and, since it is obvious that a fraction of a conductor per slot is impossible, the nearest whole number must be chosen. Where the number of conductors per slot is large, no harm results from this procedure. Its only result is that the flux (and consequently the magnetic densities) as worked out in the check calculations from the number of conductors finally chosen will differ slightly from that originally derived from the tables. Where the number of

SHALLOW (Usually Best)		DEEP		TAPER				
Type I—Rotor Slot		Type II—Rotor Slot	Type II and Type III	Type III—Rotor Slot				
Width	Depth	Width	Depth	Tooth Width	Slot Area	Depth Ratio	W_1	D_1
W_2	D_2	W_2	D_2	N_2		p	$(2-p)W_2$	pD_2
$\dfrac{1}{H_2}\left(2\cdot45B - \dfrac{11\cdot1H}{F}\right)$	$\cdot16B - \dfrac{\cdot73H}{F}$	$\dfrac{1}{H_2}\left(1\cdot57B - \dfrac{H\times7\cdot1}{F}\right)$	$\cdot25B - \dfrac{1\cdot13H}{F}$	$\dfrac{H}{\cdot07\times F\times H_2}$	$1\cdot5W_2D_2$	data		
$\dfrac{1}{36}\left(2\cdot45\times7 - \dfrac{11\cdot1\times2\cdot12}{3\cdot16}\right)$	$1\cdot12 - \dfrac{\cdot73\times2\cdot12}{3\cdot16}$							
$\cdot27$	$\cdot625$							
$\dfrac{1}{96}\left(2\cdot45\times26\cdot65 - \dfrac{11\cdot1\times20}{7\cdot9}\right)$	$\cdot16\times26\cdot65 - \dfrac{\cdot73\times20}{7\cdot9}$							
$\cdot39$	$2\cdot41$							
$\dfrac{1}{120}\left(2\cdot45\times28\cdot66 - \dfrac{11\cdot1\times24}{7\cdot9}\right)$	$\cdot16\times28\cdot66 - \dfrac{\cdot73\times24}{7\cdot9}$					$\cdot65$	$1\cdot35\times\cdot3$	$\cdot65\times2\cdot36$
$\cdot3$	$2\cdot36$						$\cdot405$	$1\cdot53$

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V. per Sec.	T. per Sec	Sections	Conductors per Slot	Best Area Wire	Nearest Area Wire Gauge	Gauge No. or Diam.
N	Q	R_1	S	F	G_1	H_1
data	$\dfrac{N \times L_1 \times 100}{4\cdot44 \times O \times P \times H}$	data	Nearest whole number to $\dfrac{2Q \times R}{I}$	$\dfrac{D_1 \times Wr \times W_1r}{S}$	Table VI or VII	Table VI or VII
57·5	$\dfrac{57\cdot5 \times 4 \times 100}{4\cdot44 \times \cdot96 \times 50 \times 2\cdot12}$	12	$\dfrac{2 \times 51 \times 12}{48}$	$\dfrac{\cdot3 \times \cdot663 \times \cdot258}{25}$	·00212	Diam. ·052
	51		25	·00205		(17½ S W G)

Rotor V. per Sec	Sec. Ch. F	Rotor T per Sec	Sections	Conductors per Slot	Best Area Wire	Nearest Area Wire Gauge	Gauge No. or Diam
E_2	O_2	Q_2	R_2	S_2	F_2	G_2	H_4
data	data	$\dfrac{E_2 \times L_1 \times 100}{4\cdot44 \times O_2 \times P \times H}$	data	Nearest whole number to $\dfrac{2Q_2 \times R_2}{H_2}$	$\dfrac{F_7}{S_2} \times \left\{ \begin{matrix} H_7D_1 \text{ or} \\ W_2D_2 \text{ or} \\ 1\cdot5W_2D_2 \end{matrix} \right\}$	Table VI or VII	Table VI or VII
27·	·96	$\dfrac{27\cdot5 \times 4 \times 100}{4\cdot44 \times \cdot96 \times 50 \times 2\cdot13}$	12	$\dfrac{2 \times 24 \times 12}{36}$	$\dfrac{\cdot3 \times \cdot27 \times \cdot625}{16}$	·00322	Diam ·064
		24		16	·00316		(No. 16 S.W.G.)

WINDING CALCULATIONS (II)

V. per Sec.	T. per Sec.	Sections	Conductors per Slot	Best Area Wire	Nearest Area Wire Gauge	Gauge No. or Diam.
N	Q	R	S	F	G_1	H_1
data	$\dfrac{N \times L_1 \times 100}{4.44 \times O \times P \times H}$	data	Nearest whole number to $\dfrac{2Q \times R}{I}$	$\dfrac{D_1 \times W_r \times W_1 r}{S}$	Table VI or VII	Table VI or VII
3?.7	$\dfrac{38.7 \times 6 \times 100}{4.44 \times .96 \times 50 \times 20}$ = 5.47 ; $\dfrac{38.7 \times 6 \times 100}{4.44 \times .96 \times .83 \times 50 \times 24}$ = 5.47 [1]	18	$\dfrac{2 \times 5.47 \times 18}{72} = 2.74$; $\dfrac{2 \times 5.47 \times 18}{96} = 2.02$			.6 × .4

Rotor V. per Sec.	Sec. Ch. F.	Rotor T. per Sec	Sections	Conductors per Slot	Best Area Wire	Nearest Area Wire Gauge	Gauge No. or Diam.
E_2	O_2	Q_2	R_2	S_2	F_2	G_2	H^4
data	data	$\dfrac{E_2 \times L_1 \times 100}{4.44 \times O_2 \times P \times H}$	data	Nearest whole number to $\dfrac{2Q_2 \times R_2}{H_2}$	$\dfrac{F_7}{S_2} \times \begin{Bmatrix} W_1 D_1 \text{ or} \\ W_2 D_2 \text{ or} \\ 1.5 W_2 D_2 \end{Bmatrix}$	Table VI or VII	Table VI or VII
48.5	.83	$\dfrac{48.5 \times 6 \times 100}{4.44 \times .83 \times 50 \times 24}$ = 6.9	18	$\dfrac{2 \times 6.9 \times 18}{120}$; $\dfrac{}{2}$			.6 × .3

[1] Take chord factor .83 pitch 1–11

conductors per slot is small, however, a different procedure must be adopted, as described below. The number of conductors per slot being determined, the best area of each conductor is derived from the slot area and the space factor determined by workshop experience. Referring to the wire table, the size wire nearest in area to that theoretically determined is chosen, the effect of this choice on the space factor becoming evident in the check calculation. For a wire-wound rotor the calculation pursues exactly the same course.

In the second example chosen, this method cannot be followed since the calculation made direct from the tables for an arbitrarily chosen number of slots (72) gives 2·74 conductors. To take the nearest even number of conductors (2) (*even*, because we wish to use a two coil per slot drum winding) would mean a very large percentage change in the flux. To accommodate these circumstances we have two resources : (*a*) to vary the number of slots, (*b*) to vary the chord factor. By adopting 96 slots the number of conductors is reduced from 2·74 to 2·0, and by reducing O from ·96 to ·83, giving a pitch of 1–11, the increased flux of 24 megalines is obtained, which is needed to give adequate overload capacity.

In calculating the rotor winding of the second example, we simply take the slot size determined as above, and place in it two conductors so proportioned as to fit it comfortably, the rotor voltage being determined simply by the ratio of the slots if we take the rotor chord factor $\times$ Br. Co. the same as the stator. When the rotor copper loss comes to be calculated we can check that the amount of rotor copper is sufficient.

Sheet VII. Commencement of Check

What may be called the " short calculation " is now finished. That is, the dimensions of the punching have been determined, and a suitable winding to fit the slots has been worked out. This may, in some cases, be enough for estimating purposes, but, before the design can be regarded as finished, a careful check must be carried out for several purposes—

1. To determine the characteristics, both primary [heating and (in the case of an induction motor) overload capacity] and secondary (power factor and efficiency).

2. To see that no arithmetical errors have crept into the calculation. The effect of an arithmetical error might be to proportion the machine in a worthless manner, so that the money expended in building it might be wholly wasted.

3. To check that all workshop requirements, such as ease of fit of windings in slots, etc., are met.

The check is commenced by calculating the magnetic leakage. Many different formulae for this purpose have been published, usually in a form giving the " leakage coefficient " or ratio $\sigma =$

SHEET VII

Leakage Function	Basal Short-circuit Watts	Check H.P.
L_3	Q_3	P_{11}
$K_1 \times L \sqrt{\dfrac{W_1{}^r}{W^r} + \dfrac{D_2}{W_2}\left(\dfrac{1}{r} + \dfrac{1}{H_2}\right)} + \dfrac{K_2 B}{F \times L}$	$\dfrac{26 \cdot 25}{L_3} \times H^2 \times \dfrac{Z_1}{F}$	$\dfrac{Q_3}{3740}$
$1 \cdot 66 \times 4 \sqrt{\dfrac{\cdot 663}{\cdot 258} + \dfrac{\cdot 625}{\cdot 27}\left(\dfrac{1}{48} + \dfrac{1}{36}\right)} + \dfrac{1 \cdot 21 \times 7}{3 \cdot 16 \times 4}$	$\dfrac{26 \cdot 25}{1 \cdot 38} \times 2 \cdot 12^2 \times \dfrac{1500}{3 \cdot 16}$	$\dfrac{40,800}{3740}$
$\cdot 71 + \cdot 67$	$40,800$	$10 \cdot 9$
$1 \cdot 66 \times 6 \sqrt{\dfrac{2 \cdot 52}{\cdot 664} + \dfrac{2 \cdot 41}{\cdot 39}\left(\dfrac{1}{72} + \dfrac{1}{96}\right)} + \dfrac{1 \cdot 02 \times 26 \cdot 65}{7 \cdot 9 \times 6}$	$\dfrac{26 \cdot 25}{1 \cdot 349} \times 20^2 \times \dfrac{1000}{7 \cdot 9}$	$\dfrac{990,000}{3740}$
$\cdot 772 + \cdot 577$	$990,000$	266
$1 \cdot 66 \times 6 \sqrt{\dfrac{1 \cdot 5}{\cdot 49} + \dfrac{1 \cdot 53}{\cdot 405}\left(\dfrac{1}{96} + \dfrac{1}{120}\right)} + \dfrac{1 \cdot 02 \times 28 \cdot 66}{7 \cdot 9 \times 6 \times 1 \cdot 6}$	$\dfrac{26 \cdot 25}{\cdot 875} \times 24^2 \times \dfrac{1000}{7 \cdot 9}$	$\dfrac{2,180,000}{3740}$
$\cdot 49 + \cdot 385$	$2,180,000$	585

(magnetizing current)/(short-circuit current). But the magnetizing current is a very variable factor, depending largely on unpredictable differences of air-gap between machine and machine. If we calculate the above leakage coefficient and arrive, say, at the result $\sigma = \cdot05$, then if the magnetizing current is 10 amp. the short-circuit current $\left(\dfrac{\text{Magn. curr.}}{\sigma}\right)$ will be 200 amp., while, if through a slight variation of air-gap it becomes 11 amp., we are led to believe that the short-circuit current will be 220 amp. which is not true at all, since the short-circuit current is scarcely at all a function of the magnetizing current. It is quite untrue, for instance, that the overload capacity can be increased by increasing the air-gap (which increases the magnetizing current). It depends almost entirely on the short-circuit current. For this reason it seems better to the writer to work out formulae giving the short-circuit watts or volt-amperes direct, and such a formula is presented in sheet 7, which may be relied on to give results in close accordance with experiment as it has been derived from a very wide range of machines.

Having calculated the short-circuit watts, a " check H.P." may be worked out from the very rough formula given. This is a H.P. rating which allows 100 per cent overload capacity after ample allowance has been made for reduction of overload capacity by primary resistance and by allowing for the magnetizing current. So long as this check H.P. is *in excess of or equal to* the H.P. given in the data sheet, the machine will have 100 per cent overload capacity or more.

In the case of the first example the check H.P. is 10·9 nearly double the rated H.P., so that the machine will have nearly 200 per cent overload capacity. Clearly, in this case, it is heating that determines the rating.

In the second case, however, the first calculation, made direct from the tables, gives a seriously defective horse-power, a figure of only 266 being reached where we should have at least 500. Examining the calculations, we see that the leakage function is if anything higher than that of the tiny machine first calculated, this being due (i) to the deep stator and rotor slots, (ii) to the small number of slots. Since the overload capacity is proportioned to H^2 a slight increase in H will clearly increase the overload capacity very much. The machine is accordingly recalculated with more and shallower slots, and a somewhat increased value of H when we get $P_{11} = 585$ which is quite satisfactory.

SHEET VIII

We first calculate the internal diameter which would be permissible if the rotor core density were equal to the stator core density. This should be large enough not only to admit the shaft,

SHEET VIII

Int. Diam. (calculated)	Int. Diam. (actual)	Stator Core Density	Stator Tooth Density	Leakage Coefficient (σ)	Effective Gap	Length Condr	V. per Condr.
Z_{33}	Z_3	T_1	C_4	$D\sigma$	A_0	M_3	E_1
$\dfrac{2(Br + W_1r)}{\div(A + 2D_2)}$	to suit Dwgs.	$\dfrac{H}{F \times L(A - Br - 2W_1r)}$	$\dfrac{H}{(3.1416Br - Wr \times I)F}$	$\dfrac{6560 \times P \times A_0 \times H^2}{Br \times F \times Q_3}$	$\dfrac{I_6 \times St}{1 - 47\dfrac{S_1 \times I}{3.1416Br} \times \sqrt{\dfrac{S_1}{I_6}}}$	$C + \dfrac{5.5Br}{L}$	$\dfrac{OPH}{45L}$
$\dfrac{2(7 + .663)}{= 10 + 2 \times .625}$ $= 15.35 - 11.25$		$\dfrac{2.12}{3.16 \times 4(10.7 - 1.326)}$	$\dfrac{2.12}{(3.1416 \times 7 - .258 \times 48)3.16}$	$\dfrac{6560 \times 50 \times .0355 \times 4.5}{7 \times 3.16 \times 40,800}$	$\dfrac{.02 \times 1.2}{1 - 47\dfrac{.125 \times 48}{3.1416 \times 7} \times \sqrt{\dfrac{.125}{.02}}}$	$3.5 + \dfrac{5.5 \times 7}{4}$	$\dfrac{.96 \times 50 \times 2.12}{45 \times 4}$
4.1		.1	.063	.0585	.0355	13.1	.557
$\dfrac{2(28.66 + 1.5)}{-(38 + 3.06)}$		$\dfrac{24}{7.9 \times 6(38 - 28.66 - 3)}$	$\dfrac{24}{7.9(3.1416 \times 28.66 - 49 \times 96)}$	$\dfrac{6560 \times 50 \times .049 \times 24^2}{28.66 \times 7.9 \times 2,180,000}$	$\dfrac{.04 \times 1.2}{1 - 47\dfrac{.04 \times 96}{3.1416 \times 28.66} \times \sqrt{\dfrac{.04}{.04}}}$	$8.75 + \dfrac{5.5 \times 28.6}{6 \times 1.6}$	$\dfrac{.96 \times 83 \times 50 \times 24}{45 \times 6}$
19.26		.08	.0705	.0187	.049	16.4	3.55

but to allow of any longitudinal vent holes which it is desired to use. The actual internal diameter is then chosen to suit the drawings. A table of shaft sizes, spider dimensions, etc., will usually be at hand to facilitate this, and should not be greater than this.

The stator core density and tooth density are next checked to make sure that they agree with those derived from the table, and the leakage coefficient calculated. This is called here $D\sigma$ for uniformity instead of (σ). This involves the calculation of the effective gap. The formula given involves the saturation factor St, which may be calculated or derived from data, and a further function which the writer has found to work well derived from Carter's well-known curve by replacing it by the nearest parabola. Having obtained the effective gap, we calculate the magnetizing volt-amperes and divide by Q_3 as calculated above.

Having obtained $D\sigma$ and the overload capacity, the power factor, which is, neglecting the primary resistance, a function of these two quantities above, may be obtained from the table.

The effect of the primary resistance, which has been neglected, is slightly to *raise* the power factor so that the value obtained from the table is slightly lower than will be realized on test if all other results are exact.

The primary copper loss and heating constant must next be worked out. The values given for the heating constant work well for a wide range of machines, but each designer will prefer to choose for himself the heating constant he prefers to work to, since on this point almost more than any other does the excellence of the design depend.

Sheets IX and X

The rotor copper loss is next calculated in the same way as the stator copper loss and the iron loss calculated, the quantities F_3 and B_3 being derived from tables when we know T_1 and C_4, with the exception of a " quality factor " q depending on the grade of iron. For instance, if $q = 1$ for an ordinary grade of iron, for silicon steel (Stalloy), we shall have $q = \cdot 5$. All that is claimed for these tables is that the result is *proportional* to the iron losses observed on test, and that by a suitable choice of q close accordance may be reached.

Besides the ordinary iron loss, there is a second " pulsation " loss due to the high frequency oscillations in the iron of the teeth owing to their rapidly passing the teeth on the other member.

This consists of two components, one consisting of an empirical factor multiplied by the gap area, and the other depending on the width of the slot opening. This second component is usually small for semi-closed slot machines, but for open slot machines it may mount up to very large figures. No great accuracy is claimed for the result of this formula, but it should always be worked

Stator Copper Area	Stator Copper Loss	Heating Constant	Power Factor	Length Rotor Conductor	Area all Rotor Conductors	Rotor Copper Loss
J_6	P_3	H_{1b}	L_1	M_4	G_7	P_4
$G_1 \times S \times I$	$.8 \times \left(\dfrac{T}{E_1}\right)^2 \times \dfrac{M_3}{J_6} \times 10^{-6}$	$\dfrac{P_3}{9.8 B_r{}^2 M_J \times Z_1}$	Table IX	$C + \dfrac{5.5 B_r}{L_2}$	$G_2 \times S_2 \times H_2$	$.8 \times \left(\dfrac{H.P \times 746}{E_1}\right)^2 \times \dfrac{M_1}{G_1} \times 10^{-6}$
$.00212 \times 25 \times 48$	$.8 \times \left(\dfrac{5050}{.557}\right)^2 \dfrac{13.1}{2.55} \times 10^{-6}$	$\dfrac{340}{9.8 \times 49 \times 13.1 \times 1500}$		$3.5 + \dfrac{55. \times 7}{4}$		$.8 \times \left(\dfrac{5 \times 746}{.557}\right)^2 \dfrac{13.1}{1.86} \times 10^{-6}$
2.55	340	$.000036$	$.86$	13.1	$.00322 \times 16 \times 36$	252
$.24 \times 2 \times 96$	$.8 \times \left(\dfrac{450{,}000}{3.55}\right)^2 \dfrac{16.4}{46.2} \times 10^{-6}$	$\dfrac{4550}{9.8 \times 28.66^2 \times 16.4 \times 1000}$		16.4	1.86	$.8 \times \left(\dfrac{500 \times 746}{3.55}\right)^2 \dfrac{16.4}{43.2} \times 10^{-6}$
46.2	4550	$.0000345$	$.93$		$.18 \times 2 \times 120$	3350
					43.2	

SHEET X

Core Loss per Cub. Inch F_3	Tooth Loss per Cub. Inch B_3	Iron Quality Factor q	Core Loss W_3 (F₃ from Table VIII)	Tooth Loss W_2 (B₃ from Table VIII)	Pulsation Loss W_4	F. and W. Factor S_3
Table VIII opposite value of T_1 above	Table VIII opposite value of C_4 above	data	$\cdot7854(A^2-(B_1+2W_{11})^2)F\times F_3\times q$	$W_{11}(3\cdot1416B_r-W_rl)F\times B_3\times q$	$\left[\cdot036R_0+360\left(\dfrac{I\times S_p\times H_0}{L\times I_6\times R_0}\right)^2\times10^{-6}\right]\times P\times FA\times q$	data
1·62	1·62	1·0	$\cdot7854[(100-(7+2\times\cdot663)^2]\dfrac{3\cdot16\times1\cdot62}{7\cdot9\times1\cdot05}$ 125	$\cdot663(3\cdot1416\times7-\cdot258\times48)\,3\cdot16\times1\cdot62$ 32.8	$\left[\cdot036\times\cdot7+360\left(\dfrac{48\times\cdot125\times\cdot067}{4\times\cdot02\times\cdot7}\right)^2\times10^{-6}\right]$ $(\cdot025+\cdot018)\times50\times3\cdot16\times10\times1\cdot0$ 68	·015
1·05	2·05	1·0	$\cdot7854(38^2-(28\cdot66+3)^2)\,7\cdot9\times1\cdot05$ 2870	$1\cdot5(3\cdot1416\times28\cdot66-\cdot49\times96)\times7\cdot9\times2\cdot05$ 1050	$\left(\cdot036\times\cdot75+360\dfrac{48\times\cdot04\times\cdot08}{6\times\cdot04\times\cdot75}\right)^2$ $[\cdot027+\cdot000260]\times50\times7\cdot9\times38$ 407	

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Stator Copper Loss	Rotor Copper Loss	Iron Loss	Puls. Loss	F. and W. Loss	Output	Input	Efficiency	Primary Full Load Current	Secondary Full Load Current
1	2	3	4	5	6		K_1	A_s	A_2
				$S_3 \times H.P. \times 746$	$H.P. \times 746$	sum of items 1 to 6	$\dfrac{Output}{Input}$	$\dfrac{H.P. \times 746}{K_1 \times L_1 \times N \times R}$	$\dfrac{H.P. \times 746}{E_2 \times R_2}$
340	252	158	68	56	3740	3740 340 252 158 68 56 — 4614	$\dfrac{3740}{4614}$ .81	$\dfrac{5 \times 746}{.86 \times .825 \times 57.5 \times 12}$ = 7.63	$\dfrac{5 \times 746}{27 \times 12}$ 11.5
4550	3350	3920	405	560	373,000	4550 3350 3920 405 560 373,000 — 385,785	.97	$\dfrac{373,000}{.93 \times .97 \times 38.7 \times 18}$ 600 amp	$\dfrac{373,000}{48.5 \times 18}$ 427

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out. If this second component works out to a large value, it is an infallible indication of danger. The writer has known large and costly machines entirely scrapped as the result of high iron losses incurred by neglecting to study the pulsation losses due to open slots before construction.

Sheet XI

The various losses are next added up and the final full load efficiency calculated. It is useful also to work out the primary and secondary currents for use in the specification.

WINDING SPECIFICATION

Date............

Ext. diam. (*A*) **10.** Core length (*C*) **3·5.** Ducts (*D* × *E*) **0.**
H.P. **5.** R.p.m. (*Z₁*) **1500.** Line volts (*V*) **400.** Cycles (*P*) **50.**
Phases **3.** Amps. (*A*) **7·1.**

Patt. **E.V.** Rating **Cont.** Wiring diag. Punch dwg. (**below**).
Amps. (*A₂*) **11·5.**

Stator Spec. No. Slots (*I*) **48.** Poles (*L*) **4.** Slots wound (*I*) **48.**
Groups (*R*) **12.** Cond. slot (*S*) **25.** Size wire (*H₁*) **17½.** Diag.

Rotor Spec. No. Slots (*H₂*) **36.** Poles (*L₁*) **4.** Slots wound (*H₂*) **36.**
Groups (*R₂*) **12.** Cond. slot (*S₂*) **16.** Size wire (*H₄*) **16.** Diag.

Check data pri. volts per sec. (*N*) **57·5.** Ch. F. (*O*) **·96.** Sec. V.
per sec. (*E₂*) **27.** Sec. Ch. F. (*O₂*) **·96.**
Length F. sec. condrs. (**L₂**) **4.** Rotor Ch. F. (*A₇*) **·96.** Stator
parallels (**W**) **1.** Rotor parallels (**W₂**) **1.**
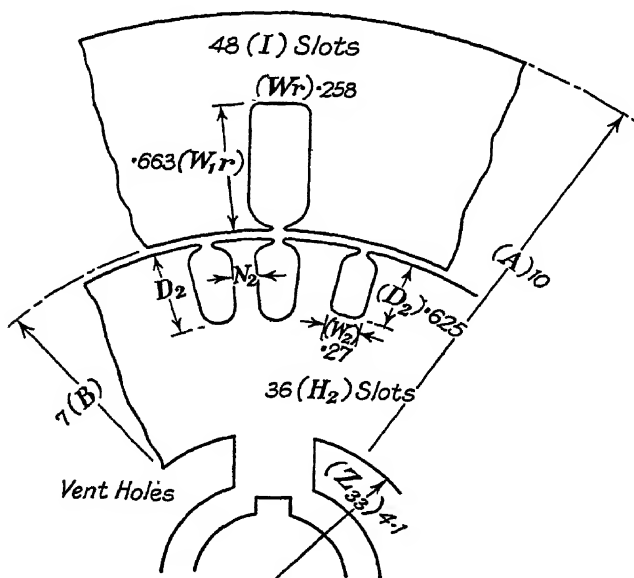
Fig. 228

SHEET XII. WINDING, SPECIFICATION, AND PUNCHING DRAWING

The data sheet for the winding specification is drawn up with the symbol for every entry in brackets next to the title of the entry as " r.p.m.(Z_1) " Similarly, the symbols for the different punching dimensions are entered on the punching drawing. If this is done, the particulars of both winding specification and punching drawing can be filled up direct from the corresponding columns of the design calculation already made.

At the bottom of the winding specification certain further data are given, the symbol for each entry being again given next to the the title of the entry. These can be entered up from the original data sheet or from the corresponding columns of the design.

When   1. The Winding Specification,
         2. The Punching Drawing,
         3. The Further Data

are entered up, it is possible to work a further check from these alone without referring in any way to the original design. This is done on the next sheet.

By this plan we work out a second time and by different formulae many of the most important quantities calculated in the original design, and do it direct from the figures that are to be used in manufacture, thus checking, not only the design, but clerical errors in filling in the sheet.

This check is, of course, applied without consulting the original calculation in any way. It might even be carried out by a different person, though this is not really necessary. A check calculation has not been worked on the second example.

CHECK SHEETS I AND II.

An endeavour should be made to keep this check calculation on a single sheet, working, if necessary, on both sides of it. The check calculations can then be filed to correspond with the winding specifications. and form a convenient compendium of the design for ready reference.

The points checked are as follows—

Stator and rotor magnetic densities in tooth and core.
      ,,   ,,   space factor.
      ,,   ,,   current densities.
      ,,   ,,   copper losses.

CHECK SHEET I

(Band 1)

Net Core Length	Volts per Section	Stator Chord Factor	Turns per Section	Flux M L	Stator Core Density	Rotor Core Density	Stator Average T Density	Rotor Tooth Width
F	N	O	Q	H	T_1	T_3	C_4	N_2
$.9(C-DE)$	data	data	$\dfrac{S\times I_a}{2R}$	$\dfrac{N\times L\times100}{4.44\times O\times P\times Q}$	$\dfrac{H}{F\times L(A-B_f-2lW_{1r})}$	$\dfrac{H}{F\times L(B_f-Z_{33}-2D_2)}$	$\dfrac{H}{(3.1416\,B_f-W_r\,l)F}$	$3.1416\left(B_f-\left\{\dfrac{2_w}{1.33}D_2\right\}\right)-W_2$ over H_2 Factor 2 gives width at root Factor 1.33 gives width ⅓ up
$3.5\times.9$	57.5	$.96$	$\dfrac{25\times48}{2\times12}$	$\dfrac{57.5\times4\times100}{4.44\times.96\times50\times50}$	$\dfrac{2.19}{4\times3.16(10-7-1.326)}$	$\dfrac{2.19}{4\times3.16(7-4.1-1.25)}$	$\dfrac{2.19}{(3.1416\times7-.258\times.48)3.16}$	$\dfrac{3.1416(7-1.33\times.625)}{36}$
3.16			50	2.19	$.098$	$.105$	$.0715$	$-.27$ $.54-.27=.27$

(Band 2)

Rotor Mean Slot Width	Rotor Space Factor	Length Stator Condrs	Rotor Length Factor	Length Rotor Condrs	Stator Area Cop	Rotor Chord Factor	Rotor Area Cop	Stator Volts per Condrs	Stator Parallels
W_2	F_7	M_3	L_2	M_4	J_6	A_7	K_6	E^1	W
$\dfrac{3.1416(B_f-D_2)-N_2}{H_2}$ Use for 1 taper slot or use W_2 (parallel slot)	$\dfrac{G_2\times S_2}{W_2\times D_2}$	$C+\dfrac{5.5\,B_f}{L}$	data	$C+\dfrac{5.5\,B_f}{L}$	$G_1\times S\times I_a$	data	$S_2\times G_2\times H_2$	$\dfrac{OPH}{45L_1}$	W
$.27$	$\dfrac{.00322\times16}{.27\times.625}$	$3.54+\dfrac{5.5\times7}{4}$	4	$3.54+\dfrac{5.5\times7}{4}$	$.00212\times25\times48$	$.96$	$\dfrac{.00322\times16}{\times36}$	$\dfrac{96\times50\times2.19}{45\times4}$	data
	$.305$	13.1		13.1	2.56		1.86	585	

(Band 3)

Rotor Avg. T Density	Stator Wire Area	Stator Space Factor	Rotor Wire Area
C_6	G_1	D_1	G_2
$\dfrac{H}{N_2\times H_2\times F}$	Tab. VI or VII	$\dfrac{G_1\times S}{W_r\times W_{1r}}$ Tab. VI or VII	Tab VI or VII
$\dfrac{2.19}{.27\times3.16\times36}$		$\dfrac{.00112\times25}{.258\times.663}$	
$.071$	$.00212$	$.31$	$.00322$

D_2 is always used for rotor slot width check for cases where other types of slot are used

<div align="center">CHECK SHEET II</div>

Stator Current Density	Rotor Current Density	Stator Copper Loss	Rotor Copper Loss
I_1	Q_2	W_5	H_7
$\dfrac{A_5}{1000w \times G_1}$	$\dfrac{A_2}{1000w_2 \times G_2}$	$\cdot 8 \times I_1{}^2 \times M_3 \times J_6$	$\cdot 8Q_2{}^2 \times M_4 \times K_6$
$\dfrac{7\,6}{1000 \times 00212}$	$\dfrac{11\cdot 5}{1000 \times 00322}$	$\cdot 8 \times (3\cdot 6)^2 \times 13\cdot 1 \times 2\,56$	$\cdot 8 \times (3\,56)^2 \times 13\cdot 1 \times 1\cdot 86$
$3\,6$	$3\,56$	$?50$	258

With a little experience any abnormalities in these quantities will at once be apparent. In general, the check sheet, *when completed but not before*, should be compared with the original calculation when any discrepancy at once gives the danger signal. A good rule is to insist that both in the original and in the check sheet the numerical values shall first be substituted in the algebraical formulae *and written down in plain figures* before any arithmetical operations are performed on them. This is an immense help in checking any discrepancy, if it occurs.

The process of calculation just given is within the capacity of anyone having enough elementary algebraical knowledge to understand the formulae, and a sound knowledge of arithmetic. Such calculations must always be undertaken under the direct supervision of an expert designer, but, subject to this condition, no electrical knowledge is needed for the calculations themselves. But it is obvious that such calculations cannot be performed without some mechanical aids—either a slide rule or a calculating machine. Now a fair amount of skill is required to use a slide rule (" it takes a good deal of learning " in everyday language) owing, as pointed out above, to its ambiguous decimal point and the difficulty of reading its fine divisions accurately.

These difficulties are both overcome by the use of a calculating machine, in which the result appears in plain figures, and with correct decimal point. If such a machine is used, and under proper supervision, the calculations are well within the capacity of a good lady-typist—especially one having a good experience of book-keeping, which teaches just those habits of accuracy which are so essential in designing. With such assistance the whole of the details of calculation are lifted from the shoulders of the designing engineer, and he is free to devote his attention to his true business of improving the product.

A few hints as to the best form of organization may be of service. Slight variations in the schedules are necessary for various purposes, for instance, for squirrel-cage motors, for strip-wound machines,

and so on. To be practically useful the schedules must be either printed or reproduced on some sort of multigraphing apparatus, slight variations being made in pen-and-ink. The engineer, of course, must always make himself responsible for these, since, however simple an algebraic transposition may be, the operator either cannot, or should not take the responsibility of doing it. A convenient plan is to compile a book of model calculations, containing an example of each variation, no matter how slight. If this is done, the simple statement on the data sheet that the calculation is to be made according to " Model $KC3$," say, will give all the information needed.

Table I gives particulars of the slot dimensions of a machine of unit external diameter for various ratios of gap diameter to external diameters.

A different table is devoted to each value of $W_o =$ core density × number of poles.

The rotor slot is of the deep and narrow type (Type II). But if it is desired to use Type I, the slot depth will be $\cdot 64 Z_m$ and the slot width $1 \cdot 56 Z_n$.

The last column gives the square root of the ratio of the stator slot area to the rotor slot area.

By examining these tables certain general conclusions may be arrived at.

1. For small values of R_o the stator slot area is much greater than the rotor slot area. This is especially marked for large values of W_o. For instance, with $W_o = 1 \cdot 2$ and $R_o = \cdot 66$ the stator slot has more than four times the area of the rotor slot, since $\dfrac{S_o}{T_o}$ is the *square root* of the ratio of slot areas.

This gives us some guidance in choosing R_o. For an induction motor in which the stator carries the magnetizing current, and may also have a lower space factor than that of the rotor, the stator slots may conveniently have about 40 per cent more area than the rotor slots. This means that $\dfrac{S_o}{T_o}$ should be about $1 \cdot 2$. For a synchronous or compensated motor with the magnetizing current carried by the rotor it should have the greater slot area.

The greater the value of W_o the larger must R_o be to give a predetermined value of $\dfrac{S_o}{T_o}$.

2. Other things being equal, the output is proportional to S_o. But S_o reaches a maximum for certain values of R_o. For instance, with $W_o = 1 \cdot 2$, a maximum is reached with R_o about $0 \cdot 68$. This gives us a second form of guidance in choosing R_o, but, of course, R_o should not be chosen to make S_o a maximum if this gives absurd slot dimensions.

3. W_o = core density (megalines per sq. in.) × number of poles clearly will be larger the greater the number of poles. Modern practice permits values for the core density up to ·1, so that the low values will only be required for very small numbers of poles (say 2 poles). For pole numbers higher than 12, extensions of the tables will be required. The tables themselves are independent of the system of units employed, but the value of W_o to which each table corresponds will be expressed by a different figure in cm. units. For instance—

Inch Units	Cm. Units	Inch Units	Cm Units
·2	·0319	·7	·111
·3	·0479	·8	·127
·4	·064	·9	·143
·5	·079	1·0	·159
·6	·096	1·1	·175
		1·2	·191

4. In all these tables the proportions have been determined on the assumption that the magnetic density at the tooth tip is ·07. If it is desired to vary the tooth density, the tables can still be used. Referring to the table for $W_o = 1·1$. If we desire to raise the tooth density, say, 10 per cent (from ·07 to ·077, say) without changing the core density, the following process must be adopted—

(*a*) Increase H_o by 10 per cent.

(*b*) Refer to a table in which W_o is 10 per cent less—$W_o = ·1$ in this case. Thus these tables are adaptable to any tooth density it is desired to adopt.

CASE OF NORMAL DENSITY ONE-THIRD UP FROM TOOTH ROOT

A case where a procedure analogous to the above is required is that in which the normal density is required, not at the narrowest part of the tooth (the tooth root), but at a radius exceeding this by one-third of the tooth length. Since the tooth is wider at this point this will permit a greater flux or, conversely for a given flux, a greater slot area.

If H_1 is the greater flux so permitted for unit external diameter, then

$$\frac{H_1}{3\cdot 1416\,(R_o + \tfrac{2}{3}V_o)} - Z_o = ·07$$

Now
$$Z_o = 3\cdot 1416 R_o - \frac{H_o}{·07}$$

$$U_o = \tfrac{1}{2}(1 - R_o) - \frac{H_o}{2W_o}$$

Substituting

$$H_1 = \cdot 07 \left(3 \cdot 1416 R_o + \frac{3 \cdot 1416 \times 2}{3} U_o - 3 \cdot 1416 R_o + \frac{H_o}{\cdot 07} \right)$$

or cancelling

$$H_1 = H_o + \cdot 147 U_o.$$

By this formula we may determine the value of H_1 required to give this density.

If it is desired that the core density should be unchanged a correspondingly lower value of W_o must be taken as discussed above.

These tables, however, are not accurate enough for all purposes. They are convenient to enable us to choose a reasonable approximate value for R_o, since they bring together all the relevant considerations. Having done this, it is better to refer to Tables II, III, IV, and V, giving H_o, U_o, Z_o, and S_o more in detail for the final values.

Next come the essential tables of wire gauge, given both for S.W.G. (English practice) and B. and S. (American practice). These do not call for extended comment. (Tables VI and VII.)

Table VIII is used in calculating iron losses.

It has already been pointed out that the power factor is a function of two variable, $D\sigma$ and the overload capacity. It is here tabulated as such. (Table IX.)

Table X is a short table to aid in calculating the saturation factor where necessary. It is the equivalent of a saturation curve, but more convenient to use on a calculating machine.

Table XI is useful in estimating the effect of a reduction of slot depth in the rotor.

TABLE I

$W_o = \cdot2$

(External Diameter, 1·0   Core Length, 1·0)

STATOR				ROTOR				GENERAL	
Gap Diam.	No Slots	Slot Depth	Slot Width	No. Slots	Slot Depth *DEEP NARROW TYPE*	Slot Width	M L Flux	Rel Putput Factor (H_b)	
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6\cdot28 Z_m = Z_n}{H_2}$	H_o		S_o over T_o
·62	I	·0780	$\dfrac{1\cdot3024}{I}$	H_2	·1045	$\dfrac{\cdot65626}{H_2}$	·0448	$\dfrac{\cdot01413}{\cdot01176}$	1·201
·66	I	·0653	$\dfrac{1\cdot4745}{I}$	H_2	·1153	$\dfrac{\cdot72408}{H_2}$	·0419	$\dfrac{\cdot01302}{\cdot01234}$	1·055
·7	I	0555	$\dfrac{1\cdot659}{I}$	H_2	·1307	$\dfrac{\cdot82205}{H_2}$	·0378	$\dfrac{\cdot01151}{\cdot01230}$	·936
·74	I	0475	$\dfrac{1\cdot8527}{I}$	H_2	·1412	$\dfrac{\cdot88673}{H_2}$	·0330	$\dfrac{\cdot00978}{\cdot01360}$	·719
·78	I	·0395	$\dfrac{2\cdot0472}{I}$	H_2	·1652	$\dfrac{1\cdot03746}{H_2}$	·0282	$\dfrac{\cdot00797}{\cdot01114}$	·715
·82	I	·0313	$\dfrac{2\cdot2403}{I}$	H_2	·1783	$\dfrac{1\cdot11982}{H_2}$	0235	$\dfrac{\cdot00624}{\cdot01054}$	·592
·86	I	·0245	$\dfrac{2\cdot4360}{I}$	H_2	1944	$\dfrac{1\cdot22083}{H_2}$	0182	$\dfrac{\cdot00442}{\cdot00689}$	·404
·9	I	·0173	$\dfrac{2\cdot5684}{I}$	H_2	·2101	$\dfrac{1\cdot319428}{H_2}$	·0131	$\dfrac{\cdot00278}{\cdot00689}$	·404

TABLE I (contd.)

$$W_o = ·3$$

(External Diameter, 1·0   Core Length, 1·0.)

Gap Diam.	STATOR			ROTOR				GENERAL
	No. Slots	Slot Depth	Slot Width	No Slots	Slot Depth	Slot Width	M L. Flux	Rel. Output Factor (H_5)
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6·28 Z_m}{H_2} = Z_n$	H_o	S_o over T_o
·62	I	·0861	$\dfrac{1·0623}{I}$	H_2	·0845	$\dfrac{·53066}{H_2}$	·0620	$\dfrac{·01899}{·01315}$ 1·436
·66	I	·0727	$\dfrac{1·2388}{I}$	H_2	·0986	$\dfrac{·61921}{H_2}$	·0584	$\dfrac{·01756}{·01445}$ 1·215
·7	I	·0602	$\dfrac{1·4290}{I}$	H_2	·1138	$\dfrac{·714664}{H_2}$	·0539	$\dfrac{·01588}{·01550}$ 1·024
·74	I	·0505	$\dfrac{1·6198}{I}$	H_2	·1308	$\dfrac{·821424}{H_2}$	·0477	$\dfrac{·01373}{·01565}$ ·877
·78	I	·0410	$\dfrac{1·8586}{I}$	H_2	·1480	$\dfrac{·92944}{H_2}$	·0414	$\dfrac{·01145}{·01536}$ ·745
·82	I	·0325	$\dfrac{2·0832}{I}$	H_2	1658	$\dfrac{1·04122}{H_2}$	·0345	$\dfrac{·00898}{·01439}$ ·625
·86	I	·0249	$\dfrac{2·3149}{I}$	H_2	·1842	$\dfrac{1·15678}{H_3}$	·0271	$\dfrac{·00647}{·01253}$ ·516
·9	I	·0175	$\dfrac{2·5484}{I}$	H_2	·2029	$\dfrac{1·27421}{H_2}$	·0915	$\dfrac{·00409}{·01037}$ ·394

TABLE 1 (contd.)

$$W_0 = \cdot 4$$

(External Diameter, 1·0 Core Length, 1·0)

STATUR				ROTOR				GENERAL	
Gap Diam.	No. Slots	Slot Depth	Slot Width	No Slots	Slot Depth	Slot Width	ML Flux	Rel. Output Factor (H_5)	
R_0	I	U_0	$\dfrac{Z_0}{I}$	H_2	Z_m	$\dfrac{6 \cdot 28 Z_m = Z_n}{H_2}$	H_0	$\dfrac{\cdots}{\cdots}$	S_0 over T_0
·62	I	·0999	$\dfrac{\cdot 9180}{I}$	H_2	·0731	$\dfrac{\cdot 45907}{H_2}$	·0721	$\dfrac{\cdot 02191}{\cdot 01322}$	1·658
·66	I	·0820	$\dfrac{1 \cdot 0673}{I}$	H_2	·0850	$\dfrac{\cdot 5338}{H_2}$	·0704	$\dfrac{\cdot 02088}{\cdot 01502}$	1·39
·7	I	·0663	$\dfrac{1 \cdot 2419}{I}$	H_2	·0988	$\dfrac{\cdot 620464}{H_2}$	·0670	$\dfrac{\cdot 01932}{\cdot 01663}$	1·162
·74	I	·0542	$\dfrac{1 \cdot 4569}{I}$	H_2	·1160	$\dfrac{\cdot 72848}{H_2}$	·0607	$\dfrac{\cdot 01703}{\cdot 01782}$	·956
·78	I	·0433	$\dfrac{1 \cdot 6874}{I}$	H_2	·1343	$\dfrac{\cdot 843404}{H_2}$	·0534	$\dfrac{\cdot 01441}{\cdot 01800}$	·8
·82	I	·0337	$\dfrac{1 \cdot 9318}{I}$	H_2	·1538	$\dfrac{\cdot 965864}{H_2}$	·0451	$\dfrac{\cdot 01150}{\cdot 01741}$	·66
·86	I	·0254	$\dfrac{2 \cdot 1920}{I}$	H_2	·1744	$\dfrac{1 \cdot 095232}{H_2}$	·0357	$\dfrac{\cdot 00841}{\cdot 01564}$	·54
·9	I	·0178	$\dfrac{2 \cdot 4585}{I}$	H_2	·1957	$\dfrac{1 \cdot 228996}{H_2}$	·0258	$\dfrac{\cdot 00536}{\cdot 01261}$	·425

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TABLE I (*contd.*)

$W_o = \cdot 5$

(External Diameter, 1·0. Core Length, 1·0)

Gap Diam.	STATOR			ROTOR				GENERAL	
	No. Slots	Slot Depth	Slot Width	No. Slots	Slot Depth	Slot Width	M.L. Flux	Rel. Output Factor (H_5)	
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6\cdot28 Z_m}{H_2} = Z_n$	H_o	S_o over T_o	
·62	I	·1101	$\dfrac{\cdot8066}{I}$	H_2	·0642	$\dfrac{\cdot40318}{H_2}$	·0799	$\dfrac{\cdot02395}{\cdot01288}$	1·86
·66	I	·0919	$\dfrac{\cdot9753}{I}$	H_2	·0763	$\dfrac{\cdot47916}{H_2}$	·0781	$\dfrac{\cdot02322}{\cdot01494}$	1·55
·7	I	·0734	$\dfrac{1\cdot1048}{I}$	H_2	·0880	$\dfrac{\cdot55264}{H_2}$	·0766	$\dfrac{\cdot02193}{\cdot01690}$	1·297
·74	I	·0587	$\dfrac{1\cdot3054}{I}$	H_2	·1040	$\dfrac{\cdot65312}{H_2}$	·0713	$\dfrac{\cdot01971}{\cdot01859}$	1·06
·78	I	·0459	$\dfrac{1\cdot5344}{I}$	H_2	·1221	$\dfrac{7\cdot6679}{H_2}$	·0641	$\dfrac{\cdot01699}{\cdot01908}$	·89
·82	I	·0350	$\dfrac{1\cdot7903}{I}$	H_2	·1425	$\dfrac{\cdot8949}{H_2}$	·0550	$\dfrac{\cdot01377}{\cdot01969}$	·6
·86	I	·0260	$\dfrac{2\cdot0735}{I}$	H_2	·1650	$\dfrac{1\cdot0362}{H_2}$	·0440	$\dfrac{\cdot01020}{\cdot01823}$	·559
·9	I	·0181	$\dfrac{2\cdot3714}{I}$	H_2	·1887	$\dfrac{1\cdot18504}{H_2}$	·0319	$\dfrac{\cdot00655}{\cdot01520}$	·43

TABLE I (*contd.*)

$W_o = .6$

(External Diameter, 1.0 Core Length, 1.0)

STATOR				ROTOR				GENERAL	
Gap Diam.	No. Slots	Slot Depth	Slot Width	No. Slots	Slot Depth	Slot Width	M.I. Flux	Rel. Output Factor (H_b)	
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6 \cdot 28 Z_m = Z_n}{H_2}$	H_o	S_o over T_o	
$\cdot 62$	I	$\cdot 1224$	$\dfrac{\cdot 7811}{I}$	H_2	$\cdot 0627$	$\dfrac{\cdot 39376}{H_2}$	$\cdot 0812$	$\dfrac{\cdot 02532}{\cdot 0127}$	$1 \cdot 99$
$\cdot 66$	I	$\cdot 1010$	$\dfrac{\cdot 8888}{I}$	H_2	$\cdot 0708$	$\dfrac{\cdot 44462}{H_2}$	$\cdot 0829$	$\dfrac{\cdot 0249}{\cdot 0147}$	$1 \cdot 69$
$\cdot 7$	I	$\cdot 0807$	$\dfrac{1 \cdot 0105}{I}$	H_2	$\cdot 0807$	$\dfrac{\cdot 5068}{H_2}$	$\cdot 0832$	$\dfrac{\cdot 02389}{\cdot 0168}$	$1 \cdot 42$
$\cdot 74$	I	$\cdot 0638$	$\dfrac{1 \cdot 1883}{I}$	H_2	$\cdot 0947$	$\dfrac{\cdot 59472}{H_2}$	$\cdot 0795$	$\dfrac{\cdot 02187}{\cdot 01889}$	$1 \cdot 157$
$\cdot 78$	I	$\cdot 0490$	$\dfrac{1 \cdot 4029}{I}$	H_2	$\cdot 1117$	$\dfrac{\cdot 70148}{H_2}$	$\cdot 0733$	$\dfrac{\cdot 01919}{\cdot 0205}$	$\cdot 935$
$\cdot 82$	I	$\cdot 0367$	$\dfrac{1 \cdot 6618}{I}$	H_2	$\cdot 1322$	$\dfrac{\cdot 83022}{H_2}$	$\cdot 0640$	$\dfrac{\cdot 01580}{\cdot 02151}$	$\cdot 734$
$\cdot 86$	I	$\cdot 0269$	$\dfrac{1 \cdot 9620}{I}$	H_2	$\cdot 1561$	$\dfrac{\cdot 98031}{H_2}$	$\cdot 0518$	$\dfrac{\cdot 01185}{\cdot 02}$	$\cdot 592$
$\cdot 9$	I	$\cdot 0185$	$\dfrac{2 \cdot 2913}{I}$	H_2	$\cdot 1823$	$\dfrac{1 \cdot 14484}{H_2}$	$\cdot 0375$	$\dfrac{\cdot 00770}{\cdot 017}$	$\cdot 413$

TABLE I (*contd.*)

$W_o = \cdot 7$

(External Diameter, 1·0. Core Length, 1·0)

STATOR				ROTOR				GENERAL	
Gap Diam.	No. Slots	Slot Depth	Slot Width	No Slots	Slot Depth	Slot Width	M L. Flux	Rel. Output Factor (H_5)	
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6 \cdot 28 Z_m = Z_n}{H_2}$	H_o		S_o over T_o
·62	I	·1305	$\dfrac{\cdot 7581}{I}$	H_2	·0603	$\dfrac{\cdot 37864}{H_2}$	·0833	$\dfrac{\cdot 02631}{\cdot 01260}$	2·088
·66	I	·1087	$\dfrac{\cdot 8460}{I}$	H_2	·0675	$\dfrac{\cdot 4239}{H_2}$	·0859	$\dfrac{\cdot 02612}{\cdot 01450}$	1·8
·7	I	·0975	$\dfrac{\cdot 9477}{I}$	H_2	·0755	$\dfrac{\cdot 47414}{H_2}$	·0876	$\dfrac{\cdot 02535}{\cdot 01659}$	1·528
·74	I	·0689	$\dfrac{1 \cdot 1012}{I}$	H_2	·0878	$\dfrac{\cdot 551384}{H_2}$	·0856	$\dfrac{\cdot 02356}{\cdot 01880}$	1·253
·78	I	·0523	$\dfrac{1 \cdot 2944}{I}$	H_2	·1031	$\dfrac{\cdot 64747}{H_2}$	·0809	$\dfrac{\cdot 02103}{\cdot 02091}$	1·005
·82	I	·0385	$\dfrac{1 \cdot 5460}{I}$	H_2	·1231	$\dfrac{\cdot 77307}{H_2}$	·0721	$\dfrac{\cdot 01759}{\cdot 02225}$	·79
·86	I	·0278	$\dfrac{1 \cdot 8563}{I}$	H_2	·1477	$\dfrac{\cdot 92756}{H_2}$	·0592	$\dfrac{\cdot 01337}{\cdot 02196}$	·608
·9	I	·0188	$\dfrac{2 \cdot 2013}{I}$	H_2	·1754	$\dfrac{1 \cdot 10151}{H_2}$	·0438	$\dfrac{\cdot 00882}{\cdot 01905}$	·463

TABLE I (*contd*)

$W_o = \cdot 8$

(External Diameter, 1·0. Core Length, 1·0)

	STATOR				ROTOR			GENERAL	
Gap Diam.	No. Slots	Slot Depth	Slot Width	No. Slots	Slot Depth	Slot Width	M.L. Flux		Rel Output Factor (H_5)
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6 \cdot 28 Z_m}{H_2}$	H_o	S_o over T_o	S_o over T_o
·62	I	·1371	$\dfrac{\cdot 7379}{I}$	H_2	·0597	$\dfrac{\cdot 37492}{H_2}$	·0847	$\dfrac{\cdot 02706}{\cdot 01252}$	2·16
·66	I	·1151	$\dfrac{\cdot 7974}{I}$	H_2	·0652	$\dfrac{\cdot 40946}{H_2}$	·0879	$\dfrac{\cdot 02702}{\cdot 01430}$	1·89
·7	I	·0934	$\dfrac{\cdot 9033}{I}$	H_2	·0719	$\dfrac{\cdot 45153}{H_2}$	·0907	$\dfrac{\cdot 02646}{\cdot 01635}$	1·62
·74	I	·0739	$\dfrac{1 \cdot 0377}{I}$	H_2	·0828	$\dfrac{\cdot 51998}{H_2}$	·0899	$\dfrac{\cdot 02489}{\cdot 01870}$	1·33
·78	I	·0558	$\dfrac{1 \cdot 2101}{I}$	H_2	·0964	$\dfrac{\cdot 60539}{H_2}$	·0868	$\dfrac{\cdot 02253}{\cdot 02098}$	1·083
·82	I	·0405	$\dfrac{1 \cdot 4446}{I}$	H_2	·1150	$\dfrac{\cdot 7222}{H_2}$	·0792	$\dfrac{\cdot 01916}{\cdot 02289}$	·83
·86	I	·0286	$\dfrac{1 \cdot 7549}{I}$	H_2	·1396	$\dfrac{\cdot 87669}{H_2}$	·0663	$\dfrac{\cdot 01482}{\cdot 02329}$	·63
·9	I	·0191	$\dfrac{2 \cdot 1199}{I}$	H_2	·1688	$\dfrac{1 \cdot 060064}{H_2}$	·0495	$\dfrac{\cdot 00988}{\cdot 02111}$	·364

TABLE I (*contd.*)

$W_o = \cdot 9$

(External Diameter, 1·0. Core Length, 1·0)

STATUS				ROTOR				GENERAL	
Gap Diam.	No. Slots	Slot Depth	Slot Width	No. Slots	Slot Depth	Slot Width	M L. Flux	Rel. Output Factor (H_b)	S_o over T_o
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6\cdot28 Z_m}{H_2}$	H_o		
·62	I	·1424	$\dfrac{\cdot 7210}{I}$	H_2	·0562	$\dfrac{\cdot 35294}{H_2}$	·0857	$\dfrac{\cdot 02762}{\cdot 01240}$	2·227
·66	I	·1204	$\dfrac{\cdot 7960}{I}$	H_2	·0635	$\dfrac{\cdot 39878}{H_2}$	·0894	$\dfrac{\cdot 02774}{\cdot 01425}$	1·946
·7	I	·0985	$\dfrac{\cdot 8747}{I}$	H_2	·0696	$\dfrac{\cdot 43709}{H_2}$	·0927	$\dfrac{\cdot 02936}{\cdot 01626}$	1·8
·74	I	·0784	$\dfrac{\cdot 9955}{I}$	H_2	·0793	$\dfrac{\cdot 49800}{H_2}$	·0930	$\dfrac{\cdot 02595}{\cdot 01842}$	1·41
·78	I	·0593	$\dfrac{1\cdot1443}{I}$	H_2	·0911	$\dfrac{\cdot 57211}{H_2}$	·0914	$\dfrac{\cdot 02385}{\cdot 02079}$	1·147
·82	I	·0426	$\dfrac{1\cdot3561}{I}$	H_2	·1081	$\dfrac{\cdot 67887}{H_2}$	·0854	$\dfrac{\cdot 02056}{\cdot 02316}$	·887
·86	I	·0296	$\dfrac{1\cdot6620}{I}$	H_2	·1324	$\dfrac{\cdot 83147}{H_2}$	·0728	$\dfrac{\cdot 01611}{\cdot 02389}$	·67
·9	I	·0195	$\dfrac{2\cdot0427}{I}$	H_2	·1626	$\dfrac{1\cdot02113}{H_2}$	·0549	$\dfrac{\cdot 01086}{\cdot 02244}$	·484

TABLE I (*contd.*)

$$W_o = 1\cdot 0$$

(External Diameter, $1\cdot 0$. Core Length, $1\cdot 0$)

STATUS	STATOR			ROTOR			GENERAL	
Gap Diam.	No. Slots	Slot Depth	Slot Width	No. Slots	Slot Depth	Slot Width	M.L. Flux	Rel. Output Factor (H_b)
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6\cdot 28 Z_m}{H_2}$	H_o	S_o over T_o
$\cdot 62$	I	$\cdot 1468$	$\dfrac{\cdot 7138}{I}$	H_2	$\cdot 0569$	$\dfrac{\cdot 35733}{H_2}$	$\cdot 0864$	$\dfrac{\cdot 02808}{\cdot 01252}$ 2·243
$\cdot 66$	I	$\cdot 1248$	$\dfrac{\cdot 7816}{I}$	H_2	$\cdot 0622$	$\dfrac{\cdot 39062}{H_2}$	$\cdot 0904$	$\dfrac{\cdot 02831}{\cdot 01421}$ 1·99
$\cdot 7$	I	$\cdot 1029$	$\dfrac{\cdot 8519}{I}$	H_2	$\cdot 0678$	$\dfrac{\cdot 42578}{H_2}$	$\cdot 0943$	$\dfrac{\cdot 02809}{\cdot 01606}$ 1·75
$\cdot 74$	I	$\cdot 0823$	$\dfrac{\cdot 9612}{I}$	H_2	$\cdot 0766$	$\dfrac{\cdot 48105}{H_2}$	$\cdot 0954$	$\dfrac{\cdot 02682}{\cdot 01833}$ 1·46
$\cdot 78$	I	$\cdot 0625$	$\dfrac{1\cdot 0929}{I}$	H_2	$\cdot 0871$	$\dfrac{\cdot 54688}{H_2}$	$\cdot 0950$	$\dfrac{\cdot 02484}{\cdot 02076}$ 1·196
$\cdot 82$	I	$\cdot 0449$	$\dfrac{1\cdot 2860}{I}$	H_2	$\cdot 1004$	$\dfrac{\cdot 63051}{H_2}$	$\cdot 0903$	$\dfrac{\cdot 02170}{\cdot 02324}$ ·93
$\cdot 86$	I	$\cdot 0307$	$\dfrac{1\cdot 5792}{I}$	H_2	$\cdot 1257$	$\dfrac{\cdot 78940}{H_2}$	$\cdot 0786$	$\dfrac{\cdot 01712}{\cdot 02486}$ ·688
$\cdot 9$	I	$\cdot 0200$	$\dfrac{1\cdot 9685}{I}$	H_2	$\cdot 1567$	$\dfrac{\cdot 98408}{H_2}$	$\cdot 0601$	$\dfrac{\cdot 01179}{\cdot 02466}$ ·478

TABLE I (*contd*)

$$W_o = 1.1$$

(External Diameter, 1·0. Core Length, 1·0)

STATUR				ROTOR				GENERAL	
Gap Diam.	No. Slots	Slot Depth	Slot Width	No. Slots	Slot Depth	Slot Width	M.L. Flux	Rel. Output Factor (H_5)	
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6\cdot28 Z_m}{H_2}$	H_o	S_o over T_o	
·62	I	·1519	$\dfrac{·7052}{I}$	H_2	·0562	$\dfrac{·35294}{H_2}$	·0870	$\dfrac{·02846}{·01225}$	2·32
·66	I	·1286	$\dfrac{·7702}{I}$	H_2	·0613	$\dfrac{·38496}{H_2}$	·0912	$\dfrac{·02876}{·01405}$	2·047
·7	I	·1067	$\dfrac{·8362}{I}$	H_2	·0666	$\dfrac{·41825}{H_2}$	·0954	$\dfrac{·02865}{·01639}$	1·748
·74	I	·0899	$\dfrac{·9355}{I}$	H_2	·0746	$\dfrac{·46849}{H_2}$	·0972	$\dfrac{·02753}{·01817}$	1·51
·78	I	·0657	$\dfrac{1·0558}{I}$	H_2	·0841	$\dfrac{·52815}{H_2}$	·0976	$\dfrac{·02566}{·02059}$	1·24
·82	I	·0472	$\dfrac{1·2289}{I}$	H_2	·0957	$\dfrac{·60100}{H_2}$	·0943	$\dfrac{·02270}{·02316}$	·98
·86	I	·0319	$\dfrac{1·5035}{I}$	H_2	·1197	$\dfrac{·75172}{H_2}$	·0839	$\dfrac{·01834}{·02521}$	·727
·9	I	·0204	$\dfrac{1·8957}{I}$	H_2	·1508	$\dfrac{·94702}{H_2}$	·0652	$\dfrac{·01268}{·02464}$	·514

TABLE I (contd)

$W_o = 1\cdot2$

(External Diameter, 1·0. Core Length, 1·0)

STATOR				ROTOR				GENERAL	
Gap Diam.	No Slots	Slot Depth	Slot Width	No Slots	Slot Depth	Slot Width	M L. Flux	Rel. Output Factor H_5	
R_o	I	U_o	$\dfrac{Z_o}{I}$	H_2	Z_m	$\dfrac{6\cdot28Z_m}{H_2}$	H_o	$\dfrac{S_o \text{ over } T_o}{}$	
·62	I	·1536	$\dfrac{·6981}{I}$	H_2	·0557	$\dfrac{·349796}{H_2}$	·0875	$\dfrac{·02878}{·01220}$	2·359
·66	I	·1318	$\dfrac{·7616}{I}$	H_2	·0606	$\dfrac{·38075}{H_2}$	·0918	$\dfrac{·02913}{·01396}$	2·086
·7	I	·1099	$\dfrac{·8233}{I}$	H_2	·0656	$\dfrac{·41197}{H_2}$	·0963	$\dfrac{·02912}{·01587}$	1·83
·74	I	·0890	$\dfrac{·9155}{I}$	H_2	·0730	$\dfrac{·45844}{H_2}$	·0986	$\dfrac{·02814}{·01807}$	1·557
·78	I	·0685	$\dfrac{1\cdot0256}{I}$	H_2	·0817	$\dfrac{·513076}{H_2}$	·0997	$\dfrac{·02639}{·02046}$	1·29
·82	I	·0494	$\dfrac{1\cdot1818}{I}$	H_2	·0941	$\dfrac{·590948}{H_2}$	0976	$\dfrac{02356}{·02305}$	1·02
·86	I	·0332	$\dfrac{1\cdot4393}{I}$	H_2	·1146	$\dfrac{·719688}{H_2}$	·0884	$\dfrac{·01926}{·02550}$	·755
·9	I	·0209	$\dfrac{1\cdot827}{I}$	H_2	·1455	$\dfrac{·91374}{H_2}$	·0700	$\dfrac{·01353}{·02557}$	·529

TABLE II

H_o—(If in doubt take next higher Flux)

R_o	$W_o=\cdot2$	·225	·25	·275	·3	·325	·35	·375	·4	·425	·45	·475	·5	·525
·62	·0448	·0496	·0534	·0577	·0620	·0645	·067	·06955	·0721	·0735	·075	·07645	·0779	·0787
·63	·04407	·04858	·0526	·05635	·0611	·0637	·0663	·0690	·0717	·07375	·0748	·0764	·078	·079
·64	·04335	·04757	·0518	·0560	·0602	·06295	·0657	·06847	·07125	·07292	·0746	·0763	·078	·079
·65	·0426	·04675	·0509	·0551	·0593	·06215	·0650	·0679	·0708	·0726	·0744	·07625	·0781	·0792
·66	·0419	·046	·0501	·05425	·0584	·0614	·0644	·0672	·0704	·0723	·0742	·07615	·0781	·0793
·67	·04087	·044935	·0490	·05315	·0573	·06035	·0634	·0665	·0696	·0766	·0736	·07565	·0777	·079025
·68	·03985	·043925	·048	·05207	·05615	·05977	·0624	·06555	·0687	·07085	·073	·075175	·07735	·07877
·69	·0388	·04275	·0469	·05095	·055	·05695	·0589	·0632	·0678	·0701	·0724	·0747	·0770	·0785
·7	·0378	·0418	·0458	·04985	·0539	·0572	·0605	·06375	·0670	·0694	·0718	·0742	·0766	·07825
·71	·0366	·0405	·0449	·04835	·0523	·0556	·0589	·06215	·0654	·06785	·0703	·0728	·0753	·07705
·72	·0354	·03925	·0431	·04695	·0508	·05405	·0573	·060575	·06385	·06685	·0689	·07195	·0740	·07585
·73	·0342	·038	·0418	·04555	·0493	·0526	·0557	·0587	·0623	·06485	·0674	·070	·0726	·0743
·74	·0331	·03675	·0404	·0441	·0478	·0510	·0542	·05745	·0607	·06335	·066	·06865	·0713	·07385
·75	·03185	·0354	·039	·0428	·0466	·04955	·0525	·0557	·0589	·06155	·0642	·06685	·0695	·0716
·76	·0306	·0341	·0376	·041025	·04455	·047625	·0508	·05395	·0571	·05975	·0624	·06505	·0677	·06995
·77	·0394	·0328	·0362	·03985	·0435	·0463	·0491	·05215	·0552	·0579	·0606	·06325	·0659	·0681
·78	·0282	·0315	·0348	·03815	·0415	·04445	·0474	·0504	·0534	·05605	·05875	·0614	·0641	·0664
·79	·0270	·03015	·0333	·0365	·0397	·0421	·0455	·0484	·0513	·0538	·0563	·05705	·0618	·0640
·8	·02585	·0289	·0319	·034975	·03795	·040775	·0436	·046450	·0493	·0516	·0539	·0567	·05955	·061675
·81	·0247	·02755	·0304	·0333	·0362	·03895	·0417	·04445	·0472	·0496	·052	·05465	·0573	·05945
·82	·0235	·02625	·029	·03175	·0345	·03715	·0398	·04245	·0451	·04755	·0500	·0525	·0550	·05725
·83	·0222	·0248	·0274	·030	·0326	·03515	·0377	·0402	·0427	·0456	·0475	·04985	·0522	·0544
·84	·0208	·0233	·0258	·0283	·0308	·0332	·0356	·0380	·0404	·04265	·04495	·0472	·0495	·0516
·85	·0195	·02185	·0242	·02655	·0289	·0312	·0335	·03575	·038	·04015	·0423	·0445	·0467	·04875
·86	·0182	·0204	·02265	·02485	·0271	·02925	·0314	·03355	·0357	·03775	·0398	·0419	·0440	·04595
·87	·0169	·01895	·0210	·0231	·0252	·0272	·0292	·0312	·0332	·0351	·037	·039	·041	·0428
·88	·01565	·01757	·0195	·0214	·0233	·02515	·0248	·0279	·0308	·03255	·03435	·03615	·0380	·0397
·89	·0144	·01615	·0179	·02465	·0214	·0231	·0248	·02665	·0283	·0299	·0315	·0332	·0349	·0366
·9	·0131	·0147	·0163	·0179	·0195	·02105	·0226	·0242	·0258	·0273	·0288	·03035	·0319	·0334

TABLE II (contd)

H_0—(If in doubt take next higher Flux)

R_0	$W_0 = \cdot55$	·575	·6	·625	·65	·675	·7	·725	·75	·775	·8	·825	·85	·875
·62	·0795	·07985	·0812	·08175	·0822	·0827	·0833	·08365	·084	·08435	·0847	·08495	·0852	·08545
·63	·0800	·0808	·0816	·08217	·08275	·08335	·08395	·08432	·0847	·0851	·0855	·08575	·086	·0863
·64	·0800	·08102	·08205	·08267	·0833	·08395	·0846	·085	·0854	·08585	·0863	·0866	·0869	·08725
·65	·0803	·0814	·0825	·08317	·08385	·08455	·08525	·08567	·0861	·086	·0871	·0874	·08775	·0881
·66	·0805	·0817	·0829	·0836	·0844	·0851	·0859	·0864	·0869	·0874	·0879	·08825	·0886	·089
·67	·08035	·08167	·083	·08382	·08465	·08545	·0863	·0867	·0874	·088	·0886	·089	·0894	·0898
·68	·0802	·08165	·08312	·084	·0849	·085825	·08675	·08737	·088	·08865	·0893	·08975	·0902	·090625
·69	·0800	·08155	·0831	·0841	·0851	·08615	·0872	·08785	·0885	·08925	·09	·09047	·09095	·091425
·7	·0799	·08155	·0832	·0843	·0854	·0865	·0876	·0883	·0891	·0899	·0907	·0912	·0917	·0922
·71	·0788	·08055	·0823	·0835	·0847	·0859	·0871	·0879	·0887	·0896	·0905	·0911	·0917	·09225
·72	·0777	·0795	·08135	·08267	·0840	·0853	·0866	·0875	·0884	·08935	·0903	·09095	·0916	·09223
·73	·076	·0782	·0804	·0818	·0832	·08465	·0861	·08705	·088	·08905	·0901	·0908	·0915	·0922
·74	·0754	·07745	·0795	·081	·0825	·0840	·0856	·0866	·0877	·0888	·0899	·09025	·0914	·0905
·75	·0737	·0758	·0779	·0795	·0811	·08275	·0844	·08555	·08675	·0879	·0891	·0887	·0884	·08875
·76	·07205	·0742	·0764	·0781	·0798	·0815	·08325	·0845	·0858	·0871	·0884	·08685	·0853	·0895
·77	·0703	·0726	·0749	·07665	·0784	·08025	·0821	·08345	·0848	·0862	·0876	·0874	·0872	·09025
·78	·0687	·0710	·0733	·0752	·0771	·0790	·0809	·0823	·0838	·0853	·0868	·08795	·0891	·08865
·79	·0662	·06845	·0707	·07495	·0792	·07895	·0787	·0802	·0817	·0833	·0849	·08615	·0874	·08705
·8	·0638	·0659	·06805	·0746	·0813	·0789	·0765	·0781	·0797	·08335	·083	·08435	·0857	·08545
·81	·0616	·0638	·066	·0753	·0846	·07945	·0743	·07595	·0776	·07935	·0811	·08205	·084	·08385
·82	·0595	·06175	·0640	·066	·0680	·07005	·0721	·07385	·0756	·0774	·0792	·08075	·0823	·0807
·83	·0566	·0588	·061	·06295	·0649	·0669	·0689	·07065	·0724	·0750	·076	·07755	·0791	·0775
·84	·0537	·0558	·0579	·05985	·0618	·06375	·0657	·06745	·0692	·0710	·0728	·07435	·0759	·0743
·85	·0508	·0528	·0548	·0567	·0586	·0605	·0624	·06415	·0659	·0677	·0695	·0711	·0727	·07115
·86	·0479	·04985	·0513	·05365	·0555	·05735	·0592	·06095	·0627	·0645	·0663	·0679	·0695	·06725
·87	·0446	·04645	·0483	·0501	·0519	·0536	·0553	·0570	·0587	·0604	·0621	·06365	·0652	·0624
·88	·0414	·0431	·04485	·0465	·0482	·04985	·0515	·0531	·0547	·0563	·0579	·0599	·0609	·05795
·89	·0381	·0396	·0411	·0428	·0445	·04605	·0476	·0491	·0506	·05215	·0537	·0551	·0565	·05355
·9	·0349	·0364	·0379	·03925	·0406	·0422	·0438	·0452	·0466	·04805	·0495	·05085	·0522	

TABLE II (contd.)

H_0—(If in doubt take next higher Flux)

R_0	$W_0 = \cdot9$	·925	·95	·975	1·0	1·025	1·05	1·075	1·1	1·125	1·15	1·175	1·2
·62	·0857	·086	·0863	·08665	·087	·08685	·0867	·08685	·087	·0871	·0872	·08735	·0875
·63	·0866	·0869	·0872	·08755	·0879	·087925	·08795	·08797	·088	·08815	·0883	·08845	·0886
·64	·0876	·0879	·0882	·0885	·0888	·0888	·0887	·0889	·0891	·089225	·08935	·0895	·08965
·65	·0885	·0888	·0891	·08942	·08975	·0898	·0899	·09	·09015	·09025	·0904	·09055	·0907
·66	·0894	·0897	·0900	·09035	·0907	·0906	·0905	·09085	·0912	·09135	·0915	·09165	·0918
·67	·0902	·09052	·09085	·09117	·0915	·09165	·0918	·092	·0922	·09235	·0925	·0927	·0929
·68	·09105	·091375	·0917	·092025	·0923	·09265	·0928	·09305	·0933	·0935	·0937	·093875	·09405
·69	·0919	·0923	·0927	·09315	·0936	·09375	·0939	·0941	·0943	·0945	·09475	·09495	·0952
·7	·0927	·0932	·0937	·09425	·0948	·09495	·0951	·09525	·0954	·0956	·09585	·09615	·0963
·71	·0928	·0933	·09385	·0944	·09505	·0952	·0954	·0956	·09585	·0962	·0964	·09665	·0969
·72	·09285	·0934	·0940	·09465	·0953	·09555	·0958	·09605	·0963	·0966	·0969	·09717	·09745
·73	·0929	·09355	·0942	·0949	·0956	·09585	·0961	·09642	·09675	·09707	·0974	·0977	·098
·74	·0930	·0936	·0944	·09515	·0959	·0962	·0965	·09685	·0972	·09755	·0979	·09825	·0986
·75	·0926	·0933	·0940	·09475	·09555	·09595	·0964	·09685	·0973	·0977	·0981	·0985	·0989
·76	·0922	·09295	·0937	·09445	·0952	·09575	·0963	·0969	·0974	·09785	·0983	·09875	·0992
·77	·0918	·0926	·09345	·09425	·0951	·09565	·0963	·09695	·0975	·09795	·0984	·0989	·0994
·78	·0914	·0923	·0932	·0941	·0950	·09555	·0963	·09605	·0976	·0981	·0986	·09915	·0997
·79	·0899	·09085	·0918	·0928	·0938	·09455	·0953	·095	·0968	·0974	·098	·0986	·0992
·8	·0884	·08995	·0905	·0916	·0927	·09335	·094	·0943	·096	·09665	·0973	·09767	·09865
·81	·0869	·08815	·0894	·0907	·0920	·09275	·0935	·09355	·0951	·09585	·0966	·09735	·0981
·82	·0854	·08685	·0883	·0898	·0913	·09205	·0928	·09075	·0943	·095125	·09595	·096775	·0976
·83	·08225	·0836	·08505	·08645	·0879	·08885	·0898	·08795	·0917	·0926	·0935	·0944	·0953
·84	·0791	·08045	·0818	·0831	·08445	·08495	·0868	·08525	·0891	·0905	·091	·092	·0930
·85	·0759	·0773	·0787	·0801	·0815	·08275	·084	·08255	·0865	·08755	·0886	·08965	·0907
·86	·0728	·07425	·0757	·07715	·0786	·0799	·0812	·08165	·0839	·0875	·0911	·08975	·0884
·87	·0683	·06975	·0712	·07265	·0741	·0766	·0791	·07325	·0842	·0841	·084	·0839	·0838
·88	·0639	·0653	·0667	·06815	·0696	·0708	·072	·0686	·0745	·07542	·07685	·078025	·0792
·89	·0594	·0608	·0622	·06365	·0651	·06625	·0674	·06405	·0698	·071	·0722	·0734	·0746
·9	·0549	·0563	·05778	·0592	·0606	·06175	·0629		·0652	·0664	·0676	·0688	·0700

TABLE III

U_o—(If in doubt take next higher value)

R_o	$W_o = .2$	.25	.3	.35	.4	.45	.5	.55	.6	.65
.62	·0780	·08205	·0861	·093	·0999	·1050	·1101	·11625	·1224	·12645
.63	·0748	·07875	·0827	·08905	·0954	·1005	·10555	·1113	·11705	·12105
.64	·07165	·0755	·0794	·0852	·09095	·0960	·101	·10635	·1117	·11565
.65	·0684	·0722	·07605	·0813	·0865	·0911	·09595	·10115	·10635	·11025
.66	·0653	·069	·0727	·07635	·0820	·08695	·0919	·09645	·1010	·10485
.67	·0627	·0661	·0695	·0738	·0781	·0827	·0873	·09165	·096	·0996
.68	·0601	·0632	·0663	·0702	·07415	·0784	·0827	·08685	·091	·0943
.69	·0575	·0603	·06315	·0667	·0702	·07415	·0781	·08205	·086	·0890
.7	·055	·0575	·06	·06325	·0663	·07	·0735	·07725	·081	·08375
.71	·05325	·0555	·05775	·0606	·0635	·0666	·06975	·0731	·0765	·07925
.72	·0515	·0535	·0555	·058	·0605	·0632	·066	·069	·072	·07475
.73	·0495	·05137	·05325	·05525	·05725	·05987	·0625	·0652	·068	·070625
.74	·0475	·04925	·051	·0525	·054	·0565	·059	·0615	·064	·0665
.75	·0455	·04687	·04825	·0499	·0515	·0536	·05575	·058	·06025	·0625
.76	·0435	·0445	·0455	·04725	·049	·05075	·0525	·0545	·0565	·0585
.77	·039	·0411	·0432	·0447	·0462	·0477	·04925	·051	·05275	·055875
.78	·0345	·039	·041	·04225	·0435	·04475	·046	·0475	·049	·05325
.79	·035	·03775	·039	·04	·041	·042	·043	·0445	·046	·04875
.8	·0355	·037	·037	·03775	·0385	·03925	·04	·0415	·043	·04425
.81	·0335	·03625	·03475	·03537	·036	·03675	·0375	·03875	·04	·041
.82	·0315	·03412	·0325	·033	·0335	·03425	·035	·036	·037	·03775
.83	·02975	·032	·0305	·031	·0315	·0321	·03275	·0335	·03425	·0351
.84	·028	·0301	·0285	·029	·0295	·03	·0305	·031	·0315	·03225
.85	·0365	·02825	·02675	·0271	·0275	·0279	·02835	·0288	·02925	·029875
.86	·0245	·02475	·025	·0252	·0255	·02575	·026	·0265	·027	·0275
.87	·02225	·02475	·02325	·0235	·02375	·024	·02425	·024625	·025	·025375
.88	·02	·02275	·0215	·02175	·022	·02225	·0225	·02275	·023	·02325
.89	·01875	·02075	·0196	·0198	·02	·02017	·02035	·02055	·02075	·02092
.9	·0175	·0176	·0177	·01785	·0180	·0181	·0182	·01835	·0185	·0186

Depth

TABLE III (contd.)

U_o—(If in doubt take next higher value)

Depth

R_o	$W_0 = .7$	.75	.8	.85	.9	.95	1.0	1.05	1.1	1.15	1.2
.62	.1305	.1338	.1371	.13975	.1424	.1446	.1468	.14935	.1519	.15275	.1536
.63	.12505	.1283	.1316	.13425	.1369	.1391	.1413	.1437	.1461	.1471	.14815
.64	.1196	.1228	.1261	.12875	.1314	.1336	.1358	.1380	.14025	.1415	.1427
.65	.11415	.1173	.1206	.12325	.1259	.1281	.1303	.13235	.1344	.1358	.13725
.66	.1087	.1119	.1151	.11775	.1204	.1226	.1248	.1267	.1286	.1302	.1318
.67	.10315	.1064	.1097	.1123	.1149	.11705	.1192	.12175	.1243	.1253	.1263
.68	.0976	.10095	.1043	.1069	.10945	.1115	.1136	.1168	.12005	.1204	.1208
.69	.09205	.0955	.0989	.10145	.1040	.1060	.10805	.1119	.1158	.1156	.11535
.7	.0865	.09	.0935	.096	.0985	.1005	.1025	.107	.1115	.1107	.1099
.71	.082	.08525	.0885	.091	.0935	.09575	.098	.10137	.10475	.1047	.1047
.72	.0775	.0805	.0835	.086	.0885	.091	.0935	.09575	.098	.0987	.09945
.73	.07325	.076	.07875	.081125	.0835	.086	.0885	.09112	.09375	.0940	.0942
.74	.069	.0715	.074	.0762	.0785	.081	.0835	.0865	.0895	.08925	.0890
.75	.06475	.06712	.0695	.07162	.07375	.076	.07825	.08087	.0835	.0837	.0839
.76	.0605	.06275	.065	.067	.069	.071	.073	.07525	.0775	.0781	.07875
.77	.059	.05975	.0605	.06225	.064	.06587	.06775	.06975	.07175	.0727	.0736
.78	.055	.05675	.056	.0575	.059	.06075	.0625	.06425	.066	.06725	.0685
.79	.0515	.05175	.052	.0535	.055	.0565	.058	.0595	.061	.06235	.0637
.8	.0453	.04675	.048	.0495	.051	.05225	.0535	.05475	.056	.057475	.05895
.81	.042	.04312	.04425	.0455	.04675	.048	.04925	.0505	.0475	.052975	.0542
.82	.0385	.0395	.0405	.0415	.0425	.04375	.045	.04625	.0475	.04845	.0494
.83	.03575	.03662	.0375	.03837	.03925	.04025	.04125	.042375	.0435	.04442	.04535
.84	.033	.03375	.0345	.03525	.036	.03675	.0375	.0385	.0395	.0404	.0413
.85	.0305	.031	.0315	.03425	.03275	.033375	.034	.03487	.0357	.0365	.03725
.86	.028	.02825	.0285	.029	.0295	.03	.0305	.03125	.0320	.0326	.0332
.87	.02575	.026	.02625	.02662	.027	.02737	.02775	.02825	.02875	.02942	.0301
.88	.0235	.02375	.024	.02425	.0245	.02475	.025	.02525	.0255	.02675	.02705
.89	.0211	.0213	.0215	.02175	.022	.02225	.0225	.02275	.023	.0235	.0240
.9	.0187	.01885	.019	.01925	.0195	.01975	.02	.02025	.0205	.0207	.0209

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TABLE IV

Z_0—(If in doubt take next lower value)

Width

R_0	$W_0=·2$	·225	·25	·275	·3	·325	·35	·375	·4	4·25	·45	·425	·5	·525
·62	1·3081	1·2467	1·1852	1·1237	1·0623	1·0262	·9902	·9541	·9181	·8902	·8623	·83445	·8066	·8002
·63	1·3497	1·28885	1·2280	1·1672	1·1064	1·0686	1·0309	·9931	·9554	·9287	·902	·8753	·8487	·8385
·64	1·3913	1·3311	1·2709	1·2107	1·1505	1·11105	1·0716	1·0321	·9927	·9672	·9418	·91635	·8909	·8769
·65	1·4329	1·3733	1·3137	1·2541	1·1946	1·1534	1·1123	1·0711	1·03	1·0059	·9815	·9573	·9331	·9152
·66	1·4745	1·41555	1·3566	1·2977	1·2388	1·1959	1·1530	1·1101	1·0673	1·0443	1·0213	·9983	·9753	·9536
·67	1·5206	1·4620	1·4034	1·3447	1·2863	1·2429	1·1986	1·1547	1·1109	1·0851	1·0593	1·0334	1·0076	·9852
·68	1·5667	1·5085	1·4503	1·3921	1·3339	1·28905	1·2442	1·1994	1·1546	1·1259	1·0973	1·0686	1·0400	1·0174
·69	1·6128	1·5549	1·4971	1·4392	1·3814	1·3356	1·2898	1·2437	1·1982	1·1662	1·1353	1·1038	1·0724	1·0493
·7	1·659	1·6015	1·544	1·4865	1·4290	1·3822	1·3354	1·2886	1·2419	1·2076	1·1733	1·139	1·1048	1·0812
·71	1·7074	1·6513	1·5949	1·5116	1·4824	1·4352	1·389	1·3423	1·2956	1·2605	1·2252	1·19	1·1549	1·1299
·72	1·7558	1·7008	1·6458	1·5908	1·5358	1·5142	1·4426	1·396	1·3494	1·3133	1·2772	1·2411	1·2051	1·1786
·73	1·8042	1·7505	1·6967	1·6429	1·5892	1·5426	1·4961	1·4496	1·4031	1·3661	1·3291	1·2921	1·2552	1·2273
·74	1·8527	1·80015	1·7476	1·6951	1·6426	1·5961	1·5497	1·5033	1·4569	1·419	1·3811	1·3432	1·3054	1·2761
·75	1·9013	1·8501	1·7989	1·7477	1·6966	1·651	1·6055	1·56	1·5145	1·4765	1·4385	1·4005	1·3626	1·3324
·76	1·9499	1·90005	1·8502	1·8004	1·7506	1·7059	1·6613	1·6167	1·5721	1·5341	1·496	1·4579	1·4199	1·3888
·77	1·9985	1·95	1·9015	1·853	1·8046	1·7608	1·7171	1·6739	1·6297	1·5915	1·5534	1·5152	1·4771	1·4451
·78	2·0472	2·00005	1·9529	1·9057	1·8586	1·8158	1·773	1·7302	1·6874	1·6491	1·6109	1·5726	1·5344	1·5015
·79	2·0954	2·0502	2·005	1·9598	1·9147	1·8731	1·8316	1·79	1·7485	1·7109	1·6734	1·6358	1·5983	1·5656
·8	2·1437	2·103	2·0573	2·0141	1·9709	1·9304	1·8902	1·8499	1·8096	1·7727	1·7359	1·6991	1·6623	1·6298
·81	2·192	2·1498	2·1097	2·0686	2·0275	1·9883	1·9491	1·9099	1·8707	1·8346	1·7985	1·7624	1·7263	1·6939
·82	2·2403	2·201	2·1617	2·1224	2·0832	2·0453	2·0075	1·9696	1·9318	1·8964	1·8610	1·8256	1·7903	1·7581
·83	2·2906	2·2532	2·2158	2·1784	2·1411	2·105	2·0689	2·0328	1·9968	1·9628	1·9289	1·895	1·8611	1·83
·84	2·3411	2·3055	2·27	2·2345	2·1990	2·1647	2·1304	2·0961	2·0619	2·0294	1·9969	1·9644	1·9319	1·8769
·85	2·3915	2·35785	2·3242	2·3176	2·2569	2·2244	2·1919	2·1594	2·1269	2·0958	2·0648	2·0337	2·0027	1·9737
·86	2·442	2·4102	2·3784	2·3466	2·3149	2·2841	2·2534	2·2227	2·1920	2·1623	2·1327	2·1031	2·0735	2·0456
·87	2·4914	2·46185	2·4323	2·4017	2·3732	2·3445	2·3159	2·2872	2·2586	2·2309	2·2032	2·1755	2·1479	2·1215
·88	2·5408	2·5185	2·4882	2·4579	2·4316	2·405	2·3784	2·3518	2·3252	2·2995	2·2738	2·2481	2·2224	2·1981
·89	2·5902	2·5601	2·5401	2·515	2·49	2·4654	2·4409	2·4163	2·3918	2·36305	2·3443	2·3206	2·2969	2·2749
·9	2·6397	2·6168	2·594	2·5712	2·5484	2·5259	2·5034	2·4809	2·4585	2·4367	2·4149	2·3931	2·3714	2·3513

TABLE IV (contd.)

Z_o—(If in doubt take next lower value)

Width

R_o	$W_o = .55$	·575	·6	·625	·65	·675	·7	·725	·75	·775	·8	·825	·85	·875
·62	·7938	·78745	·7811	·77535	·7696	·76385	·7581	·75305	·748	·74295	·7379	·73365	·7294	·7252
·63	·8283	·81815	·808	·801	·794	·787	·780	·7737	·7675	·7613	·7551	·75125	·7474	·7435
·64	·8629	·8539	·8349	·8266	·8184	·8102	·802	·7941	·7863	·7784	·7706	·7675	·7645	·7615
·65	·8974	·8746	·8618	·8523	·8429	·8334	·824	·816	·808	·8	·792	·7883	·7846	·7809
·66	·932	·9104	·8888	·8781	·8674	·8567	·8460	·8378	·8297	·8215	·8134	·809	·8047	·8003
·67	·9629	·9405	·9182	·9065	·8948	·8831	·8714	·8625	·8536	·8447	·8358	·8307	·8257	·8206
·68	·9948	·9722	·9496	·9364	·9232	·91	·8969	·8871	·8775	·8679	·8583	·8525	·8468	·84105
·69	1·0262	1·0031	·980	·9655	·9511	·9366	·9222	·9118	·9015	·8912	·8808	·8743	·8679	·8614
·7	1·0576	1·0390	1·0105	·9948	·9791	·9634	·9477	·9366	·9255	·9144	·9033	·8961	·889	·8818
·71	1·1049	1·0799	1·0549	1·0376	1·0204	1·0032	·986	·9737	·9614	·9491	·9369	·9289	·9209	·9129
·72	1·1522	1·1258	1·0994	1·0806	1·0619	1·0431	1·0244	1·0109	·9974	·9839	·9705	·9616	·9528	·9439
·73	1·1995	1·1716	1·1438	1·1235	1·1033	1·0831	1·0628	1·0481	1·0334	1·0187	1·0041	·9944	·9847	·975
·74	1·2468	1·2175	1·1883	1·1665	1·1447	1·1229	1·1012	1·0853	1·0694	1·0535	1·0377	1·0271	1·0166	1·006
·75	1·3022	1·272	1·2419	1·2188	1·1957	1·1726	1·1495	1·1323	1·1151	1·0979	1·0808	1·0687	1·0567	1·0447
·76	1·3577	1·3266	1·2956	1·2711	1·2467	1·2222	1·1978	1·1793	1·1608	1·1423	1·1239	1·1104	1·0969	1·0834
·77	1·4131	1·3811	1·3492	1·3234	1·2976	1·2715	1·2461	1·2263	1·2065	1·1867	1·167	1·152	1·137	1·122
·78	1·4686	1·4357	1·4029	1·3757	1·3486	1·3215	1·2944	1·2733	1·2522	1·2311	1·2101	1·1936	1·1772	1·1607
·79	1·5329	1·5002	1·4676	1·44	1·4124	1·3848	1·3573	1·3349	1·3125	1·2901	1·2677	1·25	1·2324	1·2148
·8	1·5973	1·5648	1·5323	1·5042	1·4762	1·4482	1·4202	1·3969	1·3737	1·3505	1·3273	1·308	1·2887	1·2694
·81	1·6616	1·6293	1·597	1·5685	1·54	1·5115	1·4831	1·4588	1·4345	1·4012	1·3859	1·3652	1·3445	1·3238
·82	1·726	1·6939	1·6618	1·6328	1·6039	1·5749	1·5460	1·5206	1·4953	1·4699	1·4446	1·4224	1·4003	1·3782
·83	1·7989	1·7678	1·7368	1·7084	1·6801	1·6518	1·6235	1·5981	1·5728	1·5474	1·5221	1·4997	1·4773	1·4549
·84	1·8719	1·8419	1·8119	1·7842	1·7565	1·7288	1·7011	1·6807	1·6504	1·625	1·5997	1·577	1·5543	1·5316
·85	1·9448	1·9158	1·8869	1·8598	1·8328	1·8057	1·7787	1·7533	1·728	1·7027	1·6774	1·6544	1·6314	1·6084
·86	2·0177	1·9898	1·9620	1·9355	1·9091	1·8827	1·8563	1·8309	1·8056	1·7802	1·7549	1·7316	1·7084	1·6852
·87	2·0951	2·0687	2·0423	2·0175	1·9924	1·9674	1·9425	1·9184	1·8943	1·8702	1·8461	1·8238	1·8016	1·7793
·88	2·1745	2·1505	2·1266	2·1021	2·0777	2·0532	2·0288	2·0059	1·9831	1·9602	1·9374	1·9161	1·8948	1·8735
·89	2·2529	2·2309	2·2089	2·1854	2·1619	2·1384	2·115	2·0934	2·0718	2·0502	2·0286	1·7951	1·5616	1·3281
·9	2·3313	2·3113	2·2913	2·2688	2·2463	2·2238	2·2013	2·1809	2·1606	2·1405	2·1199	2·1006	2·0813	2·062

TABLE IV (contd.)

Z_o—(If in doubt take next lower value)

Width

R_o	$W_0 = \cdot9$	$\cdot925$	$\cdot95$	$\cdot975$	$1\cdot0$	$1\cdot025$	$1\cdot05$	$1\cdot075$	$1\cdot1$	$1\cdot125$	$1\cdot15$	$1\cdot175$	$1\cdot2$
$\cdot62$	$\cdot7210$	$\cdot7192$	$\cdot7174$	$\cdot7156$	$\cdot7138$	$\cdot71165$	$\cdot7095$	$\cdot70735$	$\cdot7052$	$\cdot7034$	$\cdot7016$	$\cdot69985$	$\cdot6981$
$\cdot63$	$\cdot7397$	$\cdot7373$	$\cdot7352$	$\cdot7329$	$\cdot7307$	$\cdot7283$	$\cdot726$	$\cdot7237$	$\cdot7214$	$\cdot7195$	$\cdot7176$	$\cdot71575$	$\cdot7139$
$\cdot64$	$\cdot7585$	$\cdot7558$	$\cdot7531$	$\cdot7504$	$\cdot7477$	$\cdot7452$	$\cdot7427$	$\cdot7402$	$\cdot7377$	$\cdot7357$	$\cdot7337$	$\cdot73175$	$\cdot7298$
$\cdot65$	$\cdot7772$	$\cdot77405$	$\cdot7709$	$\cdot7677$	$\cdot7646$	$\cdot7619$	$\cdot7592$	$\cdot75605$	$\cdot7539$	$\cdot75185$	$\cdot7498$	$\cdot74775$	$\cdot7457$
$\cdot66$	$\cdot7960$	$\cdot7924$	$\cdot7888$	$\cdot7852$	$\cdot7816$	$\cdot7787$	$\cdot7759$	$\cdot77305$	$\cdot7702$	$\cdot76805$	$\cdot7659$	$\cdot76375$	$\cdot7616$
$\cdot67$	$\cdot8156$	$\cdot8114$	$\cdot8073$	$\cdot8032$	$\cdot7991$	$\cdot796$	$\cdot7929$	$\cdot7898$	$\cdot7867$	$\cdot78425$	$\cdot7818$	$\cdot7794$	$\cdot777$
$\cdot68$	$\cdot8353$	$\cdot8306$	$\cdot826$	$\cdot8213$	$\cdot8167$	$\cdot8133$	$\cdot8099$	$\cdot80655$	$\cdot8032$	$\cdot795$	$\cdot7978$	$\cdot7951$	$\cdot7924$
$\cdot69$	$\cdot855$	$\cdot8498$	$\cdot8446$	$\cdot8394$	$\cdot8343$	$\cdot8306$	$\cdot827$	$\cdot82335$	$\cdot8197$	$\cdot8167$	$\cdot8137$	$\cdot81075$	$\cdot8078$
$\cdot7$	$\cdot8747$	$\cdot819$	$\cdot8633$	$\cdot8526$	$\cdot8519$	$\cdot8479$	$\cdot844$	$\cdot8401$	$\cdot8362$	$\cdot83295$	$\cdot8297$	$\cdot8265$	$\cdot8233$
$\cdot71$	$\cdot9049$	$\cdot8984$	$\cdot8920$	$\cdot8856$	$\cdot8792$	$\cdot8746$	$\cdot8701$	$\cdot86555$	$\cdot861$	$\cdot8573$	$\cdot8536$	$\cdot84995$	$\cdot8463$
$\cdot72$	$\cdot9351$	$\cdot9279$	$\cdot9208$	$\cdot9136$	$\cdot9065$	$\cdot9013$	$\cdot8961$	$\cdot89095$	$\cdot8858$	$\cdot8817$	$\cdot8776$	$\cdot8735$	$\cdot8694$
$\cdot73$	$\cdot9653$	$\cdot9574$	$\cdot9495$	$\cdot9536$	$\cdot9338$	$\cdot928$	$\cdot9222$	$\cdot9164$	$\cdot9106$	$\cdot90605$	$\cdot9015$	$\cdot89675$	$\cdot8924$
$\cdot74$	$\cdot9955$	$\cdot9869$	$\cdot9783$	$\cdot9697$	$\cdot9612$	$\cdot9547$	$\cdot9483$	$\cdot9419$	$\cdot9355$	$\cdot9305$	$\cdot9255$	$\cdot9205$	$\cdot9155$
$\cdot75$	$1\cdot0327$	$1\cdot023$	$1\cdot0134$	$1\cdot0037$	$\cdot9941$	$\cdot9869$	$\cdot9798$	$\cdot97265$	$\cdot9655$	$\cdot95985$	$\cdot9542$	$\cdot9486$	$\cdot943$
$\cdot76$	$1\cdot0699$	$1\cdot0591$	$1\cdot0484$	$1\cdot0377$	$1\cdot0270$	$1\cdot0191$	$1\cdot0113$	$1\cdot00345$	$\cdot9956$	$\cdot9893$	$\cdot983$	$\cdot97675$	$\cdot9705$
$\cdot77$	$1\cdot1071$	$1\cdot0953$	$1\cdot0835$	$1\cdot0717$	$1\cdot0599$	$1\cdot0438$	$1\cdot0277$	$1\cdot0267$	$1\cdot0257$	$1\cdot01875$	$1\cdot0118$	$1\cdot0049$	$\cdot998$
$\cdot78$	$1\cdot1443$	$1\cdot1314$	$1\cdot1186$	$1\cdot1057$	$1\cdot0929$	$1\cdot0836$	$1\cdot0743$	$1\cdot06505$	$1\cdot0558$	$1\cdot04825$	$1\cdot0407$	$1\cdot03315$	$1\cdot0256$
$\cdot79$	$1\cdot1972$	$1\cdot1831$	$1\cdot1691$	$1\cdot1551$	$1\cdot1411$	$1\cdot1305$	$1\cdot12$	$1\cdot1095$	$1\cdot099$	$1\cdot0904$	$1\cdot0818$	$1\cdot0732$	$1\cdot0646$
$\cdot8$	$1\cdot2502$	$1\cdot235$	$1\cdot2198$	$1\cdot2046$	$1\cdot1894$	$1\cdot1776$	$1\cdot1658$	$1\cdot15405$	$1\cdot1423$	$1\cdot13265$	$1\cdot123$	$1\cdot11335$	$1\cdot1037$
$\cdot81$	$1\cdot3031$	$1\cdot2867$	$1\cdot2704$	$1\cdot254$	$1\cdot2377$	$1\cdot2246$	$1\cdot2116$	$1\cdot1986$	$1\cdot1856$	$1\cdot17485$	$1\cdot1641$	$1\cdot1534$	$1\cdot1427$
$\cdot82$	$1\cdot3561$	$1\cdot3385$	$1\cdot3210$	$1\cdot3035$	$1\cdot2860$	$1\cdot2717$	$1\cdot2574$	$1\cdot24315$	$1\cdot2289$	$1\cdot2171$	$1\cdot2053$	$1\cdot19355$	$1\cdot1818$
$\cdot83$	$1\cdot3425$	$1\cdot4141$	$1\cdot3958$	$1\cdot3775$	$1\cdot3593$	$1\cdot3438$	$1\cdot3284$	$1\cdot31295$	$1\cdot2975$	$1\cdot28465$	$1\cdot2718$	$1\cdot25895$	$1\cdot2461$
$\cdot84$	$1\cdot5090$	$1\cdot4899$	$1\cdot4708$	$1\cdot4517$	$1\cdot4326$	$1\cdot416$	$1\cdot3994$	$1\cdot3828$	$1\cdot3663$	$1\cdot35225$	$1\cdot3383$	$1\cdot3244$	$1\cdot3105$
$\cdot85$	$1\cdot5885$	$1\cdot5656$	$1\cdot5457$	$1\cdot5258$	$1\cdot5059$	$1\cdot489$	$1\cdot4722$	$1\cdot4535$	$1\cdot4348$	$1\cdot4198$	$1\cdot4048$	$1\cdot38985$	$1\cdot3749$
$\cdot86$	$1\cdot6620$	$1\cdot6413$	$1\cdot6206$	$1\cdot5999$	$1\cdot5792$	$1\cdot5602$	$1\cdot5413$	$1\cdot5224$	$1\cdot5035$	$1\cdot48745$	$1\cdot4714$	$1\cdot45535$	$1\cdot4393$
$\cdot87$	$1\cdot7571$	$1\cdot7369$	$1\cdot7168$	$1\cdot6966$	$1\cdot6765$	$1\cdot5327$	$1\cdot389$	$1\cdot29525$	$1\cdot6015$	$1\cdot5847$	$1\cdot5688$	$1\cdot525$	$1\cdot5512$
$\cdot88$	$1\cdot8523$	$1\cdot8326$	$1\cdot8130$	$1\cdot7934$	$1\cdot7738$	$1\cdot7553$	$1\cdot7369$	$1\cdot71825$	$1\cdot6996$	$1\cdot68295$	$1\cdot6663$	$1\cdot6497$	$1\cdot6331$
$\cdot89$	$1\cdot0947$	$1\cdot2888$	$1\cdot4829$	$1\cdot677$	$1\cdot8711$	$1\cdot8527$	$1\cdot8343$	$1\cdot81595$	$1\cdot7976$	$1\cdot7807$	$1\cdot7638$	$1\cdot7469$	$1\cdot73$
$\cdot9$	$2\cdot0437$	$2\cdot0241$	$2\cdot0056$	$1\cdot987$	$1\cdot9685$	$1\cdot9595$	$1\cdot9321$	$1\cdot91485$	$1\cdot8957$	$1\cdot8785$	$1\cdot8613$	$1\cdot84415$	$1\cdot827$

TABLE V

S_0—(If in doubt take next lower value)

R_0	$W_0 = \cdot 2$	·225	·25	·275	·3	·325	·35	·375	·4	·425	·45	·475	·5	·525
·62	·01413	·01532	·01651	·0177	·01889	·019465	·0204	·021155	·02191	·02242	·02293	·02344	·02395	·02429
·63	·01385	·01503	·01621	·01738	·018557	·01933	·02011	·02088	·02165	·02218	·02271	·02324	·02377	·02413
·64	·0138	·01474	·0159	·01706	·01823	·01902	·01981	·0206	·0214	·02194	·02249	·02309	·02359	·02397
·65	·0133	·01445	·0156	·01674	·01789	·0187	·01952	·02033	·02114	·021705	·02227	·02289	·02341	·023805
·66	·01302	·014155	·01529	·016425	·01756	·01839	·01922	·02005	·02088	·021465	·02205	·022635	·02322	·02364
·67	·01264	·013765	·01489	·016015	·01714	·01798	·01882	·01965	·02049	·02109	·0217	·0223	·0229	·02334
·68	·01227	·01358	·01449	·015605	·01672	·017565	·01841	·019255	·0201	·02072	·02134	·02196	·02258	·023035
·69	·01189	·01299	·01409	·015195	·0163	·01718	·018055	·01888	·01971	·020345	·02098	·02162	·02226	·02273
·7	·01151	·0126	·0137	·01479	·01588	·01679	·0177	·01851	·01932	·01998	·02063	·02128	·02193	·02242
·71	·0108	·012145	·01321	·014275	·01534	·01622	·017095	·01792	·01875	·01931	·019876	·020625	·02138	·02188
·72	·01065	·01169	·01273	·01377	·01481	·01565	·01649	·01733	·01818	·01865	·01913	·02006	·02082	·021335
·73	·01021	·011225	·01224	·013255	·01427	·0151	·01594	·01677	·0176	·018175	·01875	·01951	·02027	·02079
·74	·00978	·010765	·01175	·01274	·01373	·014555	·01538	·016205	·01703	·0177	·01837	·01904	·01971	·02025
·75	·00933	·010285	·01124	·0122	·01316	·013965	·01477	·01557	·01638	·01704	·0177	·018365	·01903	·01957
·76	·00888	·0098	·01073	·01166	·01259	·01337	·01416	·01494	·01572	·01638	·01704	·01769	·01835	·018895
·77	·00842	·00932	·01022	·01112	·01202	·01278	·01354	·0143	·01507	·01572	·01637	·01702	·01767	·01822
·78	·00797	·00884	·00971	·01058	·01145	·01219	·01293	·01367	·01441	·015055	·0157	·016345	·01699	·01754
·79	·00754	·00836	·009185	·01001	·01083	·011545	·01226	·01297	·01368	·01431	·01494	·01556	·016185	·016725
·8	·00711	·00788	·00866	·00944	·01022	·0109	·01159	·01227	·01296	·01356	·01417	·014775	·01538	·01591
·81	·00667	·0074	·00814	·00887	·0096	·010255	·01091	·01157	·01223	·012815	·0134	·01399	·01458	·01509
·82	·00624	·006925	·00761	·008295	·00898	·00961	·01024	·01087	·0115	·01207	·012645	·0132	·01377	·01428
·83	·00579	·00643	·00707	·00771	·00835	·008945	·00954	·010135	·01073	·011265	·0118	·01234	·01288	·013375
·84	·00533	·00593	·00653	·00713	·00773	·00828	·00884	·0094	·00996	·01046	·01097	·01148	·01199	·01247
·85	·004875	·00543	·00599	·006545	·0071	·00762	·00814	·00866	·00918	·00966	·01014	·010615	·01109	·01154
·86	·00442	·00493	·00545	·00596	·00647	·006955	·00744	·007925	·00841	·00886	·00931	·009735	·0102	·01061
·87	·00401	·0045	·00499	·00543	·00588	·00632	·00676	·007205	·00765	·008045	·00844	·008865	·00929	·00967
·88	·0036	·00402	·00444	·00486	·00528	·00568	·00608	·00648	·00689	·00723	·00758	·00798	·00838	·008725
·89	·00319	·00359	·00399	·00434	·00469	·00492	·00515	·005635	·00612	·006445	·00677	·007115	·00746	·00778
·9	·00278	·00311	·00344	·00376	·00409	·00416	·00423	·00479	·00536	·00566	·00596	·00625	·00655	·00684

TABLE V (contd.)

S_o—(If in doubt take next lower value)

R_o	$W_0 =$ .55	.575	.6	.625	.65	.675	.7	.725	.75	.775	.8	.825	.85	.875
·62	·02464	·02498	·02532	·02557	·02582	·02606	·02631	·0265	·02669	·02687	·02706	·0272	·02734	·02748
·63	·02449	·02485	·02522	·02548	·02574	·0265	·02626	·02646	·02666	·02685	·02705	·0272	·02735	·0275
·64	·02435	·02473	·02511	·02539	·02567	·02594	·026215	·02642	·02663	·02683	·02704	·0272	·02736	·02752
·65	·024205	·024605	·02501	·0253	·02559	·02588	·02617	·026385	·0266	·026815	·02703	·0272	·02737	·02754
·66	·02406	·02448	·0249	·025205	·02551	·025815	·02612	·026345	·02657	·026795	·02702	·027195	·02738	·02756
·67	·02378	·02421	·02465	·02497	·02529	·02561	·02593	·02617	·02641	·02664	·02688	·02719	·02751	·02783
·68	·02349	·02399	·0244	·02473	·02507	·0254	·02574	·02599	·02624	·02649	·02674	·027165	·02765	·0281
·69	·0232	·02367	·02414	·02449	·024845	·02519	·02554	·025805	·02607	·026335	·0266	·027185	·02773	·02834
·7	·02291	·0234	·02389	·024255	·02462	·024985	·02535	·025625	·0259	·02618	·02646	·02668	·02791	·028635
·71	·02238	·02288	·023385	·023765	·024145	·02452	·0249	·02519	·025485	·02578	·02607	·02617	·027285	·027895
·72	·02185	·022365	·02288	·023275	·02367	·02406	·02446	·02476	·02507	·02537	·02568	·02566	·02666	·027155
·73	·02132	·02185	·02238	·02278	·02319	·0236	·02401	·02433	·02465	·024965	·02528	·025155	·02604	·02642
·74	·02079	·02133	·02187	·02229	·02272	·02314	·02356	·02389	·02423	·02456	·02489	·02458	·02542	·025685
·75	·020115	·02066	·0212	·02163	·02206	·022495	·02293	·0233	·02367	·02398	·0243	·02401	·02486	·02514
·76	·01944	·019985	·02053	·02097	·02141	·02185	·0223	·02265	·023005	·02336	·02371	·023435	·02431	·0246
·77	·018765	·01931	·01986	·02031	·02076	·02121	·02167	·02203	·02239	·022755	·02312	·02286	·02375	·02406
·78	·01809	·01864	·01919	·01965	·0201	·02057	·02103	·021405	·02178	·022155	·02253	·022025	·02319	·02352
·79	·017265	·017805	·018345	·0188	·01926	·019715	·02017	·02055	·02093	·02131	·02169	·02119	·02236	·022695
·8	·01644	·01697	·0175	·01795	·01841	·01886	·01931	·019695	·02008	·02046	·02085	·02035	·02153	·02187
·81	·01561	·01615	·01669	·01712	·01755	·01825	·01895	·01909	·01923	·019615	·020	·01951	·0207	·02104
·82	·01479	·01529	·0158	·01625	·0167	·01714	·01759	·01798	·01838	·01877	·01916	·01842	·01986	·02021
·83	·01387	·01434	·01481	·01524	·01567	·0161	·016535	·01692	·017305	·01769	·018075	·017325	·01876	·019105
·84	·01295	·01339	·01383	·01424	·01465	·015065	·01548	·01586	·01624	·01661	·01699	·01623	·01766	·018
·85	·01199	·012415	·01284	·013235	·01363	·01403	·01425	·014795	·015165	·015535	·015905	·01514	·01656	·01689
·86	·01103	·01144	·01185	·01223	·01261	·01299	·01337	·01373	·0141	·01446	·01482	·01389	·015465	·01579
·87	·01005	·01043	·01081	·011165	·01152	·011875	·01223	·01257	·01291	·013247	·01359	·012635	·01419	·014495
·88	·009075	·009425	·00978	·010105	·010435	·010765	·0111	·01141	·011723	·012036	·01235	·011355	·01292	·0132
·89	·0081	·00842	·00874	·009045	·00935	·009655	·00996	·01025	·010541	·0108	·011065	·010125	·011645	·01191
·9	·00713	·00741	·00770	·00798	·00826	·00854	·00882	·009085	·00935	·009615	·00988		·01037	·010815

TABLE V (contd.)

S_c.—(If in doubt take next lower value)

R_0	·9	·925	·95	·975	1·0	1·25	1·5	1·75	1·1	1·125	1·15	1·175	1·2
·62	·02762	·027235	·02685	·027465	·02808	·028175	·02827	·028365	·02846	·02854	·02862	·0287	·02878
·63	·02765	·02752	·027395	·02777	·02814	·02824	·02834	·02844	·02854	·02862	·0287	·02878	·02886
·64	·02768	·02781	·02794	·02807	·0282	·0283	·02841	·02851	·02861	·028695	·02878	·028865	·02895
·65	·02771	·027845	·02798	·028115	·02825	·02836	·02847	·02858	·028685	·02877	·02886	·02895	·02904
·66	·02774	·02788	·02803	·02817	·02831	·02842	·02854	·02865	·02876	·02885	·02895	·02904	·02913
·67	·028145	·02817	·0282	·02823	·02826	·02837	·02849	·02861	·02873	·02883	·028915	·02903	·02913
·68	·02855	·02846	·028375	·02829	·0282	·028325	·02845	·028545	·02871	·02881	·0289	·02902	·029122
·69	·028955	·02875	·02855	·02835	·02815	·02828	·02841	·02851	·02868	·02879	·02889	·02901	·02912
·7	·02936	·02904	·02873	·02841	·02809	·02823	·02837		·02865	·02877		·029	
·71	·028505	·02832	·02814	·027955	·02777	·02792	·02807	·02822	·02837	·028495	·02862	·02875	·028875
·72	·02765	·0276	·02755	·0275	·02746	·02761	·02777	·02793	·02809	·028225	·02836	·028495	·02863
·73	·0268	·026885	·02697	·027055	·02714	·027305	·02747	·02764	·02781	·027955	·0281	·02824	·028385
·74	·02595	·02617	·02639	·0266	·02682	·027	·027178	·02735	·02753	·02768	·02784	·02799	·02814
·75	·025425	·02565	·025875	·0261	·02633	·02653	·02673	·026895	·02706	·02722	·02738	·02754	·0277
·76	·0249	·02513	·02537	·0256	·02583	·02611	·02639	·02649	·026595	·02676	·02693	·0271	·02727
·77	·024375	·024615	·024855	·025095	·02534	·02558	·02582	·025975	·02613	·026065	·026	·026415	·02683
·78	·02385	·0241	·024345	·02459	·02484	·025045	·02525	·025455	·02566	·02584	·026025	·02621	·02639
·79	·02303	·023285	·02354	·0238	·02406	·02427	·02449	·024705	·02492	·02511	·0253	·02549	·02568
·8	·02221	·022475	·02274	·023005	·02327	·0235	·02373	·02395	·02418	·02438	·02458	·02478	·02498
·81	·021385	·02166	·021935	·02221	·022485	·02273	·02296	·0232	·02344	·02365	·023855	·02406	·02427
·82	·02056	·020845	·02113	·021415	·0217	·02196	·0222	·02245	·0227	·022915	·02313	·023345	·02356
·83	·01945	·019725	·02	·02028	·02056	·02082	·02108	·021345	·02161	·02183	·02205	·02227	·022485
·84	·01834	·0186	·01887	·01914	·01941	·01969	·01997	·02024	·02052	·02074	·02097	·02119	·02141
·85	·01722	·01748	·01774	·018	·01827	·01876	·01885	·01914	·01943	·019655	·01988	·02011	·020335
·86	·01611	·01636	·01662	·01687	·01712	·017425	·01773	·018035	·01834	·01857	·0188	·01903	·01926
·87	·0148	·01493	·01506	·01542	·015785	·01607	·016355	·01664	·016925	·01715	·017375	·0176	·01783
·88	·01349	·01373	·01397	·01421	·01445	·01471	·01498	·015245	·01551	·01573	·01595	·01617	·016395
·89	·01217	·01241	·01265	·012885	·01312	·013365	·01361	·01385	·014095	·01431	·01453	·014745	·01496
·9	·01086	·01109	·01133	·01156	·01179	·01201	·01224	·01246	·01268	·01289	·01311	·01332	·01353

TABLE VI
STANDARD SIZES OF WIRE (S.W G.)

Diameter in Inches	Diameter in Millimetres	Area in Square Inches	Area in Square Millimetres	S.W G	Weight in Kilograms per Metre	Resistance per Metre	Resistance per Lb. of Wire
·021	·534	·000344	·222		·001965	·0765	17·650
·022	·559	·000377	·245	24	·00217	·0694	14·500
·023	·584	·000414	·267		·00236	·0636	12·200
·024	·610	·000450	·292	23	·00258	·0583	10·250
·026	·661	·000528	·341		·00302	·0498	7·530
·028	·711	·000605	·397	22	·00352	·0428	5·540
·030	·763	·000706	·456		·00404	·0372	4·210
·032	·818	·000800	·519	21	·00459	·0328	3·254
·034	·863	·000904	·584		·00516	·0291	2·560
·036	·914	·001015	·657	20	·00582	·0258	2·012
·038	·967	·00113	·729		·00646	·0233	1·650
040	1·02	00126	·811	19	·00718	·02096	1 3
·044	1·118	00152	·980	18½	·00868	·01735	·910
·048	1·22	·00181	1·17	18	·01035	0145	·650
·050	1 27	·00197	1 27		·01124	·0134	·540
·052	1·322	·00212	1·37	17½	·01212	·0214	·467
·054	1·372	·00228	1·47		·013	·01155	·405
·056	1·422	·00246	1·59	17	·01408	0107	·348
·058	1·473	·00264	1·702		01509	·00997	·300
·062	1 575	·00302	1·947		01723	·00875	·230
·064	1·625	·00322	2 08	16	0184	·00816	·200
·068	1·728	00362	2·34	15½	0207	·00726	·159
·072	1·83	·00407	2·63	15	·0233	·00646	·126
·076	1 93	·00453	2·93	14½	·0259	·0058	·102
·080	2·03	·00503	3·24	14	·02869	·00525	·0800
·083	2·11	·00543	3·52		·0312	·00483	·0703
·088	2·23	·00608	3·92		·0347	·00434	·0568
·092	2·34	·00664	4·28	13	·038	·00398	·0476
·095	2·41	·00707	4·56		·0404	·00373	·0419
·098	2·49	·00754	4·86	12½	·043	·0035	·0369
·104	2·64	·0085	5·48	12	·0486	·0031	·0289
·109	2·77	·0086	5·55		·0492	·00306	·0282
·116	2·95	·01053	6·82	11	·0604	·0025	·0188
·128	3·26	·0131	8·3	10	·0734	·00205	·0127
·144	3·66	·0164	10·5	9	·093	·001615	·00793

TABLE VII

BROWN AND SHARPE WIRE GAUGE

Diameters				Area of Copper		Per 1,000 ft	
	Insulated						
Bare	Single Cotton Covered	Double Cotton Covered	Triple Cotton Covered	Circular Mils	Square Inches	International Ohms at 75° F.	Weight in Lbs. Bare
·460		·478	·487	211600	·16619	·04965	640·51
·410		·428	·437	167772	·13176	·06263	507·84
365		·383	·392	133079	·10452	·07895	402·83
·325		·343	·352	105560	·08290	·09954	319·53
·289		·307	·316	83695	·06572	·1255	253·34
·258		·276	·285	6357	·05122	·1583	200·86
·229		·247	·256	52624	·04131	·1996	159 29
·204		·222	·231	41739	·03276	·2517	126·34
·182		·200	·209	33087	·02598	·3175	100·15
162		·178	·186	26244	·02061	·4005	79·44
·144		·160	·168	20822	·01635	·5046	63·03
·129		·143	·150	16512	·01297	·6363	49·98
·114		·126	·132	13087	·01011	·8039	39 61
·102	·107	·112	·117	10384	·00815	1·0119	31 43
·0907	·096	·101	·106	8227	·00646	1·2772	24·90
·0808	·086	·091	·096	6529	·00512	1·6093	19 76
·0720	·077	·081	·086	5184	·00407	2·0268	15·69
·0641	·069	·073	·078	4109	·00322	2·5572	12·44
·0571	·062	·066	·071	3260	·00256	3·2231	9 868
·0508	·056	·060	·065	2581	·00202	4 0711	7 813
0453	·050	·054	·059	2052	·00161	5·1207	6·211
·0403	·045	·049	·054	1624	·00127	6·4702	4 916
·0359	·041	·045	·050	1285	·00101	8·1773	3·889
·0320	·037	·041	·045	1024	·00080	10 261	3 099
·0285	·0335	·0375	·0425	812	·00063	12·940	2·458
·0254	·030	·034	·039	640	·00050	16·418	1·937
·0226	·0275	·0315	·0365	510	·00040	20·603	1·543
·0201	·025	·029	·034	404	·00032	26·009	1·223
·0179	·023	·027	·032	320	·00025	32·837	·968
·0159	·021	·025	·030	253	·00019	41·532	·766
·0142	·019	·023	·028	201	·00015	52·282	·608
·0126	·0175	·0215	0265	159	·00012	66·080	·481
·0113	·016	·020	·025	127	·00010	82·738	·384
·0100	·015	·019	·024	100	·00007	105·08	·303
·0089	·014	·018	·023	79·2	·00006	132·67	·269
·0080	·013	·017	·022	64	·00005	164·18	·194
·0071				50·4	·00004	208·48	·152
·0063				39·7	·00003	264·68	·120
·0056				31·4	·00002	334·66	·095

Specific gravity = 8·89.   Resistance Mil-foot soft copper = 9·59 ohms at 0°C.

TABLE VIII

Puls Const I_3	Puls Sq. In. J_3	From Calculation		From Calculation	
		Avg. Dens. C_4	T. Loss Const. B_3	Max. Core Dens. T	Core Loss Const F_3
0	·7	·04	·7	·02	2
5	·7	·045	·85	·03	3
10	·72	·05	1·05	·04	·4
15	·75	·06	1·3	·05	·55
20	·8	·065	1·625	·06	·7
25	·9	·070	2·05	·07	·85
30	1·0	·075	2·6	·08	1·05
35	1·15			·09	1·3
40	1·25			·10	1 625
45	1·4			·11	2·05
50	1·6			·12	2·75
55	1·8				
60	2·2				

N B When I_3 comes *between* values given, take the higher one.
Where C_4 or T comes *between* values given, take the higher one.

TABLE IX
POWER FACTOR TABLES
$$L_1$$
Overload Capacity

$D\sigma$	1·5	2·0	2·5	3·0	3·5	4·0
·03	·9034	·9311	·9393	·9408	·9401	·9365
·04	·8920	·9172	·9224	·9205	·9158	·9214
·0425						
·045					·9024	·8993
·0475						·8882
·05	·8801	·9021	·9040	·8979	·8891	·8772
·0525		·8981				
·055		·8941			·8746	·8604
·0575		·8901			·8673	
·06	·8676	·8861	·8840	·7806	·8601	·8436
·065		·8776			·8499	
·0675		·8734			·8368	
·07	·8546	·8692	·8628	·8479	·8297	·8082
·075		·8603				
·08	·8413	·8515	·8405	·8211	·7981	·7724
·0825						
·085		·8424				
·0875		·8378				
·09	·8275	·8333	·8176	·7936	·7663	·7367
·095		·8239	·8059			
·10	·8134	·8145	·7942	·7675	·7346	·7016
·1025		·8092				
·105		·8049				
·11	·7989	·7953	·7705	·7380	·7032	·6675
·12	·7844	·7759	·7465	·7103	·6725	·7068
·13	·7693	·7561	·7226	·6830	·6362	·6036
·14	·7544	·7363	·6989	·6564	·6140	·5733
·15	·7391	·7164	·6755	·6304	·5864	·5450
·16	·7238	·6966	·6524	·6052	·5601	·5182
·17	·7085	·6768	·6298	·5809	·5349	·4929
·18	·6930	·6575	·6079	·5573	·5110	·4691
·19	·6776	·6458	·5862	·5113	·4881	·4467
·20	·6622	·6192	·5657	·5132	·4666	·4257
·21	·6468	·6086	·5451	·4925	·4461	·4057
·22	·6315	·5824	·5254	·4726	·4266	·3869
·23	·6163	·5644	·5065	·4536	·4081	·3693
·24	·6012	·5469	·4881	·4355	·3907	·3527
·25	·5863	·5298	·4705	·4182	·3741	·3371

TABLE IX (*contd.*)

POWER FACTOR TABLES

L_1

Overload Capacity

$D\sigma$	4·5	5·0	5·5	6·0	6·5	7·0
·03	·9316	·9254	·9181	·9099	·9018	·8928
·04	·8991	·8883	·8762	·8638	·8503	·8366
·0425	·8875					
·045	·876					
·0475	·8694					
·05	·8629	·8473	·8306	·8135	·7956	·7777
·0525		·8365				
·055		·8258		·7779		
·0575		·815				
·06	·8244	·8043	·7833	·7622	·7407	·7195
·065						
·0675						
·07	·7848	·7607	·7361	·7118	·7287	·6643
·075						
·08	·⁻453	·7177	·6903	·6637	·6378	·6133
·0825		·7073				
·085		·6969				
·0875						
·09	·7063	·6761	6467	·6185	·5916	·5661
·095						
·10	·6689	·6365	·6057	·5766	5492	·5258
·1025						
·105						
·11	·6325	·5991	·5675	·5380	·5106	·4853
·12	·5983	·5640	·5805	·5027	·4755	·4507
·13	·5659	·5312	·4993	·4703	·4437	·4196
·14	·5355	·5008	·4692	·4407	·4386	·3915
·15	·5069	·4725	·4414	·4136	·3886	·3661
·16	·4802	·4462	·4159	·3888	·3647	·3431
·17	·4553	·4219	·3923	·3661	·3429	·3221
·18	·4320	·3993	·3706	·3453	·3229	·3030
·19	·4102	·3784	·3505	·3243	·3047	·2856
·20	·3898	·3589	·3320	·3084	·2876	·2696
·21	·3708	·3408	·3148	·2877	·2724	·2549
·22	·3530	·3239	·2988	·2731	·2581	·2414
·23	·3365	·3081	·2839	·2576	·2448	·2289
·24	·3206	·2934	·2701	·2500	·2326	·2173
·25	·3058	·2796	·2372	·2378	·2211	·2065

TABLE X

Density	A.T.
·02	4
·03	6
·04	8
·05	10
·06	12
·07	13
·08	14
·09	18
·1	26
·11	60
·115	120
·12	200

TABLE XI

RATIO OF AREA OF ROTOR SLOT p TIMES NORMAL DEPTH TO NORMAL AREA OF SLOT

p	D_6 Parallel Slots $p(2-p)$	D_6 Taper Slots $\dfrac{p(4-p)}{I}$
·25	·4375	·3125
·3	·51	·37
·35	·5775	·4258
·4	·64	·48
·45	·6975	·5325
·5	·75	·5833
·55	·7975	·6325
·6	·84	·68
·65	·8775	·737
·7	·91	·77
·75	·9375	·8342
·8	·96	·8533
·85	·9795	·8925
·9	·99	·93
·95	·9975	·9658
1·00	1·00	1·00

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